1. Introduction

Because of Proportional Integral and Derivative (PID) controller's widespread use in process industries, the problem of tuning PID controller parameters in single input single output (SI/SO) systems is widespread, too (Ziegler and Nichols,1942; Cohen and Coon, 1953; Lopez et al., 1967; Smith et al.,1975; Riviera wt al., 1986; Chien and Fruehauf, 1990; Tyreus and Luyben, 1992; Sung et al.,1995; Lee et al.,1996). One of the methods for Proportional Derivative and Integral controller (PID) parameter tuning is the internal model control and PID (IMC-PID) tuning method, which is based on keeping the controlled variable response close to the desired closed-loop response (Riviera et al., 1986; Morari and Zafiriou, 1989). An important advantage of this method is that the closed-loop time constant, which is the same as the internal model control (IMC) filter time constant, provides convenient tuning parameter to adjust the speed and robustness of the closed-loop system. However, this method gives derivative and integral time constants which do not depend on the closed-loop system time constant. Also, this method can not be used for every process model. Therefore; Lee and Park (1998) have made a new approach to IMC-PID tuning method and gained the PID parameters for general models by approximating the ideal controller with a Maclaurin series in s domain. With this method, controller parameters become dependent on the closed-loop time constant and the closed-loop response becomes better.

Moreover, when disturbance to the control variable or non-linear final control element are included in the system, cascade control can be preferred in order to improve the closed-loop response. Cascade control is used to improve the dynamic response of a feedback control loop to disturbances in the manipulated variable (Krishnaswamy, Jha and Deshpande; 1990).

There is information in the published literature on the tuning methods of cascade controllers (Jury, 1973; Edger et al., 1982; Krishnaswamy, 1990), but it is rather limited. Also, these methods tune the inner loop first and the outer loop, containing the inner loop, later. Lee and Park (1998) proposed a new method, finding the ideal controller that gives the desired closed-loop response and then finding the PID approximation of the ideal controller by Maclaurin series. The method, which can be applied to any open loop stable processes, enables us to tune the PID controller both for the inner loop and the outer loop simultaneously.

It is important to decide when to use cascade control, because it requires at least two measuring elements instead of one. According to Krishnaswamy, Jha and Deshpande (1990), cascade control makes progress about ITAE with PI-P controllers if

- The inner loop is faster than or as fast as the outer loop
- The disturbance effects the inner loop

In the article of Krishnaswamy, Jha and Deshpande (1990), cascade controller type is chosen as PI-P because it has only three tuning parameters and gives a good performance. Also, the systems are treated as FOPDT systems, since complex dynamic processes can be represented by FOPDT systems.

Also, Aström (1995) says that cascade control can be used when there are several measurement signals and one control variable. It is particularly useful when there are significant dynamics, e.g., long dead times or long time constants, between control variable and the process variable. Tighter control can be achieved by using an intermediate measured signal that responds faster to the control signal.

In this study, cascade and single feedback control structures are compared with each other and PID controller tuning method to obtain desired closed loop responses for cascade control systems (Lee and Park, 1998). Moreover, PID controller parameter calculations are detailed for both cascade and feedback control systems. Also, simulation studies are made to compare these two structures. In addition, both cascade and single feedback controls are applied to a real system, and the results are also compared with each other. In consequence, it is observed that cascade control is better in disturbance rejecting and it is usually better in acquiring good temporary system response.

In the second part; how the tuning algorithm for PID is obtained, is explained, simulations are made and comments are given for the simulation results. In the third part; how the tuning algorithm for general process models of cascade control systems is obtained, is explained, a parameter tuning example for first order plus dead time (FOPDT) systems is made, simulations are performed and comments are given for the simulation results. Fourth part includes the derivation of the digital controller, obtaining its parameters, simulation studies in z-domain, comments on simulation studies, period of realizing the control systems and information about application process. Fifth part is composed of the conclusion of the study.

