

# A Sliding Mode Speed and Flux Control of a Doubly Fed Induction Machine

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## Abstract

This paper, presents a Direct Field-Oriented Control (DFOC) of doubly fed induction machine (DFIM) motor mode with a variable structure control based on a sliding mode control (SMC). Our aim is to make the speed and the flux control robust to parameter variations. The use of the nonlinear sliding mode method provides very good performance for motor operation and robustness of the control law despite the external perturbation. The results show that the sliding mode controller has better performance, and it is robust against external load variation. To validate the proposed method, the simulation has been carried out using MATLAB-SIMULINK software and the results are presented.

## 1. Introduction

Known since 1899 [1], the doubly fed induction machine (DFIM) is an asynchronous machine with wound rotor which can be supplied even time by the stator and the rotor external source voltages. This solution is very attractive for the variable speed applications such as the electric vehicle and the electrical energy production [1]. Consequently, it covers all powers ranges. Obviously, the variable speed and the performances ranges depend of the application nature. With DFIM, it can possible to modulate power flow into and out the rotor winding in order to have, at the same time, a variable speed in the characterized super-synchronous or sub-synchronous modes in motor or in generator regimes. Two modes can be associated to slip power recovery: sub-synchronous motoring and super-synchronous generating operations. In general, while the rotor is fed through a cycloconverter, the power range can attain the MW order which presents the size power often reserved to the synchronous machines [1].

The DFIM has some distinct advantages compared to the conventional squirrel-cage machine. The DFIM can be fed and controlled stator or rotor by various possible combinations. Indeed, the input-commands are done by means of four precise degrees of control freedom relatively to the squirrel cage induction machine where its control appears quite simpler. The flux orientation strategy can transform the non linear and coupled DFIM-mathematical model to a linear model conducting to one attractive solution as well as under generating or motoring operations [1].

The control of the DFIM must take into account machine specificities: the high order of the model, the nonlinear functioning as well as the coupling between the different variables of control. The new industrial applications necessitate speed variators having high dynamic performances, a good precision in permanent regime, and a high capacity of overload

on all the range of speed and a robustness to the different perturbations. Thus, the recourse to robust control algorithms is desirable in stabilization and in tracking trajectories. The variable structure control (VSC) possesses this robustness using the sliding mode control that can offer many good properties such as good performance against unmodelled dynamics, insensitivity to parameter variation, external disturbance rejection and fast dynamic [2].

A SMC is basically an adaptive control scheme that gives robust performance of a drive with parameter variations. The control is nonlinear and applied to linear and nonlinear plant. In SMC, the drive response is forced to slide along a predefined trajectory in a phase plane by a switching control algorithm, irrespective drive's parameter variation and load disturbances [3]. These advantages of sliding mode control can be employed in the speed control of an alternative current servo system.

In the field-oriented control of DFIM, the knowledge of rotor speed and flux is necessary. In this work the flux is obtained by the measurement of stator and rotor (rotor winding) currents. The speed is measured.

In this paper, we begin with the DFIM oriented model in view of the vector control, next the stator flux  $\phi_s$  is estimated. We, then, present the sliding mode theory and design the sliding mode controllers of stator flux and motor speed. Finally, we give some conclusion remarks on the control proposed of DFIM using sliding mode.

## 2. Equations of DFIM in the stator flux orientation

In this section, the DFIM model can be described by the following state equations in the synchronous reference frame whose axis d is aligned with the stator flux vector, ( $\phi_{sd} = \phi_s$  and  $\phi_{sq} = 0$ ) [4], [5] :

$$i_{rd} = \frac{\phi_{sd}^*}{M} \quad (1)$$

$$i_{rq} = -\frac{L_s}{P.M\phi_{sd}^*} C e^* \quad (2)$$

$$\frac{d\theta_s}{dt} = \omega_s = \left( \frac{R_s.M}{L_s} i_{rq} + V_{sq} \right) / \phi_{sd}^* \quad (3)$$

$$\dot{i}_{rd} = -\frac{1}{\sigma} \left( \frac{1}{T_r} + \frac{M^2}{L_s T_s L_r} \right) i_{rd} - \frac{M}{\sigma L_r L_s} V_{sd} + \quad (4)$$

