Probability of Small-Signal Stability of Power Systems in the Presence of Communication Delays

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Abstract

This paper presents a probabilistic approach to evaluate the small-signal stability of power systems in the presence of communication delays. An exact method is first proposed to determine the relationship between delay margin and system parameters such as the system load. The delay margin is then modeled as a random variable and the probability density function (PDF) of the delay margin is determined based on the PDF of the load using a Monte Carlo simulation approach. The communication delays are assumed to be uniformly distributed in a practical range and the probability of system being small-signal stable for a given time delay is determined using the estimated PDF of the delay margin. The proposed method is applied to a single-machine-infinite bus (SMIB) power system with an exciter.

1. Introduction

A significant amount of time delays has been observed in power systems during the use of phasor measurement units (PMU) and various communication links in wide-area measurement/monitoring systems (WAMS). The measurement and communication delays involved between the instant of measurement and that of signal being available to the controller can typically be in the range of 100-700 msec depending on the type of the communication link [1, 2].

The inevitable large time delays in power systems may have a destabilizing impact on the system dynamics and lead to unacceptable performance such as loss of synchronism and instability. Therefore, stability analysis and controller design methods must take into account time delays and practical tools should be developed to study the complicated dynamic behavior of time-delayed power systems. The previous studies on the dynamics of time-delayed power systems have mainly focused on the following issues: i) To investigate the time delay influence on the controller design for power system stabilizers (PSS) [3], for load frequency control (LFC) or automatic generation control (AGC) [4, 5], ii) To eliminate periodic and chaotic oscillations in power systems by applying time-delayed feedback control [6]; iv) To analyze the effect of time delay on small-signal stability [7-9].

There exists little work on the small-signal stability of time-delayed power systems. In [7], a method to determine the delay margin, which was first presented in [10] for analyzing the stability of the time-delayed linear time-invariant systems based on Rekasius substitution [11], was applied to the SMIB power system to estimate delay margin. The impact of various system parameters on the delay margin was also investigated in [7]. An optimal-based algorithm was proposed in [8] to trace boundaries of small-signal stability region in the presence of communication delays.

The small-signal stability analysis of time delayed power systems may be viewed as a probabilistic rather than a deterministic problem. The need for application of probabilistic techniques for small-signal stability is motivated by the random nature of the steady-state operating conditions of the system and communication delays. The steady-state operating condition strongly depends on load that is a random process, and the delay margin that defines the stability boundary is a function of the system load as well as other parameters. Therefore, the delay margin may fluctuate randomly in a certain range and it should be regarded as a random variable. The size of communication delays in WAMS mainly depends on the physical media of communication as well as several other factors such as the phasor packet size, transmission protocol employed and communication network load (congested or idle). As a result, a system being small-signal stable for a given time delay and operating condition is a random event. The deterministic approaches presented in [7-9], which have not taken into account the probabilistic aspect of the stability problem, provide a limited description of the system stability characteristics. Therefore, there is a strong need for a probabilistic approach to evaluate the small-signal stability of power systems in the presence of time delays, and to determine the probability of stability for a given steady-state operating condition.

This paper proposes a probabilistic approach to determine the probability of stability, a stability index, for a given steady-state operating condition by considering the random nature of both system load and communication delays. Firstly, an analytical formula to compute the delay margin is developed based on an exact and direct method presented in [9, 12]. Secondly, a Monte Carlo simulation approach is used to estimate the PDF of delay margin in terms of the PDF of system load. Using the estimated PDF of the delay margin, the probability of system being small-signal stable (stability index) for a given time delay is determined. Such a stability index could be used to measure the degree of stability of the system for a given steady-state operating condition and time delay. The proposed method is applied to a SMIB power system with an exciter. The effect of system load on the delay margin and on the probability of stability is investigated.

