A GENERALISED VARIANT OF ORTHOGONAL INTERCONNECTING NETWORKS

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Abstract

The paper deals with a variant of direct interconnection network, Generalised Hyper Structure – GHS. *A GHS has a non-homogeneous orthogonal topology in which* dimension i *is connected to an* interconnecting vector *represented by an* union of elementary interconnection structures, EIS-s. *Constituent EIS-s are homogeneous topologies, for example, tori, grids or completely connected networks. Definition of the GHS-s is an attempt to specify* a consistent method to design interconnection structures with variable interconnection spatial locality properties, *known criterion of the topological design for the interconnection networks used in parallel systems.*

Index Terms: Direct interconnection networks, Parallel systems, Topology, Interconnection locality

1. THE HOMOGENOUS TOPOLOGIES

Most implemented *Direct Interconnection Networks*, DIN, have an orthogonal topologies [3]. Among them are *generalised hypercubes*, GHC, [2], or *alpha networks* [1]. These structures interconnect N nodes in r dimensions where $N=m_r \cdot m_{r-1} \cdot \ldots \cdot m_i \cdot \ldots \cdot m_1$. In every dimension i there are m_i nodes interconnected all by all.

Definition 1.1: A GHC is a DIN in which every node represented by an address written in a *Mixed Radix Number System*, MRNS, $X=(x_r\, x_{r-1} \ldots x_{i+1}\, x_i\, x_{i-1} \ldots x_1)$ is connected with the nodes addressed by $X' = (x_r\, x_{r-1} \ldots x_{i+1} x'_i x_{i-1} \ldots x_1)$, where $1 \le i \le r$, $0 \le x'_i \le m_i - 1$ and $x'_i \ne x_i$.

If a GHC has a single dimension we obtain the known *Fully* or *Completely Connected Network*, CCN, in which the nodes ($N=m$) are tied all by all. Let us notice that a GHC can be now understood as a DIN in which the nodes of any dimension are linked by a mono-dimensional *Elementary Interconnection Structure, EIS*, of CCN type.

Other elementary interconnection structures can define other generalised DIN. For example, if *EIS* is a torus (T), we will obtain the *generalised hypertorus*, GHT, and if *EIS* is a grid (G), we will obtain *generalised hypergrids*, GHG. We will give the two more definitions, we use later, based on the torus and grid *EIS*-s.

Definition 1. 2: A GHT is a DIN in which every node X represented by an address written in a MRNS is connected with the nearest neighbour nodes addressed by $X'=(x_r x_{r-1} \ldots x_{i+1} x'_i x_{i-1} \ldots x_1)$, where $1 \le i \le r$, $x'_i = |x_i \pm 1|_{modulo\ m_i}$.

Definition 1. 3: A GHG is a DIN in which every node X represented by an address written in a MRNS is connected in a grid with the nodes addressed by $X'=(x_r\, x_{r-1} \ldots x_{i+1}\, x'_i\, x_{i-1} \ldots x_1)$, where $1 \le i \le r$; $x'_i = x_i \pm 1 \mid x_i \ne 0$ and $x_i \ne m_i - 1$; $x'_i = x_i + 1 \mid x_i = 0$; $x'_i = x_i - 1 \mid x_i = m_i - 1$.

Resulted networks interconnect a number of nodes by a number of links in a multidimensional structure, in every dimension the nodes being connected by *a specified EIS*. With these examples, we defined orthogonal topologies with a *constant EIS*. That means the *EIS* is the *same for all dimensions* and is *constant in every dimension*. Let us call them *homogenous networks*. Examples are GHC-s, GHT-s and GHG-s.

2. DEFINITION OF THE GENERALISED HYPER STRUCTURES

If we vary the *EIS*, we will obtain *non-homogenous* networks or what we named hybrid hypercubes [6] or *generalised hyper structures*, GHS-s:

Definition 2. 1: A GHS is an interconnection network in which every node X represented by an address written in a MRNS is connected in the dimension i, $1 \le i \le r$, with the nodes addressed by

$$\bigcup_{j=1}^{k_i} X^{ij} = (x_r x_{r-1} \ldots x_{i+1} x'_i\, x_{i-1} \ldots x_1).$$

X' is substituted by a *union*, $\bigcup_{j=1}^{k_i} X^{ij}$. Therefore, the union $\bigcup_{j=1}^{k_i} X^{ij}$ specifies that GHS is connected by a vector of elementary interconnection structures - *interconnecting vector*, which has r *elements*, $\bigcup_{j=1}^{k_i} X^{ij}$, $1 \le i \le r$. So, this *interconnecting vector* is defined, on the one hand, by the number of dimensions, r, and, on the other hand, by k_i elementary interconnection structures for which the dimension i is specified, X^{ij}, $j=1, 2, \ldots, k_i$. X^{ij} are homogenous networks, like those described in the introduction, and must not be disjoint for a dimension.