$$\frac{M}{\sigma L_r L_s T_s} \phi_{sd} + (\omega_s - \omega) i_{rq} + \frac{1}{\sigma L_r} V_{rd}$$

$$\dot{i}_{rq} = -\frac{1}{\sigma} \left( \frac{1}{T_r} + \frac{M^2}{L_s T_s L_r} \right) i_{rq} - \frac{M}{\sigma L_r L_s} V_{sq} +$$

$$\frac{M}{\sigma L_r L_s} \omega \phi_{sd} - (\omega_s - \omega) i_{rd} + \frac{1}{\sigma L_r} V_{rq}$$

$$\dot{\phi}_{sd} = V_{sd} + \frac{M}{T_s} i_{rd} - \frac{1}{T_s} \phi_{sd}$$

$$\dot{\phi}_{sq} = V_{sq} + \frac{M}{T_s} i_{rq} - \omega_s \phi_{sd}$$

$$\dot{\Omega} = -\frac{P \cdot M}{J L_s} (i_{rq} \phi_{sd}) - \frac{C_r}{J} - \frac{f}{J} \Omega$$

With:

$$T_r = \frac{L_r}{R_r}; T_s = \frac{L_s}{R_s}; \sigma = 1 - \frac{M^2}{L_s L_r}$$

Where:

$i_{rd}$ ,  $i_{rq}$  are rotor current components,  $\phi_{sd}$ ,  $\phi_{sq}$  are stator flux components,  $V_{sd}$ ,  $V_{sq}$  are stator voltage components,  $V_{rd}$ ,  $V_{rq}$  are rotor voltage components.  $R_s$  and  $R_r$  are stator and rotor resistances,  $L_s$  and  $L_r$  are stator and rotor inductances,  $M$  is mutual inductance,  $\sigma$  is leakage factor and  $P$  is number of pole pairs.  $C_e$  is the electromagnetic torque,  $C_r$  is the load torque,  $J$  is the moment of inertia of the DFIM,  $\Omega$  is mechanical speed,  $\omega_s$  is the stator pulsation,  $\omega$  is the rotor pulsation,  $f$  is the friction coefficient,  $T_s$  and  $T_r$  are statoric and rotoric time-constant.

### 3. Stator flux estimator

For the DFOC of DFIM, accurate knowledge of the magnitude and position of the stator flux vector is necessary. In a DFIM motor mode, as stator and rotor currents are measurable, the stator flux can be estimated (calculated). The flux estimator can be obtained by the following equations [1]:

$$\phi_{sd} = L_s i_{sd} + M i_{rd} \quad (9)$$

$$\phi_{sq} = L_s i_{sq} + M i_{rq} \quad (10)$$

The position stator flux is calculated by the following equations:

$$\theta_r = \theta_s - \theta \quad (11)$$

In which:

$$\theta_s = \int \omega_s dt, \theta = \int \omega dt, \omega = P \Omega.$$

Where:

$\theta_s$  is the electrical stator position,

$\theta$  is the electrical rotor position.

### 4. Sliding mode control

A Sliding Mode Controller (SMC) is a Variable Structure Controller (VSC). Basically, a VSC includes several different continuous functions that can map plant state to a control surface, whereas switching among different functions is determined by plant state represented by a switching function [6].

The design of the control system will be demonstrated for a following nonlinear system [2]:

$$\dot{x} = f(x, t) + B(x, t) \cdot u(x, t) \quad (12)$$

(5) Where  $x \in \mathbb{R}^n$  is the state vector,  $u \in \mathbb{R}^m$  is the control vector,  $f(x, t) \in \mathbb{R}^n$ ,  $B(x, t) \in \mathbb{R}^{n \times m}$ .

From the system (12), it possible to define a set  $S$  of the state trajectories  $x$  such as:

$$S = \{x(t) \mid \sigma(x, t) = 0\} \quad (13)$$

Where:

$$\sigma(x, t) = [\sigma_1(x, t), \sigma_2(x, t), \dots, \sigma_m(x, t)]^T \quad (14)$$

and  $[\cdot]^T$  denotes the transposed vector,  $S$  is called the sliding surface.