2. PID Controller Tuning for Desired Closed-Loop Responses

In this part; the method of PID tuning for closed-loop responses for SI/SO systems (Lee and Park, 1998) is analyzed, the way of obtaining controller parameters is detailed and simulation studies are made.

2.1 Derivation of General Tuning Algorithm for PID Controllers

The block diagram of the system is given in Figure 2.1. In this control system, process is considered to be stable and the disturbance is considered to affect as an input of the process. The reason for choosing the disturbance as an input of the process is to form a disturbance that adds new dynamics to the process. It is harder to cope with this type of disturbance.

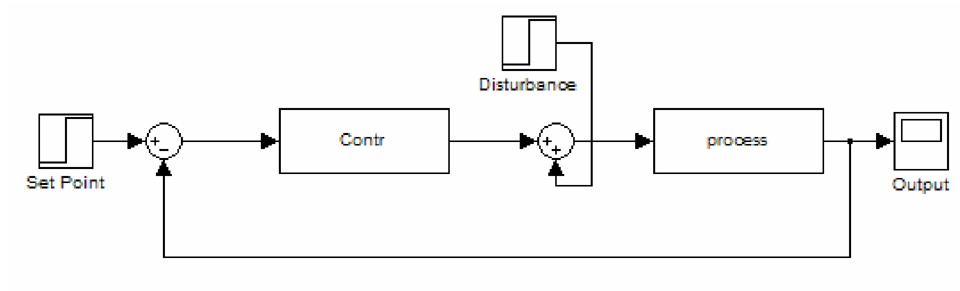

Figure 2.1: Single-Feedback Control System

The form of the stable process model is considered as;

$$G(s) = p_m(s)p_A(s) \tag{2.1}$$

In Eq. 2.1 $p_m(s)$ is the minimum phase portion of the model which can be inverted by the controller and $p_A(s)$ is the non-minimum phase portion of the model, which can not be inverted by the controller because it has right-plane zeros and/or dead times.

Usually $p_A(s)$ is chosen in the form

$$p_A(s) = \prod_{i,j}\left(\frac{-\tau_i s + 1}{\tau_i s + 1}\right)\left(\frac{\tau_j^2 s^2 - 2\tau_j \xi_j s + 1}{\tau_j^2 s^2 - 2\tau_j \xi_j s + 1}\right)e^{-Ts} \tag{2.2}$$

because this form lets $p_A(0)=1$, which is the necessary condition for the controlled variable to track its set point.

Since $p_A(s)$ is a portion that can not be inverted by the controller, the desired closed-loop response can be selected as

$$\frac{C}{R} = \frac{p_A(s)}{(\lambda s + 1)^r} \tag{2.3}$$

where the term functions as a filter with an adjustable time constant λ and an order r that is chosen to make the controller realizable.

If the required calculations are made, the ideal controller is found in the form

$$G_c(s) = \frac{q}{1 - Gq} = \frac{p_m^{-1}(s)}{(\lambda s + 1)^r - p_A(s)} \tag{2.4}$$

where q is called the IMC controller and it is given in the form

$$q = p_m^{-1}(s)/(\lambda s + 1)^r \tag{2.5}$$

Although the ideal controller in Eq. 2.4 is physically realizable, it is not in the standard PID form. So, the right PID parameters have to be found that makes PID controller behave like the ideal controller.

Because of the structure of $p_A(s)$, $p_A(0)=1$. So, the desired closed-loop response yields 1 at s=0, which means it has no steady state error. Therefore, the ideal controller must have an integral term to get rid of steady state error. The ideal controller can be represented as

$$G_c = f(s)/s \tag{2.6}$$

f(s) is the part of G_c that has no poles at zero, because its denominator ($((\lambda s + 1)^r - p_A(s))/s$) or the derivative of its denominator never becomes zero at s=0 if r is greater than zero. So, G_c can be expanded in a Maclaurin series in s in the form

$$G_c(s) = \frac{1}{s}\left[f(0) + f^{'}(0)s + \frac{f^{''}(0)s^2}{2} + \ldots \right] \tag{2.7}$$