2. Small-Signal Stability with Time Delay

When a time delay is observed in the system, power system dynamics should be described by the following time-delayed differential-algebraic equation (DAE) model [7-9]:

$$\begin{cases} \dot{x} = f(x, y, x_\tau, y_\tau, \beta) \\ 0 = g(x, y, \beta) \\ 0 = g(x_\tau, y_\tau, \beta) \end{cases} \quad (1)$$

where $x \in \Re^n$ and $x_\tau = x(t-\tau) \in \Re^n$ are the vectors of delay-free and time-delayed state variables, respectively such as rotor angles and control states of exciter and speed governor. $y \in \Re^m$ and $y_\tau = y(t-\tau) \in \Re^m$ are the vectors of delay-free and time-delayed algebraic variables, respectively such as voltage magnitude and phase angles at the load buses; $\tau > 0$ is the constant time delay observed in the system, and $\beta \in \Re^k$ is the vector of parameters such as real/reactive power demand at the buses, transmission line parameters, and control set points and gains. The dynamics of generators, control devices (exciter, speed governor, stabilizer) and load dynamics together define the set of differential equations. The algebraic equations are the power flow equations representing real and reactive power balances at the load buses.

The small-signal stability is the ability of the power system to maintain synchronism under small disturbances that occur continually on the system because of small variations in loads and generation. The disturbances are considered sufficiently small for linearization of system equations around an equilibrium point to be permissible for the purpose of stability analysis. By linearizing (1) at an equilibrium point (x_0, y_0), we can easily obtain the following incremental DAE:

$$\begin{aligned} \Delta\dot{x} &= [A_0(\beta)]\Delta x + [A_\tau(\beta)]\Delta x_\tau \\ &\quad + [B_0(\beta)]\Delta y + [B_\tau(\beta)]\Delta y_\tau \\ 0 &= [C_0(\beta)]\Delta x + [D_0(\beta)]\Delta y \\ 0 &= [C_\tau(\beta)]\Delta x_\tau + [D_\tau(\beta)]\Delta y_\tau \end{aligned} \quad (2)$$

where

$$\begin{aligned} &A_0(\beta) = \left.\frac{\partial f}{\partial x}\right|_\beta ; \; B_0(\beta) = \left.\frac{\partial f}{\partial y}\right|_\beta ; \; C_0(\beta) = \left.\frac{\partial g}{\partial x}\right|_\beta ; \\ &D_0(\beta) = \left.\frac{\partial g}{\partial y}\right|_\beta ; A_\tau(\beta) = \left.\frac{\partial f}{\partial x_\tau}\right|_\beta ; \; B_\tau(\beta) = \left.\frac{\partial f}{\partial y_\tau}\right|_\beta ; \\ &C_\tau(\beta) = \left.\frac{\partial g}{\partial x_\tau}\right|_\beta ; \; D_\tau(\beta) = \left.\frac{\partial g}{\partial y_\tau}\right|_\beta \end{aligned} \quad (3)$$

are the Jacobian matrices with respect to the state variables and the time-delayed state variables evaluated at the equilibrium point (x_0, y_0). When the algebraic Jacobian matrices D_0, D_τ are non-singular, the incremental DAE of (1) could be reduced to a set of incremental ordinary differential equations (ODEs), and local dynamics in the neighborhood of the equilibrium point could be investigated by the time-delayed ODEs:

$$\Delta\dot{x}(t) = \left[\tilde{A}_0(\beta)\right]\Delta x(t) + \left[\tilde{A}_\tau(\beta)\right]\Delta x_\tau(t-\tau) \quad (4)$$

where

$$\left[\tilde{A}_i(\beta)\right] = [A_i(\beta)] - [B_i(\beta)][D_i(\beta)]^{-1}[C_i(\beta)]; \; i = 0, \tau$$

The stability of time-delayed system given in (4) is determined by the location of system eigenvalues that can be obtained from the following characteristic equation:

$$\Delta(s,\tau) = det\left[sI - \tilde{A}_0(\beta) - \tilde{A}_\tau(\beta)e^{-s\tau}\right] = P(s) + Q(s)e^{-s\tau} = 0 \quad (5)$$

where $P(s)$, $Q(s)$ are polynomials in s with real coefficients determined by the elements of $\left[\tilde{A}_0\right]$ and $\left[\tilde{A}_\tau\right]$ matrices. It is obvious that the roots of (5) are a function of the time delay τ. Let's denote these roots by $s^\tau = \left[s_1^\tau, s_2^\tau, \ldots, s_n^\tau\right]$. Similar to the delay-free system, if the following condition is hold, then the system is small-signal stable.

$$max\left(real\left(s_i^\tau\right)\right) < 0 \text{ for } \forall s_i^\tau \in s^\tau \quad (6)$$

In other words, if all the roots are in the negative half part of the complex plane, the system is small-signal stable.