To bring the state variable to the sliding surfaces, the following two conditions have to be satisfied:

$$\sigma(x, t) = 0, \quad \dot{\sigma}(x, t) = 0 \quad (15)$$

The control law satisfies the precedent conditions is presented in the following form:

$$u = u^{eq} + u^n \quad (16)$$

$$u^n = -k_f \operatorname{sgn}(\sigma(x, t))$$

Where  $u$  is the control vector,  $u^{eq}$  is the equivalent control vector,  $u^n$  is the switching part of the control (the correction factor),  $k_f$  is the controller gain.  $u^{eq}$  can be obtained by considering the condition for the sliding regimen,  $\sigma(x, t) = 0$ . The equivalent control keeps the state variable on sliding surface, once they reach it. For a defined function  $\varphi$  [7], [8]:

$$\operatorname{sgn}(\varphi) = \begin{cases} 1, & \text{if } \varphi > 0 \\ 0, & \text{if } \varphi = 0 \\ -1, & \text{if } \varphi < 0 \end{cases} \quad (17)$$

The controller described by the equation (16) presents high robustness, insensitive to parameter fluctuations and disturbances, but it will have high-frequency switching (chattering phenomena) near the sliding surface due to  $\operatorname{sgn}$  function involved. These drastic changes of input can be avoided by introducing a boundary layer with width  $\varepsilon$  [9]. Thus replacing  $\operatorname{sgn}(\sigma(t))$  by  $\operatorname{sat}(\sigma(t)/\varepsilon)$  (saturation function), in (16), we have

$$u = u^{eq} - K_f \operatorname{sat}(\sigma(x, t)) \quad (18)$$

Where

$$\varepsilon > 0$$

$$\operatorname{sat}(\varphi) = \begin{cases} \operatorname{sgn}(\varphi), & \text{if } |\varphi| \geq 1 \\ \varphi, & \text{if } |\varphi| < 1 \end{cases} \quad (19)$$

Consider a Lyapunov function [7]:

$$V = \frac{1}{2} \sigma^2 \quad (20)$$

From Lyapunov theorem we know that if  $\dot{V}$  is negative definite, the system trajectory will be driven and attracted

toward the sliding surface and remain sliding on it until the origin is reached asymptotically [9]:

$$\dot{V} = \frac{1}{2} \frac{d}{dt} \sigma^2 = \sigma \dot{\sigma} \leq -\eta |\sigma| \quad (21)$$

Where  $\eta$  is a strictly positive constant.

In this paper, we use the sliding surface proposed par J.J. Slotine,

$$\sigma(x, t) = \left( \frac{d}{dt} + \lambda \right)^{n-1} e \quad (22)$$

Where

$x = [x, \dot{x}, \dots, x^{n-1}]^T$  is the state vector,  $x^d = [x^d, \dot{x}^d, \dots, x^{d,n-1}]^T$  is the desired state vector,  $e = x^d - x = [e, \dot{e}, \dots, e^{n-1}]$  is the error vector, and  $\lambda$  is a positive coefficient, and  $n$  is the system order.

Commonly, in DFIM control using sliding mode theory, the surfaces are chosen as functions of the error between the reference input signal and the measured signals [2].

#### 4.1. Speed control

The speed error is defined by:

$$e = \Omega_{ref} - \Omega \quad (23)$$

For  $n=1$ , the speed control manifold equation can be obtained from equation (22) as follow:

$$\sigma(\Omega) = e = \Omega_{ref} - \Omega \quad (24)$$

$$\dot{\sigma}(\Omega) = \dot{\Omega}_{ref} - \dot{\Omega} \quad (25)$$

Substituting the expression of  $\dot{\Omega}$  equation (8) in equation (25), we obtain:

$$\dot{\sigma}(\Omega) = \dot{\Omega}_{ref} - \left( -\frac{P.M}{J.L_s} (i_{rq} \cdot \phi_{sd}) - \frac{C_r}{J} - \frac{f}{J} \Omega \right) \quad (26)$$

We take:

$$i_{rq} = i_{rq}^{eq} + i_{rq}^n \quad (27)$$

During the sliding mode and in permanent regime, we have:

$$\sigma(\Omega) = 0, \dot{\sigma}(\Omega) = 0, i_{rq}^n = 0$$

Where the equivalent control is:

$$i_{rq}^{eq} = -\frac{J.L_s}{P.M \cdot \phi_{sd}} \left( \dot{\Omega}_{ref} + \frac{C_r}{J} + \frac{f}{J} \Omega \right) \quad (28)$$

Therefore, the correction factor is given by:

$$i_{rq}^n = K i_{rq} \text{sat}(\sigma(\Omega)) \quad (29)$$

$K i_{rq}$  : negative constant.

