Two Dimensional Fluid Flow in the Channel of a Magnetohydrodynamic Pump

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Abstract

This paper deals with the study of an unsteady fluid flow in a MHD pump fed with sinusoidal current at variable frequency for the mercury pumping using an alternate coupling finite element and volume computation with an adaptation of the nodes. By introducing vector potentials $\left(\vec{A},\vec{\psi}\right)$ and the vorticity $\vec{\xi}$, a system of equations which are of the elliptic type for describing a velocity and pressure distribution in the channel is obtained. In this analysis, the width of the channel is taken into account and the results are compared to those obtained in the limit case when the width is much smaller or much bigger than a characteristic dimension of the channel. In this case, the influence of the frequency on the fluid flow of the MHD pump is investigated.

1. Introduction

This paper deals with the study of an unsteady fluid flow in a MHD pump fed with sinusoidal current at variable frequency for the mercury pumping using an alternate coupling finite element and volume computation with an adaptation of the nodes.

By introducing vector potentials $\left(\vec{A},\vec{\psi}\right)$ and the vorticity $\vec{\xi}$, a system of equations which are of the elliptic type for describing a velocity and pressure distribution in the channel is obtained.
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The pumping of liquid metal may use an electromagnetic device, which induces eddy currents in the metal. These induced currents and their associated magnetic field generate the Laplace force whose effect can be actually the pumping of the liquid metal [2].

The purpose of this paper is to determine the velocity and pressure profiles in the channel, in the case of laminar magnetohydrodynamic flow and to study the influence of certain parameters like the frequency on the fluid flow. Also, the full problem, in which the width of the channel is taken into account is analysed.

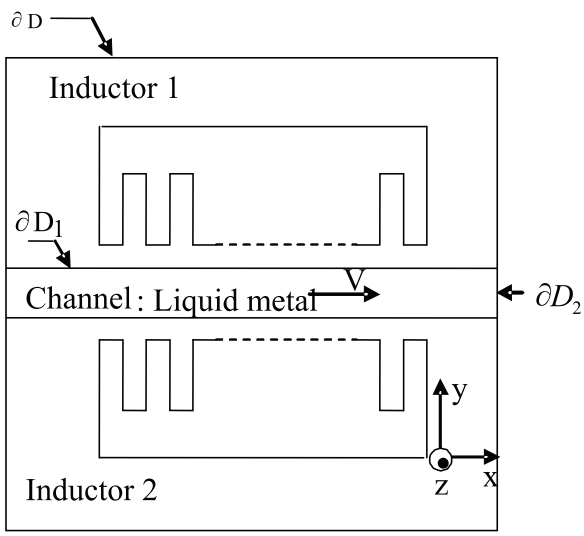

Fig. 1 Schematic view of the MHD pump

2. The MHD equations

A schematic view of the pump is shown in fig. 2. An electromagnetic pump using mercury as the liquid metal. The liquid metal flows along a channel and a ferromagnetic core is placed on the inner and the outer side of the channel. The conducting fluid is assumed to be viscous and incompressible. A three balanced system of currents supplies the windings.

$$J_1 = J_0 \sin(\omega t) \tag{1}$$

$$J_2 = J_0 \sin\left(\omega t - \frac{2\pi}{3}\right) \tag{2}$$

$$J_3 = J_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \tag{3}$$

The currents of the windings generate the travelling magnetic field which produces a current in the liquid metal. As a consequence a Laplace force acting on the fluid is obtained.

2.1 The electromagnetic model

The equations describing the pumping process in the channel are the Maxwell equations and Ohm's law for laminar incompressible flows.

$$div\,\vec{D} = 0 \qquad (4)$$

$$\overrightarrow{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (5)$$

$$div\,\vec{B} = 0 \qquad (6)$$

$$\overrightarrow{rot}\,\vec{H} = \vec{J} \qquad (7)$$

$$\vec{J_i} = \sigma\left(\vec{E} + \vec{V} \wedge \vec{B}\right) \qquad (8)$$

$$\vec{B} = \mu\,\vec{H} \qquad (9)$$

$$\vec{D} = \varepsilon\,\vec{E} \qquad (10)$$

The current density vector $\vec{J}$ is made up of by two components $\vec{J} = \overrightarrow{J_{ex}} + \vec{J_i}$, where $\overrightarrow{J_i}$ is the eddy current density flowing in the fluid and $\overrightarrow{J_{ex}}$ is the current density in the windings. In (8) σ is the electrical conductivity and $\vec{V}$ is the velocity of the fluid. The other symbols are conventional.

