SLIDING MODE CONTROL OF AUTONOMOUS UNDERWATER VEHICLE

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Abstract

In this study, an autonomous underwater vehicle (AUV) model with six degrees of freedom is presented to be shown having been linearized under several working conditions. Sliding Mode Control Law which is derived from linear theory is applied autonomous underwater vehicle for yaw steering plane. Simulation study is given to show that sliding mode controller designed assuming small states variation and decoupled plane cope with modeling non-linearity, uncertainty, disturbance effect.

1. Introduction

Autonomous Underwater Vehicles (AUVs) are unmanned robotic platforms that can be preprogrammed to achieve accurate navigation, control, and guidance tasks. Nowadays, AUV(Autonomous Underwater Vehicle) are widely used military, commercial and scientific purpose. These:

- Commercial: The oil and gas industry uses AUVs to make detailed maps of the seafloor before they start building subsea infrastructure; pipelines and sub sea completions can be installed in the most cost effective manner with minimum disruption to the environment.
- Military: A typical military mission for an AUV is to map an area to determine if there are any mines, or to monitor a protected area (such as a harbor) for new unidentified objects. AUVs are also employed in anti-submarine warfare, to aid in the detection of manned submarines.
- Scientific: Scientists use AUVs to study lakes, the ocean, and the ocean floor. A variety of sensors can be affixed to AUVs to measure the concentration of various elements or compounds, the absorption or reflection of light, and the presence of microscopic life.

The dynamics AUV is very complex, highly nonlinear and depend on several parameters like added-mass, hydrodynamics damping coefficients. [1]. For this reason, The designed controller must be robust against the uncertainties and environmental distributions The variable structure control theory has provided effective means to design robust state feedback controllers for uncertain dynamic systems. [2] The central feature of the variable structure system is the so-called sliding mode on the switching surface within which the system remains insensitive to internal parameter variations and extraneous disturbance satisfying the so-called matching condition.

In this article, we were linearized AUV dynamics at different operation point after we summarized AUV dynamics. The sliding mode approach used here appears to offer simplicity for vehicle in floating conditions and to be robust against uncertainty and disturbances.

2. AUV Dynamics

An AUV 6-DOF (Degree of Freedom) non-linear dynamics model is given as follows (Fossen [3]) '

$$\mathbf{M}\dot{\boldsymbol{v}} + \mathbf{C}(\boldsymbol{v})\boldsymbol{v} + \mathbf{D}(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{G}(\boldsymbol{\eta}) = \boldsymbol{\tau} \quad (1)$$

where $\mathbf{M}$ is the inertia and added inertia matrix, $\mathbf{C}$ is the matrix of Coriolis and Centrifugal terms, $\mathbf{D}$ is the matrix of hydrodynamics terms, $\boldsymbol{G}$ is the vector of gravity and buoyant forces, $\boldsymbol{\tau}$ is control-input vector describing forces and moment efforts with moveable parts of AUV according to body frame

$$\boldsymbol{\tau} = \left[f(\delta_r)\, f(\delta_s)\, f(n) \right]^T \quad (2)$$

where δ_s is the fin angle of AUV , δ_r is the rudder angle of AUV, n is the revolution of propeller as thrusters of AUV.

$\boldsymbol{v}$ is the vector of linear and angular velocity of the AUV in the body-fixed coordinate frame, $\boldsymbol{\eta}$ is the vector of position and attitude of the AUV in the earth fixed coordinate frame as follows

$$\boldsymbol{v} = [u\ v\ w\ p\ q\ r]^T \quad (3)$$

$$\boldsymbol{\eta} = [X_e\ Y_e\ Z_e\ \varphi\ \theta\ \psi]^T \quad (4)$$

Transformation matrix from body-fixed to earth fixed coordinate frame is

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} \mathbf{J}_1(\boldsymbol{\eta}) & 0 \\ 0 & \mathbf{J}_2(\boldsymbol{\eta}) \end{bmatrix} \boldsymbol{v}$$

$$\mathbf{J}_1(\boldsymbol{\eta}) = \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi & \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{bmatrix} \quad (5)$$

$$\mathbf{J}_2(\boldsymbol{\eta}) = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\varphi & \sin\varphi\cos\theta \\ 0 & -\sin\psi & \cos\varphi\cos\theta \end{bmatrix}$$