#### 4.2. Stator flux control

In the proposed control, the manifold equation can be obtained by:

$$\sigma(\phi_{sd}) = \phi_{sd}^{ref} - \phi_{sd} \quad (30)$$

$$\dot{\sigma}(\phi_{sd}) = \dot{\phi}_{sd}^{ref} - \dot{\phi}_{sd} \quad (31)$$

Substituting the expression of  $\dot{\phi}_{sd}$  equation (6) in equation (31), we obtain:

$$\dot{\sigma}(\phi_{sd}) = \dot{\phi}_{sd}^{ref} - \left( V_{sd} + \frac{M}{T_s} i_{rd} - \frac{1}{T_s} \phi_{sd} \right) \quad (32)$$

The control current  $i_{rd}$  is defined by:

$$i_{rd} = i_{rd}^{eq} + i_{rd}^n \quad (33)$$

During the sliding mode and in permanent regime, we have:

$$\sigma(\phi_{sd}) = 0, \dot{\sigma}(\phi_{sd}) = 0, i_{rd}^n = 0$$

The equivalent control is:

$$i_{rd}^n = \left( \dot{\phi}_{sd}^{ref} - V_{sd} + \frac{1}{T_s} \phi_{sd} \right) \frac{T_s}{M} \quad (34)$$

Where, the correction factor is given by:

$$i_{rd}^n = K i_{rd} \text{sat}(\sigma(\phi_{sd})) \quad (35)$$

$K i_{rd}$  : positive constant.

#### 4.3. Rotor direct current control and limitation

In order to limit all possible overshoot of the current  $i_{rd}$ , we add a limiter of current defined by:

$$i_{rd}^{\lim} = i_{rd}^{\max} \text{sat}(i_{rd}) \quad (36)$$

The direct current error is defined by:

$$e = i_{rd}^{\lim} - i_{rd} \quad (37)$$

For  $n=1$ , the direct current control manifold equation can be obtained by:

$$\sigma(i_{rd}) = i_{rd}^{\lim} - i_{rd} \quad (38)$$

$$\dot{\sigma}(i_{rd}) = \dot{i}_{rd}^{\lim} - \dot{i}_{rd} \quad (39)$$

Substituting the expression of  $\dot{i}_{rd}$  equation (4) in equation (39), we obtain:

$$\dot{\sigma}(i_{rd}) = \dot{i}_{rd}^{\lim} - \left( -\frac{1}{\sigma} \left( \frac{1}{T_r} + \frac{M^2}{L_s T_s L_r} \right) i_{rd} - \frac{M}{\sigma L_r L_s} V_{sd} \right. \quad (40)$$

$$\left. + \frac{M}{\sigma L_r L_s T_s} \phi_{sd} + (\omega_s - \omega) i_{rq} + \frac{1}{\sigma L_r} V_{rd} \right)$$

The control voltage  $V_{rd}^{ref}$  is defined by

$$V_{rd}^{ref} = V_{rd}^{eq} + V_{rd}^n \quad (41)$$

During the sliding mode and in permanent regime, we have:

$$\sigma(i_{rd}) = 0, \dot{\sigma}(i_{rd}) = 0, V_{rd}^n = 0$$

Where the equivalent control is

$$V_{rd}^{eq} = \left( \dot{i}_{rd}^{\lim} + \frac{1}{\sigma} \left( \frac{1}{T_r} + \frac{M^2}{L_s T_s L_r} \right) i_{rd} + \frac{M}{\sigma L_r L_s} V_{sd} \right. \quad (42)$$

$$\left. - \frac{M}{\sigma L_r L_s T_s} \phi_{sd} - (\omega_s - \omega) i_{rq} \right) \sigma L_r$$

Substituting (1) in (42) gives

$$V_{rd}^{eq} = \left( \lim_{\sigma} \left( i_{rd} + \frac{1}{\sigma L_r} i_{rd} + \frac{M}{\sigma L_r L_s} V_{sd} - (\omega_s - \omega) i_{rq} \right) \sigma L_r \right) \quad (43)$$

Therefore, the correction factor is given by:

$$V_{rd}^n = K_{V_{rd}} \text{sat}(\sigma(i_{rd})) \quad (44)$$

$K_{V_{rd}}$  : positive constant.