3. Delay Margin Computation

The main goal of the stability studies of delayed systems is to compute the delay margin for stability. As with the delay-free system (i.e., $\tau = 0$), the stability of the time-delayed system of (4) depends on the locations of roots the characteristic equation of (5). A necessary and sufficient condition for the system to be asymptotically stable is that all the roots of the characteristic equation of (5) lie in the left half of the complex plane. As seen in (5), there exists an exponential type transcendental term in the characteristic equation. The transcendentality brings infinitely many characteristic roots, which makes the stability problem a complex task. However, the delay margin problem is to find values τ_c for which the characteristic equation of (5) has roots (if any) on the imaginary axis. Clearly, $\Delta(s,\tau) = 0$ is an implicit function of s and τ which may, or may not, cross the imaginary axis. Assume for simplicity that $\Delta(s,0) = 0$ has all its roots in the left half-plane. That is, the delay-free system is stable. If for some τ_c, $\Delta(s,\tau_c) = 0$ has root on the imaginary axis at $s = j\omega_c$ (where subscript c refers to "crossing" the imaginary axis), so does $\Delta(-s,\tau_c) = 0$, for the same value of τ_c and ω_c. Hence, looking for roots on the imaginary axis reduces to finding values of τ_c for which $\Delta(s,\tau_c) = 0$ and $\Delta(-s,\tau_c) = 0$ have a common root. That is,

$$\begin{aligned} P(s) + Q(s)e^{-s\tau_c} &= 0 \\ P(-s) + Q(-s)e^{s\tau_c} &= 0 \end{aligned} \quad (7)$$

By eliminating the exponential term in (7), we get the following polynomial:

$$P(s)P(-s) + Q(s)Q(-s) = 0 \quad (8)$$

If we replace s by $j\omega_c$ in (8), we have the following polynomial in ω_c^2:

$$W(\omega_c^2) = P(j\omega_c)P(-j\omega_c) - Q(j\omega_c)Q(-j\omega_c) = 0 \quad (9)$$

Please note that *n*th degree transcendental characteristic equation with delay given in (5) is now converted into a *2n*-degree polynomial without transcendentality given by (9) and its real roots coincide with the imaginary roots of (5) exactly. It must be noted that these real roots, if exist, will be dependent on the system parameters β implicitly since the coefficients of $P(s)$ and $Q(s)$ of (5) are functions of the system parameters β. Depending on the roots of (9), the following situation may occur:

i) The polynomial of (9) does not have any positive real roots, which implies that the characteristic equation of (5) does not have any roots on the $j\omega$ -axis. In that case, the system is stable for all $\tau \geq 0$, indicating that the system is *delay-independent stable*.

ii) The polynomial of (9) has at least one positive real root, which implies that the characteristic equation of (5) has at least a pair complex eigenvalues on the $j\omega$ -axis. In that case, the system is *delay-dependent stable*.

For a positive real root ω_c, the corresponding value of delay margin τ_c can be easily obtained using (7) [9]:

$$\tau_c = \frac{1}{\omega_c} Tan^{-1}\left(\frac{Im\left\{ \frac{P(j\omega_c)}{Q(j\omega_c)} \right\}}{Re\left\{ -\frac{P(j\omega_c)}{Q(j\omega_c)} \right\}} \right) + \frac{2r\pi}{\omega_c}; \quad (10)$$
$$r = 0,1,2,\ldots,\infty$$

4. Probability of Stability

Time delays and delay margin for an operating point are probabilistic in nature. The size of communication delays in WAMS mainly depends on the physical media of communication (i.e., fiber-optic cables, digital microwave links, power lines, telephone lines and satellite links) as well as several other factors such as the size phasor packet, transmission protocol employed and communication network load (congested or idle). The experimental results show that communications delays range from milliseconds to several 100 msec, indicating its randomness. Similarly, the delay margin may fluctuate randomly in a certain range since it is a function of system parameters that continuously change in daily operation of the system. Note that the analytical expression for the delay margin given by (10) indicates that the delay margin is determined by the coefficients of the polynomials $P(s)$, $Q(s)$ and by the positive real root ω_c of the polynomial $W(\omega_c^2)$ given in (9). These coefficients and the root depend on the steady-state condition and system parameters such as system load level, transmission line and generator reactances, generator damping, and controller parameters. Therefore, the delay margin τ_c will be a function of system parameters and could be synthetically described as

$$\tau_c = F(p_i(\beta), q_j(\beta), \omega_c(\beta), \beta) \quad (11)$$

where p_i and q_j represent the coefficients of $P(s)$ and $Q(s)$, respectively.