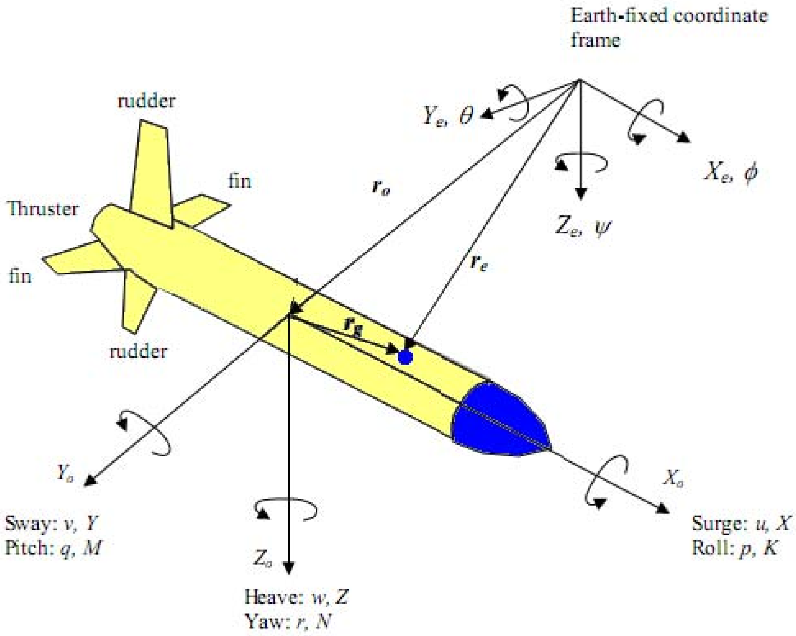

Fig. 1. AUV in body fixed frame and Earth fixed frame

In this article, NPS AUV II model formed by Healey and Lineard is used [2]. Only rear fin, rudder, propeller revolution of NPS AUV II model have been used to control the system. Dynamic equation of NPS AUV II which include hydrostatic and hydrodynamic forces and moments have been modeled by using Euler Integration Method on MATLAB S-Function with programming language "C". Hence, this model may be run in the real time for an embedded system on DSP or microcontroller.

3. Linearizing AUV Model

The use of a linear model to design a control law has the advantage to be easier than a nonlinear model. By assuming to move along on the x axis with a constant speed throughout body axis, the linearization of AUV model on the different body plane can be obtained by taking account of a little variation of state vector. The main point of linearizing AUV with constant speed is to neglect the state vector components which effects a little on desired linearizing plane. The steps of linearizing an autonomous underwater vehicle on diving plane and also steering plane have been studied on numerous resources like Fossen [4], Healey [2].

The control law that is derived by the assumption of AUV is moving along on the body axis with a constant speed, restricts AUV's maneuverability. For instance an AUV should move on diving plane at same time on yaw steering plane. For this purpose, maneuverability should strengthen to add coupled effect between controllers to be designed.

Firstly, for linearizing AUV model on two operation speeds, we calculate control input values which hold system on the equilibrium point. We have two different linear NPS AUV II model obtained from the operation point of 0.75 m/sn and 1.5 m/sn. At the equilibrium point, the derivatives of some state vector such as $\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}, \dot{\phi}, \dot{\theta}, \dot{\psi}$ given in **Fig. 1.** must be '0'. It is a numerical optimization problem to calculate control input vector which guarantee AUV to stay at equilibrium point with speeds of $u = 0.75m/sn, u = 1.5m/sn$ and desired state vector variation to be '0'. When this optimization problem is solved by "trim" command in MATLAB with initial condition of, control input vector given as:

$$u_{trim\_0.75m/sn} = [0\ rad \quad 0\ rad \quad 596\ rpm]$$
$$u_{trim\_1.5m/sn} = [0\ rad \quad 0\ rad \quad 1193\ rpm] \quad (6)$$

In this article we will give linear model for yaw steering plane of AUV. One can realize from **Fig. 1** that some state vectors related the diving steering plane are considered as null for yaw steering plane,

$w = p = q = \varphi = \theta = 0$

By using (5)

$r = \dot{\psi}$

Linearized NPS AUV II model (Healey [2]) has been formed at yaw steering plane as follows

$$\begin{aligned}
&(m - \bar{Y}_{\dot{v}}).\dot{v}(t) + (m.x_G - \bar{Y}_{\dot{r}}).\dot{r}(t) = \\
&\bar{Y}_v.u_0.v(t) + (\bar{Y}_r - m).u_0.r(t) + \bar{Y}_{dr}.\delta_r(t), \\
&(m.x_G - \bar{N}_{\dot{v}}).\dot{v}(t) + (I_z - \bar{N}_{\dot{r}}).\dot{r}(t) = \\
&\bar{N}_v.u_0.v + (\bar{N}_r).u_0.r(t) + \bar{N}_{dr}.\delta_r(t), \\
&\dot{\psi}(t) = r(t)
\end{aligned} \tag{7}$$