The Maxwell's equations applied to a pump so that the magnetic vector potential $\vec{A}$ has only one component in the z direction are characterised by :

$$\overrightarrow{rot}\left(\frac{1}{\mu}\overrightarrow{rot}\vec{A}\right) + \sigma\left(\frac{\partial \vec{A}}{\partial t} - \vec{V} \wedge \overrightarrow{rot}\vec{A}\right) = \overrightarrow{J_{ex}} \qquad (11)$$

The eddy currents are computed by :

$$\vec{J_i} = -\sigma\left(\frac{\partial \vec{A}}{\partial t} - \vec{V} \wedge \overrightarrow{rot}\vec{A}\right) \qquad (12)$$

And the body forces are given by :

$$\vec{F} = \vec{J_i} \wedge \overrightarrow{rot}\vec{A} \qquad (13)$$

2.2 The hydrodynamic model

The equations of the flow for unsteady state motion of a constant density ρ fluid are [3]:

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)\vec{V} = -\frac{1}{\rho}\overrightarrow{grad}\,p + \upsilon\Delta\vec{V} + \frac{\vec{F}}{\rho} \qquad (14)$$

$$div\,\vec{V} = 0 \qquad (15)$$

with : p : pressure of the fluid (Pa) ;

υ : kinematic viscosity of the fluid $\left(m^2/S\right)$;

$\vec{F}$: electromagnetic thrust $\left(N/m^3\right)$;

ρ : fluid density (kg/m^3).

The difficulty is that in the previous equations there are two unknown : the pressure and the velocity. The elimination of pressure from the equations leads to a vorticity-stream function which is one of the most popular methods for solving the 2-D incompressible Navier-Stokes equations [3], [4].

$$\vec{\xi} = \overrightarrow{rot}\,\vec{U} \qquad (16)$$

$$\frac{\partial \psi}{\partial y} = u, \qquad \frac{\partial \psi}{\partial x} = -v \qquad (17)$$

Using these new dependent variables, the two momentum equations can be combined (thereby eliminating pressure) to give:

$$\frac{\partial \xi}{\partial t} + u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} = \nu\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right) + \frac{1}{\rho}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) \qquad (18)$$

An additional equation involving the new dependant variables ξ and ψ can be obtained by substituting (18) into (16) which gives :

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\xi \qquad (19)$$

In order to determine the pressure, it is necessary to solve an additional equation which is referred to as the Poisson equation for pressure ; the latter is obtained by differentiating equation (14).

$$\Delta p = 2\,\rho\left(\frac{\partial^2 \psi}{\partial x^2}\,\frac{\partial^2 \psi}{\partial y^2}\right) \qquad (20)$$

3. Results and discussion

The Petrov Galerkin method is utilised for the resolution of the magnetodynamic model [5],[6] and to solve the Navier Stokes equations, a control-volume procedure is used [7]. The code generated is based on an unstructured mesh-generation. For accuracy in application of boundary conditions, triangular type mesh is utilised. The nodes of the mesh for the coupling model magnetodynamic-hydrodynamics are the same as shown in the following figure (fig.2). For the model of the fluid flow, there is one control volume surrounding each node (fig.2) and the differential equation is integrated over each control volume.

At each time step magnetodynamics and hydrodynamics can be solved alternatively and iteratively until convergence is reached. The process is repeatedly until $t \geq t_f$.

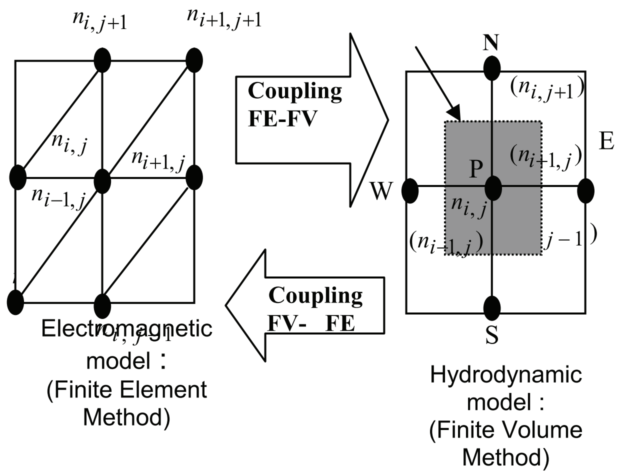

Fig. 2 The adaptation of the nodes for the coupling

F.E-F.V

The potential vector $\vec{A}$ is calculated for each finite element node, by means of finite element method. Hydrodynamic calculations supply the pressure and the velocity components which must be known at each integration point of the finite volume. Concerning the coupling F.E.M – F.V.M, it is necessary to ensure an adaptation of the grid mesh, i.e. we must find the same nodes for the two methods.

