# Assessment of Voltage Stability of Electrical Power Systems: A Simulational Survey

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### **Abstract**

In this paper, we analyze important aspects related to voltage stability indices in electrical power systems, which allows operators to determine the weakest bus of a system as well as its critical line settings of each bus. Conventional techniques are analyzed and a comparison of the performance of several indices is presented. The feasibility of the reported methods is shown by numerical studies in IEEE 30 busbar test system.

#### 1. Introduction

Voltage instability is a critical problem that can lead to a total system collapse.

Voltage stability refers to the ability of a power system to maintain steady voltages at all nodes of the system after being subjected to a disturbance from a given initial operating condition [1].

A system is voltage unstable if, for at least one bus in the system, bus voltage magnitude decreases as the reactive power injection at the same bus is increased.

One causes of the occurrence of voltage collapse is due to the excess of power transferred through a line or the excessive absorption of power by the line.

The study of voltage stability has been analyzed under different approaches that can be basically classified into dynamic and static analysis.

Dynamic analysis uses time-domain simulations to solve nonlinear system differential and algebraic equations, which include generators dynamics, tap changing transformers, etc.

Static analysis is based on the solution of conventional or modified powerflow equations. Static analysis is ideal for the bulk of studies in which voltage stability limits for many precontingency and post-contingency cases must be determined [2,3].

There are different methods used to study the voltage collapse phenomenon.

In this paper we used voltage stability indices for identification of the critical line referred to a bus and to reveal the weakest bus of the IEEE 30 busbar test system.

We verify that voltage stability indices can provide accurate information about the stability condition of a system.

#### 2. Indices Formulation

## 2.1. P-V and Q-V Curves

The P-V curves are the most used method of predicting voltage security. They are used to determine the loading margin of a power system. The power system load is gradually increased and, at each increment, is necessary recomputed power flows until the nose of the PV curve is reached. The margin between the voltage collapse point and the current operating point is used as voltage stability criterion.

With Q-V curve is possible, for the operators, to know which is the maximum reactive power that can be achieved or added to the weakest bus before reaching minimum voltage limit. Such curves indicate the sensitivity and variation of bus voltages with respect to reactive power injections.

The reactive power margin is the MVar distance from the operating point to the bottom of the Q-V curve. The Q-V curve can be used as an index for voltage instability. The point where dQ/dV is zero is the point of voltage stability limit.

## 2.2. Local load margin index

The local load margin index ( $P_{Lmg}$ ) is based on the distance from the base case ( $P_{0i}$ , MW) to the point of voltage collapse ( $P_{CRi}$ , MW):

$$P_{Lmg} = \frac{P_{CRi} - P_{0i}}{P_{CRi}} \tag{1}$$

where  $P_{0i}$  is the active power at the base case of bus i and  $P_{CRi}$  is the maximum power transmitted in the node i.

The equation 1 indicates the local load margin for the PQ busbar. The local load margin index,  $P_{Lmg}$ , presents a value between 0 (voltage collapse) and 1 (no load).

## 2.3. V/V0 Index

Assuming the bus voltage values (V) to be known from load flow or state estimation studies, new bus voltages (V0) are obtained solving a load flow for the system at an identical state

but with all loads set to zero. The ratio V/Vo at each node yields a voltage stability map of the system, allowing for immediate detection of weak and effective countermeasure spots [4].

## 2.4. Modal Analysis

Gao et al. [5] proposed a method that computes the smallest eigenvalue and associated eigenvectors of the reduced Jacobian matrix of the power system based on the steady state system model. The eigenvalues are associated with a mode of voltage and reactive power variation. If all the eigenvalues are positive, the system is considered to be voltage stable. If one of the eigenvalues is negative, the system is considered to be voltage unstable. A zero eigenvalue of the reduced Jacobian matrix means that the system is on the border of voltage instability. The potential voltage collapse situation of a stable system can be predicted through the evaluation of the minimum positive eigenvalues. The magnitude of each minimum eigenvalue provides a measure to know how close the system is to voltage collapse.

By using the bus participation factor, the weakest bus can be determined, which is the greatest contributing factor for a system to reach voltage collapse situation.

## 2.5. Line stability Index L<sub>mn</sub>

M. Moghavemmi *et al.* [6] established a criterion of stability which shows the proximity to voltage collapse of each line of a network. The  $L_{mn}$  index can have a maximum value of 1 if the system is about to suffer a voltage collapse and a minimum value of 0 when there is no load on the system.

$$L_{mn} = \frac{4XQ_{j}}{\left[V_{i}sen(\theta - \delta)\right]^{2}}$$
 (2)

## 2.6. Line Stability Index FVSI

The line stability index  $FVSI_{ij}$  proposed by I. Musirin *et al.* [7] is based on a concept of power flow through a single line.