We take constant speed $u = u_0$ throughout body axis. The $\bar{Y}, \bar{N}, \bar{X}$ coefficients are in dimensional character, they can be calculated from dimensionless parameters which are in pu (or percent) of base given by Healey [2] as follows:

$$\begin{aligned}
&\bar{\mathbf{Y}}_{\dot{\mathbf{v}}} = \frac{1}{2}\mathbf{Y}_{\dot{\mathbf{v}}}\rho L^3 \quad \bar{\mathbf{Y}}_{\dot{\mathbf{r}}} = \frac{1}{2}\mathbf{Y}_{\dot{\mathbf{r}}}\rho L^4 \quad \bar{Y}_v = \frac{1}{2}Y_v\rho L^2 \\
&\bar{Y}_r = \frac{1}{2}Y_r\rho L^3 \quad \bar{X}_{\dot{u}} = \frac{1}{2}X_{\dot{u}}\rho L^3 \quad \bar{Y}_{dr} = \frac{1}{2}Y_{dr}\rho L^2 u_0^2 \\
&\bar{\mathbf{N}}_{\dot{\mathbf{v}}} = \frac{1}{2}\mathbf{N}_{\dot{\mathbf{v}}}\rho L^4 \quad \bar{\mathbf{N}}_{\dot{\mathbf{r}}} = \frac{1}{2}\mathbf{N}_{\dot{\mathbf{r}}}\rho L^5 \quad \bar{N}_v = \frac{1}{2}\mathbf{N}_{\mathbf{v}}\rho L^3 \\
&\bar{N}_r = \frac{1}{2}\mathbf{N}_{\mathbf{r}}\rho L^4 \quad \bar{N}_{dr} = \frac{1}{2}N_{dr}\rho L^3 u_0^2
\end{aligned} \tag{8}$$

Now, if we rewrite (7) to use the numeric values of NPS AUV II for $u_0 = 0.75m/s$, state space form is obtained as follows

$$\begin{bmatrix} \dot{v}(t) \\ \dot{r}(t) \\ \dot{\psi}(t) \end{bmatrix} = \begin{bmatrix} -0.1114 & -0.2647 & 0 \\ -0.0225 & -0.2331 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ r(t) \\ \psi(t) \end{bmatrix} + \begin{bmatrix} 0.0211 \\ -0.0258 \\ 0 \end{bmatrix} \delta_r(t) \tag{9}$$

Linear Model of AUV at the equilibrium points is derived by using "linmod" command in MATLAB instead of using the small variation method which neglect insignificant terms at linearizing plane shown above. The obtained Linear State Space Model of AUV is

$$\begin{bmatrix} \dot{\mathbf{v}}(\mathbf{t}) \\ \dot{r}(t) \\ \dot{\psi}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{-0.1114} & \mathbf{-0.2654} & \mathbf{0} \\ \mathbf{-0.0226} & \mathbf{-0.2348} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}(\mathbf{t}) \\ r(t) \\ \psi(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0.0212} \\ \mathbf{-0.0258} \\ 0 \end{bmatrix} \delta_r(t) \tag{10}$$

(10) shows that the MATLAB calculation with "linmod" command verify the equation (7). Hence AUV linear model (10) for yaw steering plane can be used to design control.

4. Sliding Mode Control

As a nonlinear control technique, the sliding mode control has provided an effective means of designing robust state feedback controllers for uncertain dynamic systems. The sliding mode control makes system states stay on a switching surface on which the system remains insensitive to internal parameter variations and extraneous disturbances. [5]

Assume that linear model of AUV for any steering plane can be written as single-input multi-output model

$$\dot{\boldsymbol{x}} = \mathbf{A}x + \boldsymbol{b}u + \boldsymbol{f}(\boldsymbol{x}) \tag{11}$$