#### 4.4. Rotor quadrature current control and limitation

In order to limit all possible overshoot of the current  $i_{rq}$ , we add a limiter of current defined by:

$$i_{rq}^{\lim} = i_{rq}^{\max} \text{sat}(i_{rq}) \quad (45)$$

The current control manifold is

$$\sigma(i_{rq}) = i_{rq}^{\lim} - i_{rq} \quad (46)$$

$$\dot{\sigma}(i_{rq}) = \dot{i}_{rq}^{\lim} - \dot{i}_{rq} \quad (47)$$

Substituting the expression of  $\dot{i}_{rd}$  equation (5) in equation (47), we obtain:

$$\dot{\sigma}(i_{rq}) = \dot{i}_{rq}^{\lim} - \left( -\frac{1}{\sigma} \left( \frac{1}{T_r} + \frac{M^2}{L_s T_s L_r} \right) i_{rq} - \frac{M}{\sigma L_r L_s} V_{sq} \right. \quad (48)$$

$$\left. + \frac{M}{\sigma L_r L_s} \omega_s \phi_{sd} - (\omega_s - \omega) i_{rd} + \frac{1}{\sigma L_r} V_{rq} \right)$$

The control voltage  $V_{rq}^{ref}$  is defined by

$$V_{rq}^{ref} = V_{rq}^{eq} + V_{rq}^n \quad (49)$$

During the sliding mode and in permanent regime, we have:

$$\sigma(i_{rq}) = 0, \dot{\sigma}(i_{rq}) = 0, V_{rq}^n = 0$$

Where the equivalent control is

$$V_{rq}^{eq} = \left( \lim_{\sigma} \left( i_{rq} + \frac{1}{\sigma} \left( \frac{1}{T_r} + \frac{M^2}{L_s T_s L_r} \right) i_{rq} + \frac{M}{\sigma L_r L_s} V_{sq} \right) \right)$$

(50)

$$- \frac{M}{\sigma L_r L_s} \omega_s \phi_{sd} + (\omega_s - \omega) i_{rq} \bigg) \sigma L_r$$

Therefore, the correction factor is given by:

$$V_{rq}^n = K_{V_{rq}} \text{sat}(\sigma(i_{rq})) \quad (51)$$

$K_{V_{rq}}$  : positive constant.

## 5. Simulation results

### 5.1. Description of the system

The control scheme for DFIM using the sliding mode controllers is presented in figure 1, the blocks SMC1, SMC2, SMC3 and SMC4 represent the proposed sliding mode controllers. The block limiter limits the current within the limits values. The block 'Coordinate transform' makes the conversion between the synchronously rotating and stationary reference frame. The block 'PWM Inverter' shows the control by technique PWM whose is realized for the inverter control, which feeds the rotor through a converter.

The estimator-block represents respectively the estimated stator and rotor current and the stator flux. The block 'DFIM' represents the doubly fed induction motor. The DFIM used in this work is a 0.8 kW, whose rated data of the simulated doubly fed induction motor: 0.8 KW; 220/380 V-50 Hz; 3.8/2.2 A, 1420 rpm  $R_s = 11.98 \Omega$ ,  $R_r = 0.904 \Omega$ ,  $L_s = 0.414 \text{ H}$ ,  $L_r = 0.0556 \text{ H}$ ,  $M = 0.126 \text{ H}$ ,  $P = 2$ ,  $J = 0.01 \text{ Kg.m}^2$ ,  $f = 0.00 \text{ I.S}$ .

### 5.2. Test results

The speed and flux regulation performance of the proposed sliding mode is checked in terms of load torque variations.

In the following study, the variations of the motor parameters is confined to load change and speed reference reversal only, while the other parameters, e.g., friction coefficient  $f$ , mutual inductance  $M$  and stator inductance  $L_s$  are held constant.