For a typical transmission line, the stability index is calculated by:

$$FVSI_{ij} = \frac{4Z^2Q_j}{V_i^2X}$$
 (3)

The line that gives index value closest to 1 will be the most critical line of the bus and may lead to the whole system instability.

The calculated FVSI can also be used to determine the weakest bus on the system. The determination of the weakest bus is based on the maximum load allowed on a load bus. The most vulnerable bus in the system corresponds to the bus with the smallest maximum permissible load.

## 2.7. Line Stability Index LQP

The LQP index derived by A. Mohamed et al. [8] is obtained

when the discriminant of the power quadratic equation is set to be greater or equal than zero.

The LQP is obtained as follows:

$$LQP = 4 \left( \frac{X}{V_{i}^{2}} \right) \left( \frac{X}{V_{i}^{2}} P_{i}^{2} + Q_{j} \right)$$
 (4)

To maintain a secure condition, the value of LQP index should be maintained less than 1.

# 2.8. Line Stability Indices VCPI

The VCPI indices proposed by M. Moghavvemi *et al.* [9] investigates the stability of each line of the system and they are based on the concept of maximum power transferred through a line.

The stability indices of the line i-j are defined as:

$$VCPI(Power) = \frac{P_R}{P_{R(max)}}$$
 (5)

$$VCPI(Losses) = \frac{P_{Losses}}{P_{Losses(max)}}$$
(6)

With the increasing power flow transferred by transmission lines, the values of VCPI (Power) and VCPI (losses) increase gradually, and when they reach 1, the voltage collapse occurs. So, if any line of network reach that value, it is possible to predict the voltage collapse. Therefore, the VCPI indices varies from 0 (no load condition) to 1 (voltage collapse).

## 3. Test Results and Discussion

In order to verify the effectiveness of the voltage stability indices, numerical studies have been made in 30 IEEE busbar test system.

The IEEE 30 busbar test system has 6 generator busbars, 24 load busbars and 41 interconnected branches.

The voltage stability margin was determined with P-V curves and each point of these curves was obtained from load flow solution, using the conventional Newton-Raphson method.

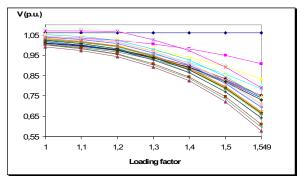


Fig. 1 P-V curves for IEEE 30 busbar test system

Figure 1 illustrates that when load increase, the voltage at all busbar decreased and it was observed that bus 30 had the minimum voltage. Therefore, bus 30 is more disposed to voltage collapse.

The voltage stability margin of the IEEE 30 busbar test system is approximated 55%.

Q-V curves can give the reactive power load margin at a bus from a stable operating point to the point of voltage instability. However, such curves give no insight into the causes of the voltage stability, such as participating nodes and branches.

The Q-V curve at a test bus is generated by placing a variable reactive power source with infinite limits at the bus.

Successive power flows are performed for different scheduled values of bus voltage, and the required reactive power injection is measured.

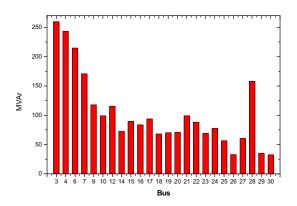


Fig. 2 Margin of reactive power for IEEE 30 busbar test system

Figure 2 shows that bus 30 has the lowest margin of reactive power, so bus 30 is the critical bus of the IEEE 30 busbar test system.

Figure 3 presents the values of the local load margins for PQ busbar of the IEEE 30 test system. To calculate this index, it was necessary draw P-V curves for each of the PQ busbars of the system.

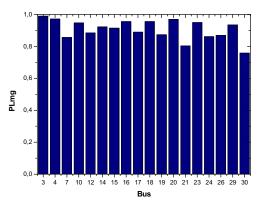


Fig. 3 Local load margin index for IEEE 30 busbar test system

Observing Figure 3, it was concluded that the bus that has the lowest local load margin, ie, the bus that has the highest

probability to present problems if subjected to a disturbance is the bus 30.

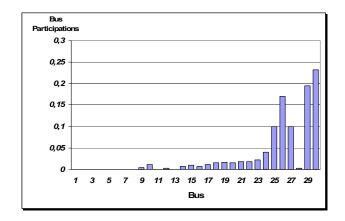
The modal analysis was applicated to IEEE 30 busbar test system for three different operating conditions: the base case, the case of loading factor 1,3 and to the critical operating case (loading factor 1,549).