Sliding surface in the state error space for tracking problem is defined and now the state errors are:

$$\tilde{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}_d = [x_1 - x_{1d} \quad \dots \quad x_n - x_{nd}]^T \tag{12}$$

where x_d desired tracking state. Now, sliding surface can be defined in the error state space form as follows

$$\sigma = \boldsymbol{S}^T\tilde{\boldsymbol{x}} = [s_1 \dots s_n] \begin{bmatrix} x_1 - x_{1d} \\ \vdots \\ x_n - x_{nd} \end{bmatrix} = 0 \tag{13}$$

$\boldsymbol{S}$ must be chosen to guarantee the convergence of the error $\lim_{t\to\infty} \tilde{\boldsymbol{x}} \to 0$ in state space. [6] The positive definite Lyapunov function candidate is

$$V(\sigma) = \frac{1}{2}\sigma^T\sigma = \frac{1}{2}\tilde{\boldsymbol{x}}^T\boldsymbol{S}\boldsymbol{S}^T\tilde{\boldsymbol{x}} \tag{14}$$

$V(\sigma)$ guarantees that the error state converges to sliding surface exists if following condition is provided

$$\dot{V}(\sigma) = \sigma\dot{\sigma} < 0 \tag{15}$$

Specifically, If we choose

$$\dot{\sigma} = -\eta.sign(\sigma) \qquad \eta > 0 \tag{16}$$

Differentiating the sliding surface (13)

$$\begin{aligned}
\dot{\sigma} &= \boldsymbol{S}^T\dot{\tilde{\boldsymbol{x}}} = \boldsymbol{S}^T(\mathbf{A}\boldsymbol{x} + \boldsymbol{b}u + \boldsymbol{f}(\boldsymbol{x}) - \dot{\boldsymbol{x}}_d) \\
&= -\eta sign(\sigma)
\end{aligned} \tag{17}$$

Let us extract control input u from (17)

$$\begin{aligned}
&u = (\boldsymbol{S}^T\boldsymbol{b})^{-1}[\boldsymbol{S}^T\dot{x}_d - \boldsymbol{S}^T\hat{\boldsymbol{f}}(\boldsymbol{x}) - \eta sign(\sigma) + \beta - \boldsymbol{S}^T\mathbf{A}\mathbf{x}], \\
&(\boldsymbol{S}^T\boldsymbol{b}) \neq 0 \\
&u = -(\boldsymbol{S}^T\boldsymbol{b})^{-1}\boldsymbol{S}^T\mathbf{A}\mathbf{x} + (\boldsymbol{S}^T\boldsymbol{b})^{-1} \\
&[\boldsymbol{S}^T\dot{x}_d - \boldsymbol{S}^T\hat{\boldsymbol{f}}(\boldsymbol{x}) - \eta sign(\sigma) + \beta] \\
&u = -\boldsymbol{K}^T x + (\boldsymbol{S}^T\boldsymbol{b})^{-1}[\boldsymbol{S}^T\dot{x}_d - \boldsymbol{S}^T\hat{\boldsymbol{f}}(\boldsymbol{x}) - \eta sign(\sigma)]
\end{aligned} \tag{18}$$

Where $\hat{\boldsymbol{f}}(\boldsymbol{x})$ is a estimate of $\boldsymbol{f}(\boldsymbol{x})$.

if we use real $\boldsymbol{f}(\boldsymbol{x})$ value, $\dot{\sigma}$ is

$$\begin{aligned}
&\dot{\sigma} = \boldsymbol{S}^T(\mathbf{A} - \boldsymbol{b}\boldsymbol{K}^T)\boldsymbol{x} - \eta sign(\sigma) - \boldsymbol{S}^T(\boldsymbol{f}(\boldsymbol{x}) - \hat{\boldsymbol{f}}(\boldsymbol{x})) \\
&\dot{\sigma} = \boldsymbol{S}^T\mathbf{A}_c\boldsymbol{x} - \eta sign(\sigma) - \boldsymbol{S}^T(\Delta\boldsymbol{f}(\boldsymbol{x}))
\end{aligned} \tag{19}$$

Where $\Delta\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) - \hat{\boldsymbol{f}}(\boldsymbol{x})$, $\mathbf{A}_c = \mathbf{A} - \boldsymbol{b}\boldsymbol{K}^T$

Now, we study the subject how to choose $\boldsymbol{S}^T$. If one of the eigenvalues of $\mathbf{A}_c$ is specified to be zero, $\boldsymbol{S}^T\mathbf{A}_c\boldsymbol{x}$ terms can be equal to zero by choosing $\boldsymbol{S}^T$ as the right eigenvector of $\mathbf{A}_c^T$ for eigenvalue $\lambda = 0$. That is,

$$\dot{\sigma} = -\eta sign(\sigma) - \boldsymbol{S}^T\Delta\boldsymbol{f}(\boldsymbol{x}) \tag{20}$$