where it has infinite number of elements. Since this type of controller is not realizable and the low, middle frequencies are more important than high frequencies, this controller can be approximated to the standard PID given by

$$G_c(s) = K_c(1 + \frac{1}{\tau_I s} + \tau_D s) \tag{2.8}$$

where

$$K_c = f^{'}(0) \tag{2.9.1}$$

$$\tau_I = f^{'}(0)/f(0) \tag{2.9.2}$$

$$\tau_D = f^{''}(0)/2f^{'}(0) \tag{2.9.3}$$

In order to obtain the PID controller parameters, we calculate the derivatives of the denominator of f(s) at s=0 first.

$$D(s) = ((\lambda s + 1)^r - p_A(s))/s \tag{2.10.1}$$

$$D(0) = r\lambda - p_A^{'}(0) \tag{2.10.2}$$

$$D^{'}(0) = [r(r-1)\lambda^2 - p_A^{''}(0)]/2 \tag{2.10.3}$$

$$D^{''}(0) = [r(r-1)(r-2)\lambda^3 - p_A^{'''}(0)]/3 \tag{2.10.4}$$

Then, the function f(s) and its first and second derivatives are evaluated at s=0.

$$f(0) = \frac{1}{KpD(0)} \tag{2.11.1}$$

$$f^{'}(0)=\frac{-[p_m^{'}(0)D(0)+K_pD^{'}(0)]}{(KpD(0))^2} \tag{2.11.2}$$

$$f^{''}(0)=f^{'}(0)\times[\frac{[p_m^{''}(0)D(0)+2p_m^{'}(0)D^{'}(0)+K_pD^{'}(0)]}{p_m^{'}(0)D(0)+K_pD^{'}(0)}+2\frac{f^{'}(0)}{f(0)}] \tag{2.11.3}$$

If we want the FOPDT process model in Eq. 2.12 behave like the reference trajectory in Eq. 2.13 and the detailed calculations are made, the controller parameters below are obtained. The detailed calculations are represented in Appendix A.

$$G(s)=\frac{K}{\tau s+1}e^{-\theta s} \tag{2.12}$$

$$\frac{C}{R}=\frac{e^{-\theta s}}{(\lambda s+1)} \tag{2.13}$$

$$K_C=\frac{\tau_I}{K(\lambda+\theta)} \tag{2.14}$$

$$\tau_I=\tau+\frac{\theta^2}{2(\lambda+\theta)} \tag{2.14.1}$$

$$\tau_D=\frac{\theta^2}{6(\lambda+\theta)}[3-\frac{\theta}{\tau_I}] \tag{2.14.2}$$

If we want the SOPDT process model in Eq. 2.15 behave like the reference trajectory in Eq. 2.13 and the detailed calculations are made, the controller parameters below are obtained.

$$G(s)=\frac{Ke^{-\theta s}}{(\tau^2s^2+2\xi\tau s+1)} \tag{2.15}$$

$$K_C=\frac{\tau_I}{K(\lambda+\theta)} \tag{2.16.1}$$

$$\tau_I=2\xi\tau+\frac{\theta^2}{2(\lambda+\theta)} \tag{2.16.2}$$

$$\tau_D=\frac{\tau^2-\frac{\theta^3}{6(\lambda+\theta)}}{\tau_I}+\frac{\theta^2}{2(\lambda+\theta)} \tag{2.16.3}$$

2.2 Simulation Study

For the simulation study, two processes

$$G_1(s) = \frac{1}{(s+1)^3} \tag{2.17.1}$$

and

$$G_2(s) = \frac{0.97}{0.73s+1} e^{-0.35s} \tag{2.17.2}$$

are chosen. If the system in Eq. 2.17.1 is modeled as FOPDT system (Aström, 1995), the new model of the system is obtained as in the Eq. 2.18. The detailed calculations are represented in Appendix B.

$$G_1(s) = \frac{1}{2.69s+1} e^{-0.589s} \tag{2.18}$$

If the systems in Eq. 2.17.2 and 2.18 are serially connected, process model is evaluated as

$$G(s) = \frac{0.97}{1.963s^2 + 3.42s + 1} e^{-0.939s} \tag{2.19}$$

and the controller parameter calculations are made with respect to parameters of the system in Eq. 2.19, but the systems in Eq. 2.17.1 and 2.17.2 are used in simulation study. Also, the controller parameters are calculated with the help of the program, prepared in MATLAB. This program is represented in Appendix C.