Among system parameters, the system load level heavily affects the delay margin and it is probabilistic in nature since it keeps changing in actual operation of power systems. Therefore, deterministic approaches as to determine delay margin do not reflect the true characteristics of the system stability boundary, and the delay margin should be considered as a random variable in order to take into account of the random nature of the system load. Based on this argument, a system being small-signal stable for a given time delay and operating point is a random event. The information on the probability of this random event is crucial for power system planning studies and safe operation. Under the assumption that the probability density function (PDF) of the delay margin is known, the probability of the system being small-signal stable can be determined by

$$P\{stability\} = P\{\tau < \tau_c\} \quad (12)$$

This probability value is used as a stability index to measure the degree of small-signal stability of the system for a given loading condition and time delay.

There exist two main methods in the literature for probabilistic assessment of the stability, namely conditional probability and Monte Carlo simulation approaches. The delay margin is a complicated function of the system load as well as other parameters including voltages, reactances, generator parameters and controller gains, etc. In other words, the function that relates the delay margin to the system load is not analytically invertible. Therefore, the PDF of the delay margin cannot be obtained analytically in a closed-form expression using conditional probability method. In such cases, Monte Carlo simulation method should be used to estimate the PDF of the delay margin and thus, to compute the probability of small-signal stability. In this paper, the Monte Carlo method is adopted to estimate the PDF of the delay margin.

The Monte Carlo simulation approach applied into the small-signal stability consists of three parts. The first part simulates the occurrence of the uncertainties in time delay τ and in the system load. A random number generator is employed to generate random samples for the time delay and the system load from their corresponding distributions. The second part is to compute the corresponding random samples of delay margins using the analytical relation of (10) or (11) based on every load value that is generated in the first part and determines the estimated PDF of the delay margin τ_c using a histogram. In the third part, based on the simulated values of τ and τ_c, an estimate of probability of stability is computed. In order to estimate the probability of stability, a random sample $\tau_c = \{\tau_{c1}, \tau_{c2}, \ldots, \tau_{cN}\}$ from the delay margin, and a random sample $\tau = \{\tau_1, \tau_2, \ldots, \tau_N\}$ from the time delay have been simulated. Given the two independent samples, an estimate of the probability of stability is then obtained as follows:

$$P\{stablity\} = P\{\tau \leq \tau_c\} = \lim_{N \to \infty} \frac{N_s}{N^2} \quad (13)$$

where N is the total number of Monte Carlo trials and N_s is the number of couples ($\tau_{ci}, \tau_{j;}\ i, j = 1, \ldots, N$) for which $\tau_j \leq \tau_{ci}$.

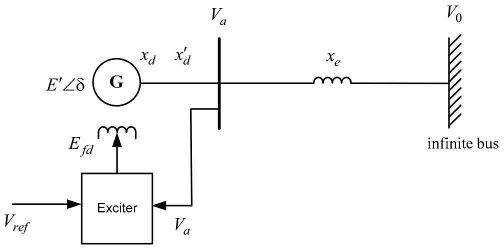

Fig. 1. Single-machine-infinite-bus power system with an exciter

5. SMIB System Application

The SMIB system with an exciter whose one-line diagram given in Fig. 1 is used to illustrate the application of the proposed method to stability analysis of the delayed power systems. Nominal values of system parameters are given in [7]. The dynamics of the SMIB system can be described by the following set of differential equations. It is supposed that there exists a time delay in the measurement of generator terminal voltage $V_a(t)$ [7].