Differentiation of Lyapunov function candidate with respect to time is

$$\begin{aligned}
\dot{V}(\sigma) &= \sigma\dot{\sigma} = -\eta.\sigma.sign(\sigma) + \sigma.\boldsymbol{S}^T.\Delta\boldsymbol{f}(\boldsymbol{x}) \\
&= -\eta.|\sigma| + \sigma.\boldsymbol{S}^T.\Delta\boldsymbol{f}(\boldsymbol{x})
\end{aligned} \tag{21}$$

σ -dynamics can be made globally asymptotically stable from (21) if η is chosen as follows

$$\eta > \|\boldsymbol{S}^T\|.\|\Delta\boldsymbol{f}(\boldsymbol{x})\| \tag{22}$$

Choosing η according to (21) ensures that $\dot{V}(\sigma) < 0$. Hence, According to Barbalat' lemma σ converges to zero in finite time if η is chosen large enough to overcome the destabilizing effects of unmodelled dynamics $\Delta\boldsymbol{f}(\boldsymbol{x})$. The choice of η will be trade of between robustness and performance (Fossen [3])

5. Simulations

Let's consider the linear system (10) which is linearized with the numerical method on yaw steering plane. MATLAB-Simulink Model is given as **Fig. 2**.

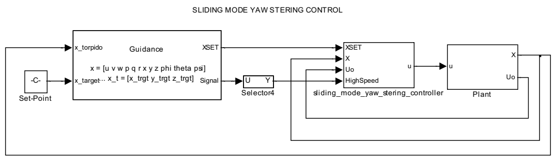

Fig. 2: Sliding Mode Yaw Steering Control Simulink Diagram

If the closed loop dynamics of the yaw steering sub-system (10) are chosen to be {0, -0.41, -0.42} for the sliding motion, $\boldsymbol{K}^T$ can easily be obtained by using the Ackermann formula as

$$\boldsymbol{K}^T = [9.8307 \quad -10.6740 \quad 0] \tag{23}$$

$\mathbf{A}_c$ thus becomes as the following matrix below

$$\mathbf{A}_c = \mathbf{A} - \boldsymbol{b}\boldsymbol{K}^T = \begin{bmatrix} -0.3709 & -0.0065 & 0 \\ 0.2956 & -0.4591 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{24}$$

$\boldsymbol{S}^T$ is the right eigenvector of $\mathbf{A}_c^T$ for eigenvalue $\lambda = 0$, that is found to be as

$$\boldsymbol{S}^T = [0.5859 \quad 0.7351 \quad 0.3413] \tag{25}$$

Sliding mode control law is

$$u = -\boldsymbol{K}^T x + (\boldsymbol{S}^T \boldsymbol{b})^{-1} [\boldsymbol{S}^T \dot{x}_d - \boldsymbol{S}^T \hat{\boldsymbol{f}}(\boldsymbol{x}) - \eta sign(\sigma)] \tag{26}$$

To prevent chattering because of the discontinuity in control law with ideal switching function $sign(\sigma)$ alternatively, sign function could be replaced by the continuous function $\tanh(\sigma/\phi)$ which is given in **Fig. 3**, where ϕ is the sliding surface boundary layer thickness.

Reducing ϕ increases the nonlinear gain, while increasing ϕ introduces a filtering effect if measurement is noisy. Hence equation (27) can be rewritten to prevent chattering as below

$$u = -\boldsymbol{K}^T x + (\boldsymbol{S}^T \boldsymbol{b})^{-1} [\boldsymbol{S}^T \dot{x}_d - \boldsymbol{S}^T \hat{\boldsymbol{f}}(\boldsymbol{x}) - \eta \tanh(\sigma/\phi)] \tag{27}$$