(i) When we choose the desired closed-loop system like $\frac{C}{R} = \frac{e^{-0.939s}}{0.9s+1}$, the controller parameters are evaluated as Kc=2.0516, Ti=3.6597, Td=0.7558.

In Figure 2.2, the simulation block diagram of the single-feedback control system is shown with a unit step input as a reference and a step input with a magnitude of 0.5 as disturbance. Also, the output of the system in Figure 2.2 is shown in Figure 2.3 and the control signal of the same system is shown in Figure 2.4.

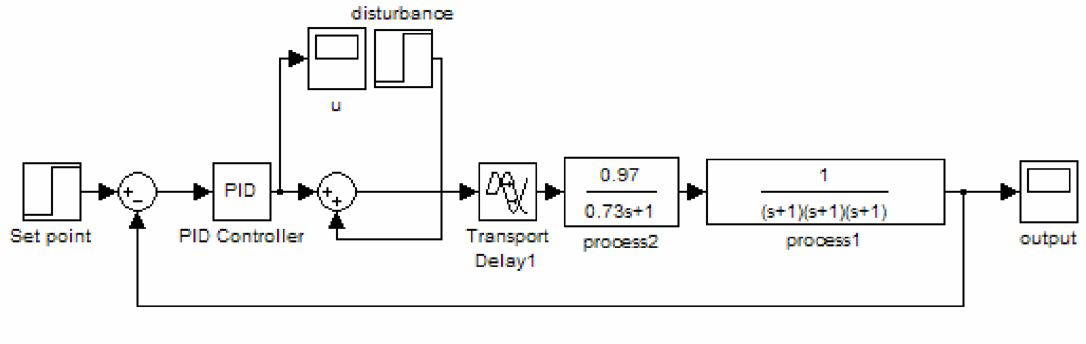

Figure 2.2: Simulation Diagram of the System

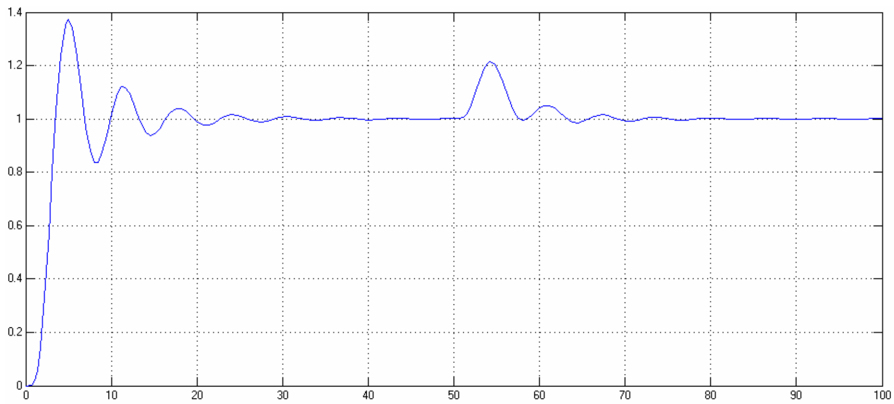

Figure 2.3: Unit Step Response of the Controlled System

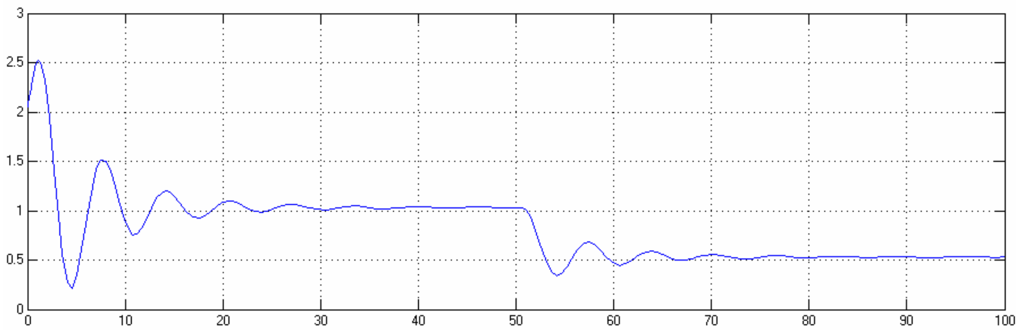

Figure 2.4: Control Signal

(ii) When we choose the desired closed-loop system like $\frac{C}{R} = \frac{e^{-0.939s}}{2.5s+1}$, the controller parameters are evaluated as Kc=1.0637, Ti=3.5482, Td=0.6703.

The output of the system in Figure 2.2 with the PID parameters given above is shown in Figure 2.5 and the control signal of the same system is shown in Figure 2.6.