$$\begin{aligned}
\dot{\delta} &= \omega_s \omega \\
\dot{\omega} &= -\frac{D}{M}\omega + \frac{1}{M}(P_m - P_G) \\
\dot{E}' &= -\frac{1}{T'_{do}}E' - \frac{1}{T'_{do}}(x_d - x'_d)I_d + \frac{1}{T'_{do}}E_{fd} \\
\dot{E}_{fd} &= -\frac{K_A}{T_A}(V_a(t-\tau) - V_{ref}) - \frac{1}{T_A}(E_{fd} - E_{fd0})
\end{aligned} \tag{14}$$

where the electrical output power, generator terminal voltage and the direct-axis current are given as follows:

$$\begin{aligned}
&P_G = \frac{E'V_0 \sin\delta}{x_e + x'_d};\ V_a = \frac{\sqrt{(x'_d + x_e E'\cos\delta)^2 + (x_e E'\sin\delta)^2}}{x_e + x'_d}; \\
&I_d = \frac{E' - V_0\cos\delta}{x_e + x'_d}
\end{aligned} \tag{15}$$

The meaning of symbols in (14) and (15) are given in [7, 9]. We can easily rewrite (14) into the general form given by (1) by defining the dynamic and algebraic state variables as follows: $x = \begin{bmatrix}\delta & \omega & E' & E_{fd}\end{bmatrix}^T$ $y = \begin{bmatrix}P_G & V_a & I_d\end{bmatrix}^T$ and $y(t-\tau) = \begin{bmatrix}V_a(t-\tau\end{bmatrix}$. The following are the parameters that will have an impact on the delay margin and probability of stability: Generator mechanical input power P_m, generator damping D, generator transient reactance x'_d, transmission line reactance x_e, the exciter gain K_A. The parameter vector is $\beta = \begin{bmatrix}P_m & D & x'_d & x_e & K_A\end{bmatrix}^T$. Linearization of system equations of (14) at an equilibrium point (x_0, y_0) for a fixed system parameter β_0 results in the following time-delayed state-space equation

$$\Delta\dot{x}(t) = \begin{bmatrix}\tilde{A}_0(\beta)\end{bmatrix}\Delta x(t) + \begin{bmatrix}\tilde{A}_\tau(\beta)\end{bmatrix}\Delta x_\tau(t-\tau) \tag{16}$$

where the reduced system matrices are as follows

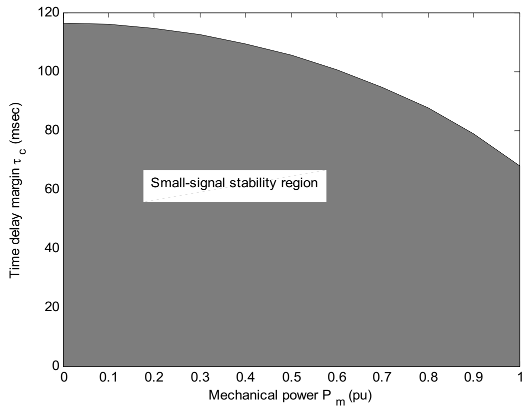

Fig. 2. Variation of delay margin with respect to mechanical power

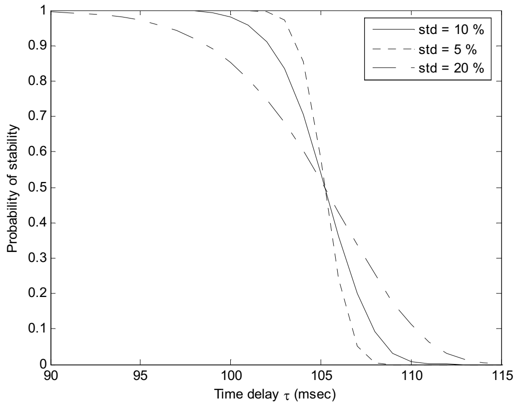

Fig. 3. Effect of load standard deviation on the probability of stability

$$\tilde{A}_0 = \begin{bmatrix} 0 & \omega_s & 0 & 0 \\ -\frac{K_1}{M} & -\frac{D}{M} & -\frac{K_2}{M} & 0 \\ -\frac{K_3}{T'_{d0}} & 0 & -\frac{K_4}{T'_{d0}} & \frac{1}{T'_{d0}} \\ 0 & 0 & 0 & -\frac{1}{T_A} \end{bmatrix};\ \tilde{A}_\tau = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{K_5}{T_A} & 0 & -\frac{K_6}{T_A} & 0 \end{bmatrix}$$