All the eigenvalues of reduced reactive Jacobian matrix are positive, indicating that the system is voltage stable at all tested load conditions. The magnitudes of the eigenvalues decrease as the system approaches to instability.

At the critical operating point, the smallest eigenvalue (least stable mode) was calculated and it was used to determine the bus participation factors.

Figure 4 shows the bus participation factors in the least stable mode for the critical operating point. As we can see, the bus that presents the highest participation factor is bus 30. So this is the bus that contributes more to voltage collapse.

Through the application of modal analysis, is possible the identification of the best places to proceeding to the installation of static compensators in order to improve the voltage margins of a system.



**Fig. 4** Bus participation factos in the least stable mode for critical operating case for IEEE 30 busbar test system

In order to demonstrate the usefulness of V/Vo index, three separate test cases were used: for a condition near the base case (Situation A), for a case near the critical point (Situation B) and for the critical operating case (Situation C).

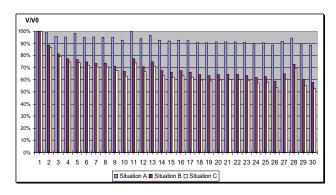


Fig. 5 V/V0 index for IEEE 30 busbar test system

Observing figure 5, we notice that as we are approaching to the voltage stability limit, the voltage at the busbar present values very far away of the ones obtained with the system with all loads a zero (V0), in opposition to situation A.

Analyzing V/V0 index values we concluded that the critical bus is bus 30 because it presents the smallest index value.

To illustrate the application of some line stability indices, we gradually increased the reactive load, only in one bus of the IEEE 30 busbar test system at a time, from the base case until its maximum allowable load, keeping the load at the other busbars fixed at base load.

Figure 6 illustrates the response of FVSI index with the reactive load variation.

The individual FVSI curves present in Figure 6 are the most critical lines referred to a bus.

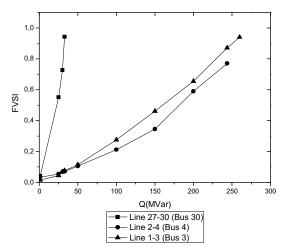


Fig. 6 FVSI vs. reactive load variation for IEEE 30 busbar test system

Table I shows the stressed conditions of the lines of IEEE 30 busbar system for the maximum loadability of the busbar.

It is observed that line stability indices have the highest value at the maximum loadability of each load bus and when they are close to one the system reached its stability limit.

Table I - Line Stability Indices for IEEE 30 Busbar Test System						
Load (p.u.)	Line	L <sub>mn</sub>	FVSI	LQP	VCPI(P)	VCPI(L)
Q <sub>3</sub> =2,599	1-3	0,98	0,94	0,96	0,99	0,78
	4-3	0,42	0,44	0,4	0,48	0,07
Q <sub>4</sub> =2,437	2-4	0,78	0,77	0,75	0,82	0,33
	3-4	0,36	0,36	0,34	0,41	0,05
	4-6	0,25	0,27	0,26	0,34	0,03
	4-12	0,21	0,22	0,48	0,63	0,51
Q <sub>30</sub> =0,327	27-30	0,897	0,94	0,78	0,88	0,42
	29-30	0,65	0,68	0,54	0,62	0,15

Table I shows that the performance of the line stability indices studied has high degree of accuracy, reliability and the

results are very closed in agreement.

From Table I we concluded that the line that connect bus 1 to bus 3 is the most critical line referred to bus 3 because presents the largest index value for the maximum loadability of the bus, the line 2-4 is the most critical line with respect to bus 4 and line 27-30 is the most critical line of bus 30.

Line stability indices can also determine the weakest bus in the system and it is based on the maximum permissible load.

In Figure 7, buses 3, 4 and 30 indicate 2,599 p.u., 2,437 p.u. and 0,327 p.u. as the maximum permissible reactive load, respectively.

Since the most vulnerable bus in the system corresponds to the bus with the smallest maximum permissible load, bus 30 is the critical bus of IEEE 30 busbar test system.

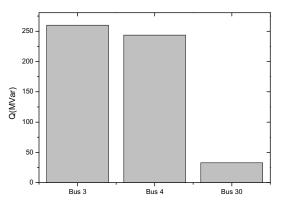


Fig.7 Maximum permissible reactive load on IEEE 30 Busbar Test System

#### 4. Conclusions

In this paper we described and implemented voltage stability techniques that can predict how far is the point of operation to the limit of the voltage instability, identifying the weakest busbar and the most critical line of a system. The results obtained by simulations were mutually consistent and provided excellent indications of which were the critical areas of IEEE 30 busbar tested system.

# 5. References

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