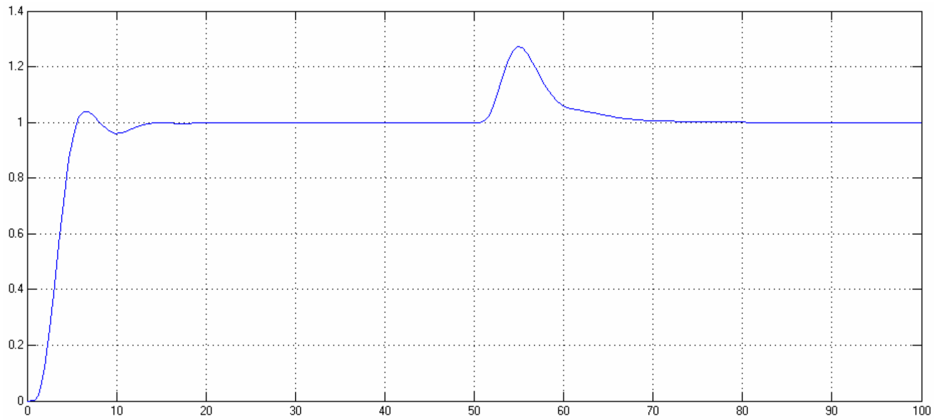

Figure 2.5: Unit Step Response of the Controlled System

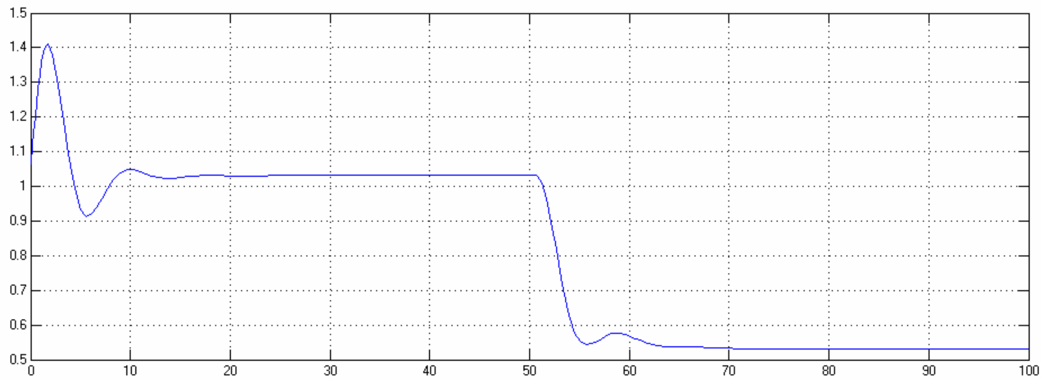

Figure 2.6: Control Signal

2.3. Comments on PID Parameter Tuning Method

First of all, the method enables us to tune PID controller by choosing only λ. As it can be seen from the simulation outputs; when the time constant of the desired closed-loop system becomes less, the overshoot of the system increases since the gain of the PID controller becomes greater. On the other hand; when the time constant of the desired closed-loop system becomes less, the disturbance effect on the system output becomes less because the system response to the disturbance is adjusted faster. Although disturbance or overshoot is adjusted individually, we can not adjust two of them simultaneously with this method. We should choose the optimal λ value in order to obtain the output we want.

3. PID Controller Tuning To Obtain Desired Closed-Loop Responses for Cascade Control Systems

In this part; the method of PID tuning to obtain desired closed-loop responses for cascade control systems (Lee and Park, 1998) is analyzed, the way of obtaining controller parameters is detailed and simulation studies are made.