In order to understand the Generalised Hyper Structures we give two examples.

3. TWO EXAMPLES OF GHS

Example 3. 1: Let GHS with $N=5 \cdot 4$ nodes using for each dimension the *EIS*-s defined by the definitions 1.1 and, respective, 1.2.

The address of the node X represented in a MRNS will be $X=(x_2\ x_1)$, where $x_2 \in \{0,1,2,3,4\}$ and $x_1 \in \{0,1,2,3\}$.

In the first dimension the X node will be tied with X^1 nodes in a torus, T, in accordance with the definition 1.2, $X^1=(x_2\, x'_1)$, where $x'_1 = |x_1 \pm 1|_{modulo\ 4}$.

In the second dimension the X node will be tied with X^2 nodes in a CCN pattern defined by the definition 1.1, $X^2=(x'_2\, x_1)$, where $0 \le x'_2 \le 4$, $x'_2 \ne x_2$.

The interconnecting vector is $\{X^2, X^1\}$. Above GHS is coded in accordance to the interconnecting vector by {CCN, T} and is represented in the figure 1a.

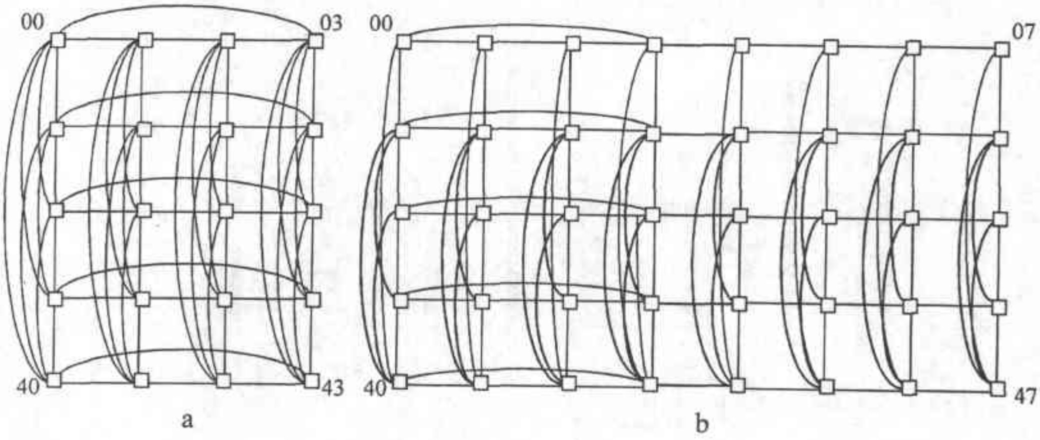

Figure 1. Two examples of GHS-s with $N=m_2 \times m_1=5 \times 4$: {CCN, T} – (a) and with $N=m_2 \times m_1=5 \times 8$: $\{T_{0\div4} \cup CCN_{1\div4},\ T_{0\div3} \cup G_{3\div7}\}$ - (b)

Example 3. 2: Let GHS with $N=5 \cdot 8$ nodes. In the first dimension, first four nodes will be connected in a torus and last five in a grid (definition 1.3). In the second dimension, first five nodes (all) will be interconnected in a torus, and last four in a CCN.

The address of the node X represented in a MRNS will be $X=(x_2\ x_1)$, where $x_2 \in \{0,1,2,3,4\}$ and $x_1 \in \{0,1,2,3,4,5,6,7\}$.

In the first dimension the X node will be tied with $X^{11} \cup X^{12}$ nodes, X^{11} representing a torus, T, and X^{12} representing a grid, G. Therefore, $X^{11}=(x_2\, x'_1)$, where $x'_1 = |x_1 \pm 1|_{modulo\ 4}$, $x_1 \in \{0,1,2,3\}$, and $X^{12}=(x_2\, x'_1)$, where $x'_1 = x_1 \pm 1 \,|\, x_1 \ne 3$ and $x_1 \ne 7$; $x'_1 = x_1 + 1 \,|\, x_1 = 3$; $x'_1 = x_1 - 1 \,|\, x_1 = 7$; $x_i \in \{3,4,5,6,7\}$.

In the second dimension the X node will be tied with $X^{21} \cup X^{22}$ nodes, X^{21} representing a torus, T, and X^{22} representing a CCN pattern. On this dimension, *patterns are not disjoint*, $X^{21}=(x'_2\, x_1)$, where $x'_2 = |x_2 \pm 1|_{modulo\ 5}$, $x_2 \in \{0,1,2,3,4\}$, and $X^{22}=(x'_2\, x_1)$, where $1 \le x'_2 \le 4$, $x'_2 \ne x_2$, $x_2 \in \{1,2,3,4\}$.