All the coefficients of the reduced system matrices given above could be found in [9]. The characteristic equation of the linearized system of (16) can be easily obtained using (5). The characteristic equation has the following form:

$$\begin{aligned}
\Delta(s,\tau) &= (s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0) \\
&\quad + (q_2 s^2 + q_1 s + q_0)e^{-s\tau} = 0
\end{aligned} \tag{17}$$

The coefficients p_i, q_i of the polynomials $P(s), Q(s)$ in terms of system parameters are given in [9]. Substituting $P(s)$ and $Q(s)$ polynomials into (9) and we obtain an 8[th] order polynomial as follows:

$$W(\omega^2) = \omega^8 + (p_3^2 - 2p_2)\omega^6 + (p_2^2 + 2p_0 - 2p_1p_3 - q_2^2)\omega^4 + (p_1^2 - 2p_0p_2 - q_1^2 + 2q_0q_2)\omega^2 + p_0^2 - q_0^2 = 0 \quad (18)$$

Once the positive real roots of this polynomial (ω_c) are obtained, the delay margin for each root can be determined by the following expression that is obtained by substituting $P(s = j\omega_c)$ and $Q(s = j\omega_c)$ polynomials given in (17) into (10):

$$\tau_c = \frac{1}{\omega_c} Tan^{-1}\left(\frac{t_5\omega_c^{\,5} + t_3\omega_c^{\,3} + t_1\omega_c}{t_6\omega_c^{\,6} + t_4\omega_c^{\,4} + t_2\omega_c^{\,2} + t_0} \right) + \frac{2r\pi}{\omega_c}; \quad r = 0,1,2,\ldots,\infty \quad (19)$$

where the coefficients $t_0, t_1, t_2, t_3, t_4, t_5$ and t_6 are real-valued and could be found in [9].

6. Results

6.1. Delay Margins

In this section, we compute the delay margin for a wide range of system load to investigate its effects on the delay margin. P_m value is varied in the range of $P_m = 0 - 1.0\,pu$ while other system parameters are kept constant at their nominal values. Using the proposed method, the delay margins τ_c are computed. Fig. 2 shows the variation of the delay margin with respect to P_m. It is clear that the delay margin decreases as P_m increases. The area under the curve defines small-signal stability region of the system.

6.2. Probability of Stability

This section presents results on the probability of stability and the effects of changes in SMIB system parameters on the probability of stability. Recall that the parameters of interest are $\beta = \begin{bmatrix} P_m & D & x'_d & x_e & K_A \end{bmatrix}^T$. Among these parameters, the system load P_m is assumed to be normally distributed with a mean value of $P_{m0} = 0.5\,pu$ (base case) and a standard deviation of 10% of its mean value while the rest of parameters are first kept unchanged at their nominal value. Similarly, time delays τ are assumed to be uniformly distributed around a mean value $\tau_{mean} = 105$ msec with a deviation of $\Delta\tau = 25$ msec. The mean value of the delay is selected as the delay margin τ_c computed for the mean value of the load $P_{m0} = 0.5\,pu$. To estimate the PDF of the delay margin, Monte Carlo simulation has been performed with $N = 10^4$ independent random samples of the system load and time delay, so obtaining 10^8 independent couples of the load and time delay for computation of the probability of stability using (13). This number assures a very reliable estimate of the probability of stability.

Fig. 3 shows the probability of stability at different values of the time delay τ. The probabilities of stability are given for three different standard deviations of the system load, namely 5 %, 10 % (base case) and 20 % of its mean value in order to investigate the effect of uncertainty associated with load on the probability of stability. Note that when the time delay is less than its mean value ($\tau \le 105$ *msec*), the probability of stability decreases with an increase in the uncertainty associated with the load. On the other hand, the probability of stability increases when the time delay is larger than its mean value.

7. Conclusions

This paper has addressed the necessity of modeling the delay margin as a random variable in small-signal stability studies of time-delayed power systems and has illustrated the development of a probabilistic stability index of small-signal stability in the presence of communication delays. An efficient method for computation of the delay margin in terms of system parameters has been presented and a Monte Carlo simulation method has been proposed to estimate the PDF of the delay margin and the probability of stability. It is found that the probability of stability values reduce with increasing the load uncertainty, resulting in a less stable system.

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