As a result Fig. 3 and fig. 4 show the magnetic vector potential distribution and the magnetic induction in the MHD pump.

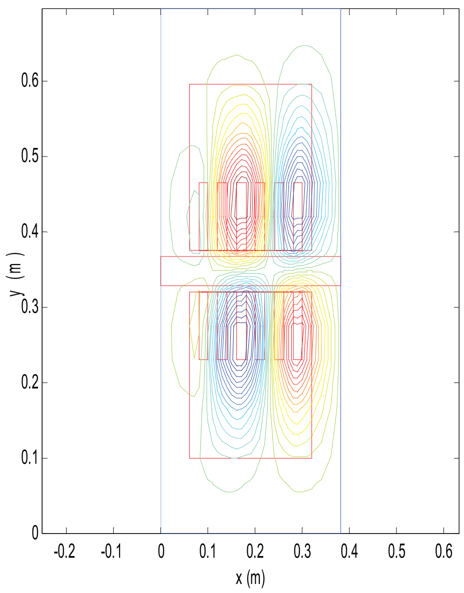

Fig. 3 Magnetic vector potential distribution in the MHD pump

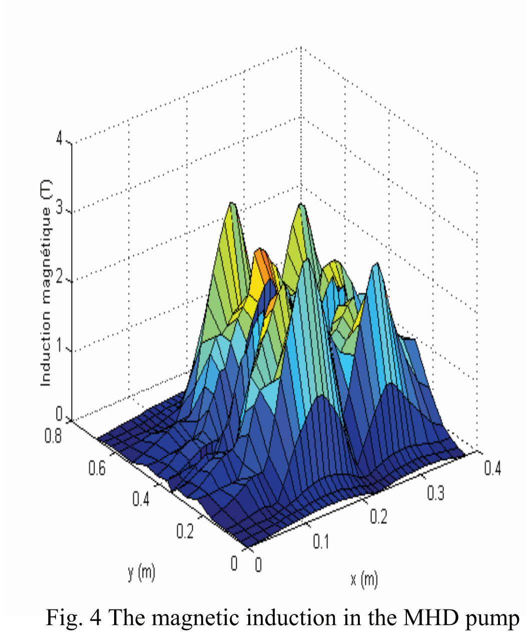

Fig. 4 The magnetic induction in the MHD pump

Fig. 5 show the eddy current variations at the starting of the pump for f=50 Hz. It is noticed that the transient state is between the interval 0-0.15 seconds where the induced current has a maximum value of 2500 A/m² .

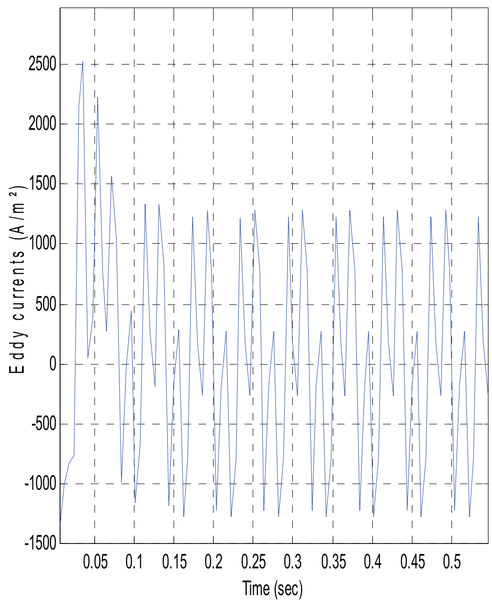

Fig. 5 Eddy currents at the starting for f=50 Hz

Fig. 6 shows the electromagnetic thrust Fx variations of the MHD pump.

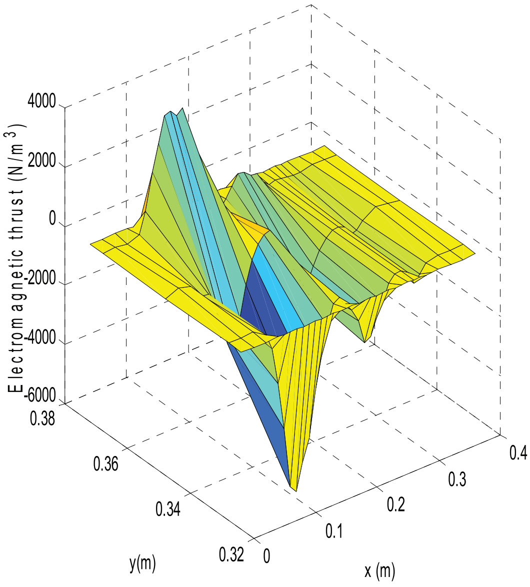

Fig. 6 Electromagnetic thrust in MHD pump

Fig. 7 represents the variation of the velocity at the starting of the pump for several frequencies. It is shown that the velocity increases as the frequency increases and the steady state is obtained approximately after three seconds.