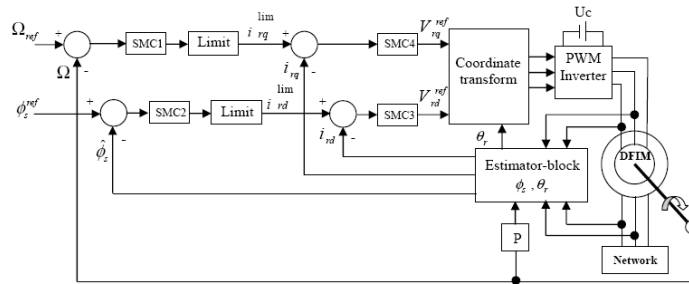


Fig. 1. Control scheme for DFIM using the sliding mode controllers

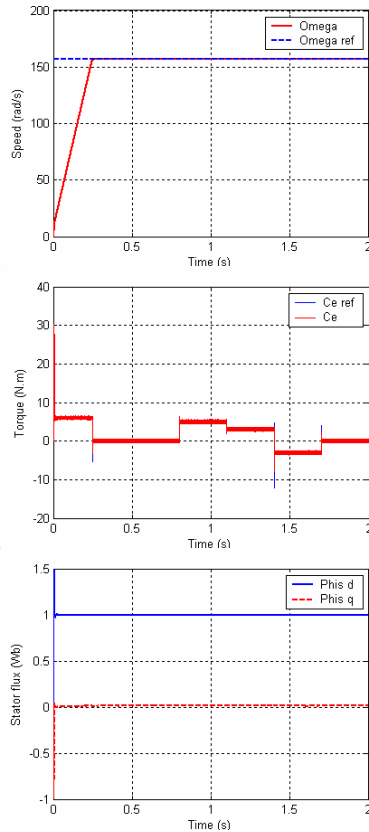
### A. Load variation

In the first test, a cyclic change of different load torque levels are subjected to the machine at certain times and as followings:

Time = [0 0.8 0.8 1.1 1.1 1.4 1.4 1.7 1.7];

Torque = [0 0 5 5 3 3 -3 -3 0];

The speed and stator flux responses of figure 2 shows no detected change, without static error and the perturbation reject is instantaneous in the speed also in stator flux at these changes of loads.

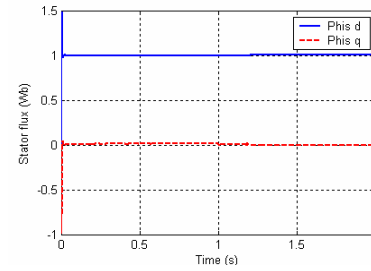
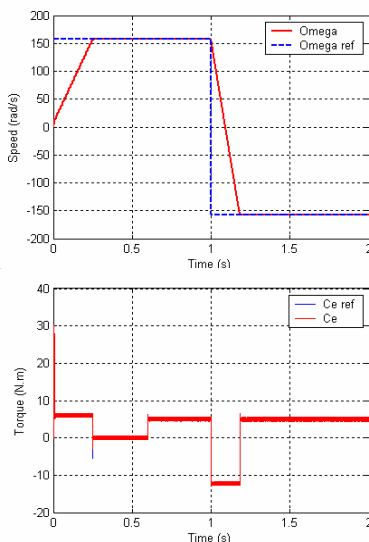


**Fig. 2.** Speed, Torque, Stator Flux obtained with a SMC under load variation.

## B. Speed reference reversal of rated value

In the second test, speed reference reversal of (157, -157 rad/s), with a load torque of 5N.m applied at  $t = 0.6$  s.

Figure 3 shows the speed, torque, stator flux, the speed follows the reference variable without overshooting. The stator flux almost insensitive during the reversal speed and is given better performance.



**Fig. 3.** Speed, Torque, Stator Flux obtained with a SMC under reversal speed

## 6. Conclusion

The field-oriented control using a sliding mode was employed to obtain the better performance from the DFIM motor mode in a speed and flux control. The robustness quality of the proposed controllers appears clearly in the test results by load variation and speed reference reversal. Finally, the proposed controller provides drive robustness improvement in direct field-oriented control of doubly fed induction machine (DFIM) motor mode.

## 7. References

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