3.1 Derivation of Tuning Rules for General Process Models

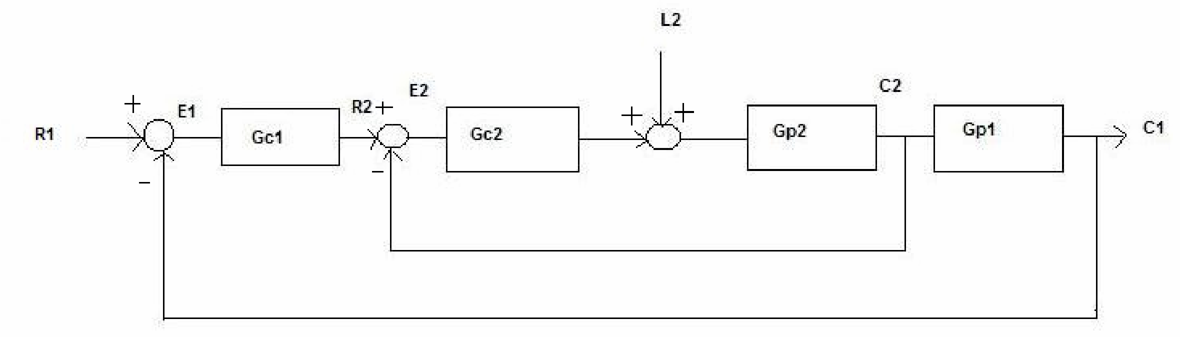

Figure 3.1: Block Diagram of the Cascade Control System

In Figure 3.1, Gc1 and Gc2 are the controllers of the primary loop and secondary loop respectively. Also, Gp1 and Gp2 are the processes of the primary loop and secondary loop respectively. The outer loop is called primary loop, because outer loop deals with the primary measured signal. L2 is the disturbance.

The closed-loop transfer functions of the primary and secondary loops are

$$\frac{C_2}{R_2} = \frac{G_{c2}G_{P2}}{1 + G_{c2}G_{P2}} \tag{3.1}$$

$$\frac{C_1}{R_1} = \frac{G_{C1}G_{P1}\dfrac{C_2}{R_2}}{1+G_{C1}G_{P1}\dfrac{C_2}{R_2}} = \frac{G_{C1}G_{P1}\dfrac{G_{c2}G_{P2}}{1+G_{c2}G_{P2}}}{1+G_{C1}G_{P1}\dfrac{G_{c2}G_{P2}}{1+G_{c2}G_{P2}}} = \frac{G_{C1}G_{c2}G_{P1}G_{P2}}{1+G_{c2}G_{P2}+G_{C1}G_{c2}G_{P1}G_{P2}} \tag{3.2}$$

For this system, G_{c2} and G_{c1} should be tuned in order to satisfy set point value R_1 and regulate the disturbance L_2.

(i) Secondary Controller's Design

Our aim while designing the secondary controller is to reject the disturbance as fast as possible. Therefore, C_2 should trace its set point as quickly as possible but not with much overshoot and any oscillation.