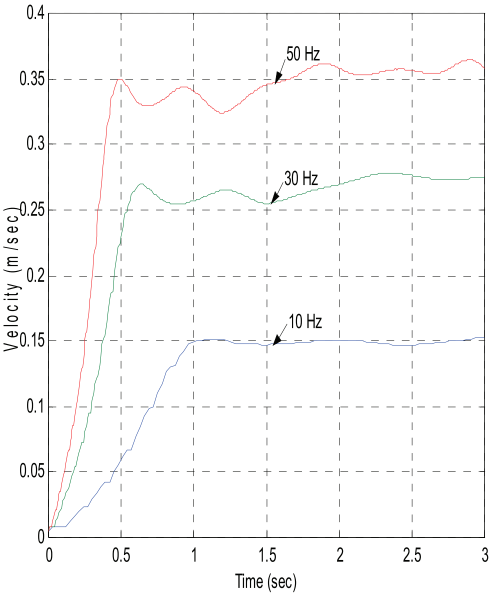

Fig. 7 Velocity in the channel for several frequencies

Fig. 8 represents the comparison of the variation of the velocity at the starting of the pump for f=10 Hz for several widths of the channel. It is shown on the figure (10) where the width of the channel is increased or decreased that the velocity is less and does not reach the steady state, this is either with the load losses or to frictions.

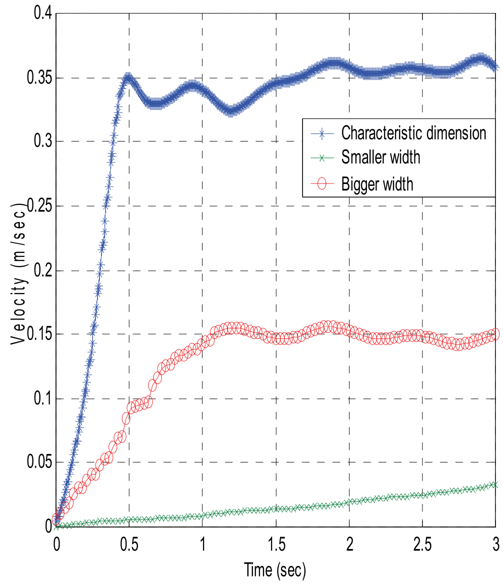

Fig. 8 Velocity in the channel of the MHD pump

Fig. 9 shows the pressure variations at the starting for several frequencies. It is found that the pressure increases as the frequency increases. It is important to notice that the amplitudes of the pressure oscillations increase with the increasing of the frequency. Moreover, the "shock" values become more significant with a shorter transient state.

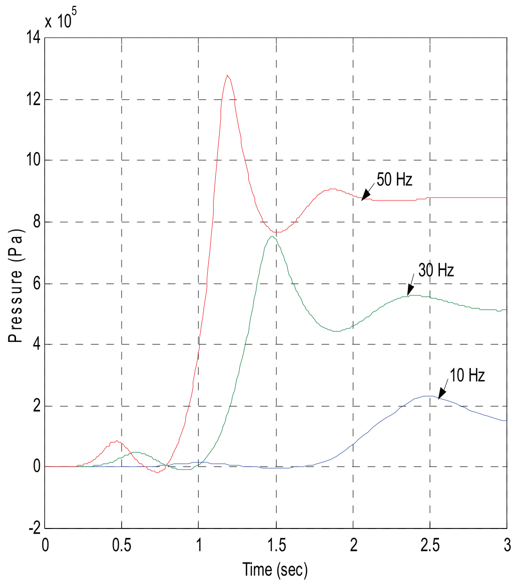

Fig. 9 Pressure in the channel for several ***frequencies***

4. Conclusion

The results of the magnetohydrodynamic analysis of a mercury linear double induction pump taking into account the movement of the fluid are obtained by using 2D finite element-finite volume method . The velocity, the pressure and its oscillation amplitudes increase with the increasing of the frequency. Moreover, the " shock" values become more significant with a shorter transient state.

Moreover, it is shown that an optimal width of the channel allows the improvement of the performances of MHD pump.

The obtained results confirm an influence of the frequency on the velocity and pressure distribution in the investigated flow and are in perfect agreement with those carried out by [8] and [9], expected that the results are obtained in different working conditions.

5. References

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