Above GHS is coded $\{T_{0+4} \cup CCN_{1+4},\ T_{0+3} \cup G_{3+7}\}$ corresponding to the interconnecting vector $\{X^{21} \cup X^{22},\ X^{11} \cup X^{12}\}$ and is represented in figure 1b.

4. INTERCONNECTION LOCALITY. AS DESIGN CRITERION. EVALUATION AND CONCLUSION

Locality is one of main criteria to design computers [3], [5], [4], [9], [6], [7]. What we tried by defining the GHS was *to specify a method to design interconnection structures with variable interconnection spatial locality properties*. Comparing with the GHC-s, GHT-s or GHG-s, the GHS-s appear as more flexible networks. Our intention is to design networks fitted to the locality requirements of different communication processes. In general, these requirements are not constant and we must be able to design interconnection structures with variable locality characteristics. The GHS-s are a possible way to obtain such interconnections especially from the point of view of the neighbourhoods and the functional average distances.

One of the synthetic measures of the interconnection locality of a DIN is the *functional average distance*. The functional average distance, covering average message distance [2], number of link visits or mean internode distance [9], is given by

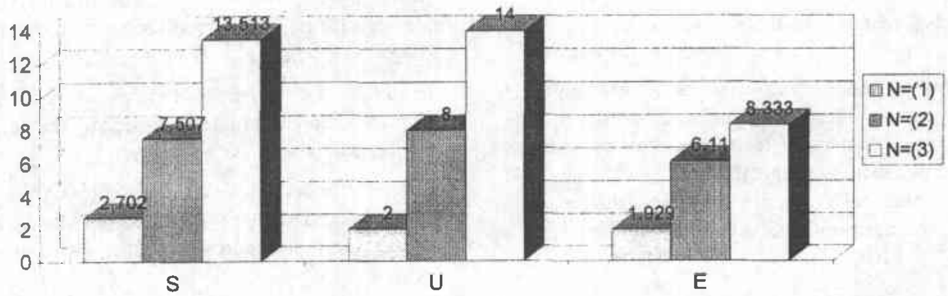

Figure 2. Functional average distances for structural (S), uniform (U) and exponential (E) distributions for structures (1), (2) and (3)

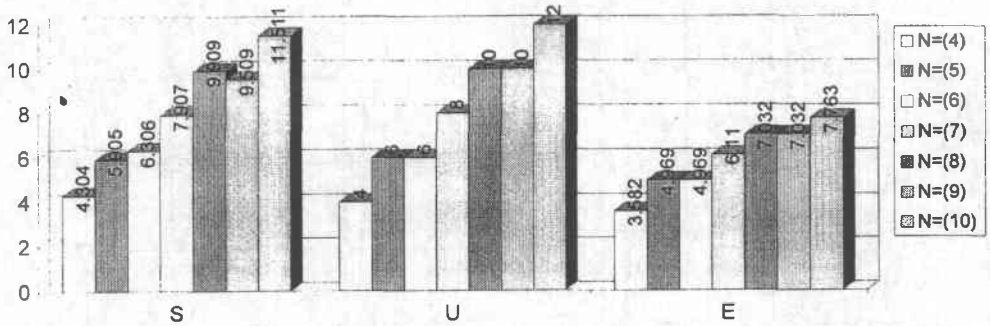

Figure 3. Functional average distances for structural (S), uniform (U) and exponential (E) distributions for structures (4)-(10)

$\overline{d_F} = \sum_{d=1}^{D} d \times \Phi(d)$, where $\Phi(d)$ is the message routing distribution at the distance d [9].

Taken, as examples, three *EIS*-s, Completely Connected Network (CCN), Torus (T) and Grid (G), we evaluated by the functional average distance *all the GHS-s for $k_i=1$* (example 3.1) with three dimensions and $N=10 \times 10 \times 10$ nodes [8]. In the figures 2 and 3 the Generalised Hyper Structures are: {CCN, CCN, CCN} (1), {T, T, T} (2), {G, G, G} (3), {CCN, CCN, T} (4), {CCN, T, T} (5), {CCN, CCN, T} (6), {CCN, T, G} (7), {CCN, G, G} (8), {T, T, G} (9), {T, G, G} (10). In the same figures the routing distributions are: $\Phi(d) = N_d/(N-1)$ – structural (S), $\Phi(d)=\varphi$ – uniform (U), and $\Phi(d)=K \cdot \lambda^d$ – exponential (E).

In the figures 2 and 3 we demonstrated that GHS-s, filling the gaps between topologies (1), (2) and (3), are a good candidate for a method of designing topologies having the interconnection spatial locality as parameter, at least measured by $\overline{d_F}$.

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