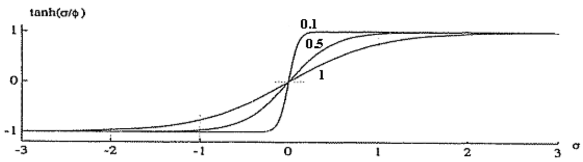

Fig. 3: $\tanh(\sigma/\phi)$ function for $\phi \in (0.1, \quad 0.5, \quad 1)$

Assume that nonlinearity is $\hat{\boldsymbol{f}}(\boldsymbol{x}) = 0, \| \Delta\boldsymbol{f}(\boldsymbol{x}) \| < 0.002$ and trajectory is constant ($\dot{\boldsymbol{X}}_d = 0$). By using (22) η must be equal and greater than 0.002. If specifically, $\eta = 0.002$ and $\phi = 0.01$ are chosen, then the control law for rudder is

$$u_{rudder} = -[9.8847 \quad -10.6867 \quad 0]x + 0.2594 * \tanh([0.54 \quad 0.7407 \quad 0.3996]x/0.01) \tag{28}$$

We have already calculated sliding mode control law for $u_0 = 0.75 m/s$. If all linearizing process given in Section 3 and process of substituting numeric value with using same value which is given in this section is repeated for $u_0 = 1.5 m/s$ and $\eta = 0.004$, sliding mode control law for rudder is

$$u_{rudder} = -[0.7791 \quad -0.7191 \quad 0]x + 0.05054 * \tanh([0.1050 \quad 0.8521 \quad 0.5127]x/0.01) \tag{29}$$

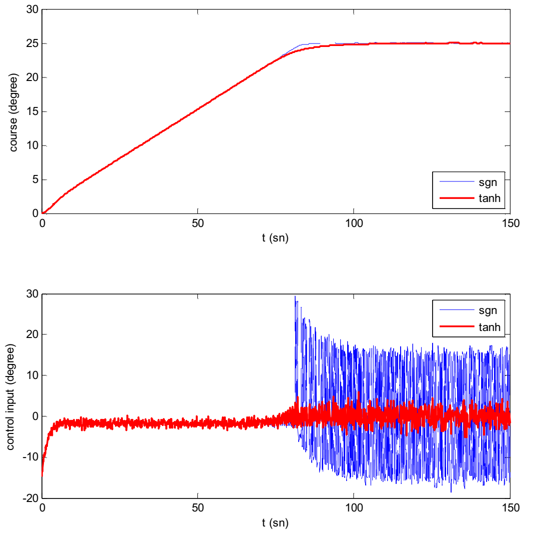

Fig. 4: Course angle and control input for $u_0 = 0.75 m/s$

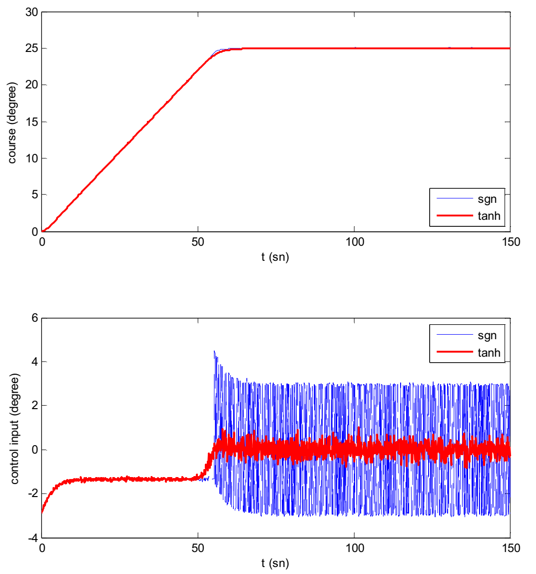

Fig. 5: Course angle and control input for $u_0 = 1.5 m/s$

We can see in **Fig. 4** and **Fig. 5** that desired values is reached for two operation speeds even if disturbances exist. After the system states reach on sliding surface, the sign functions effect in control signal forces control surface-rudder to rapid change. The perpetual on-off control signal couldn't applied to control surface-rudder during long period and damage system. Furthermore, the chattering effect cause an over consumption energy. Even though "sign" control is shown as simulation result in **Fig. 5**, it is meaningful to use “tanh” control for real system to prevent chattering effect.

6. Conclusions

In this paper, an autonomous underwater vehicle (AUV) model with six degrees of freedom is presented to be shown having been linearized under several working conditions. Sliding Mode Control Law is derived to be able to apply to autonomous underwater vehicle.

A control law derived by assuming to move along on the x axis with a constant speed for yaw steering plane applied to the nonlinear model. The system response for course command is so good and satisfactory even if model is non-linear.

The Sliding mode control with sign function consumes a lot of control energy and damages AUV. By using hyperbolic tangent function reduce control energy and prevent the chattering effect.

7. References

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