If the stable process model of the secondary loop is chosen as

$$G_{P2}(s) = p_{2m}(s)\, p_{2A}(s) \tag{3.3}$$

where $p_{2m}(s)$ is the minimum phase portion of the process model that can be inverted by the controller and $p_{2A}(s)$ is the non-minimum phase portion of the model that can not be inverted by the controller because it has right plane zeros or delay times. $p_{2A}(s)$ can be given in the form

$$\begin{aligned} &p_{2A}(s) = \prod_{i,j} \left(\frac{-\tau_i s + 1}{\tau_i s + 1}\right)\left(\frac{\tau_j^2 s^2 - 2\tau_j \xi_j s + 1}{\tau_j^2 s^2 - 2\tau_j \xi_j s + 1}\right) e^{-Ts} \\ &\tau_j, \tau_i > 0; \\ &0 < \xi_j < 1 \end{aligned} \tag{3.4}$$

As it can be seen from the Eq. 3.4, $p_{2A}(0)=1$. This indicates that $p_{2A}(s)$ traces its set point without steady state error. Since this portion can not be inverted with a controller, this form is an advantage for the secondary system to trace its set point.

Desired closed-loop response of the secondary system is given as

$$\frac{C_2}{R_2} = \frac{p_{2A}(s)}{(\lambda_2 s + 1)^{r2}} \tag{3.5}$$

where λ_2 is the time constant of the desired closed-loop and r_2 is the parameter to make the controller realizable. Also $1/(\lambda_2 s + 1)^{r2}$ structure is the IMC filter with adjustable parameters.

As long as the secondary process model and the secondary desired closed-loop response are known, the controller G_{c2} can be calculated. After all the required calculations, it can be given as

$$G_{c2}(s) = \frac{q_2}{1 - G_{P2}q_2} = \frac{p_{2m}^{-1}(s)}{(\lambda_2 s + 1)^{r2} - p_{2A}(s)} \tag{3.6}$$

Because of the structure of $p_{2A}(s)$, $p_{2A}(0)=1$. So, the desired closed-loop response in Eq. 3.5 yields 1 at s=0, which means it has no steady state error.

Therefore, the ideal controller must have an integral term to get rid of steady state error. The ideal controller can be represented as

$$G_{c2} = f_2(s)/s \tag{3.7}$$

$f_2(s)$ is the part of G_{c2} that has no poles at zero, because its denominator $(((\lambda_2 s + 1)^{r2} - p_{2A}(s))/s)$ or the derivative of its denominator never becomes zero at s=0 if r is greater than zero. So, Gc2(s) can be expressed in Maclaurin series in the form

$$G_{c2}(s) = \frac{1}{s}\left[f_2(0) + f_2^{'}(0)s + \frac{f_2^{''}(0)s^2}{2} + \ldots \right] \tag{3.8}$$

The controller in Eq. 3.7 and 3.8 are the same, as long as the controller in Eq. 3.8 includes all the terms but in practice this can not be implemented. Due to this constraint and the low importance of high frequency terms, only the first three terms of the controller in Eq. 3.8 is taken into consideration. The controller is given in the form

$$G_{c2}(s) = K_{c2}(1 + \frac{1}{\tau_{I2}s} + \tau_{D2}s) \tag{3.9}$$

From this equation, the controller parameters can be expressed as

$$K_{c2} = f_2^{'}(0) \tag{3.10.1}$$

$$\tau_{I2} = f_2^{'}(0)/\, f_2(0) \tag{3.10.2}$$

$$\tau_{D2} = f_2''(0)/2f_2'(0) \qquad (3.10.3)$$

(ii) Primary Controller's Design

If it can be assumed that the secondary closed loop response is the same as the secondary desired closed-loop response, primary desired closed-loop response can be given as

$$\frac{C_1}{R_1} = \frac{G_{C1}G_{P1}\dfrac{p_{2A}(s)}{(\lambda_2 s+1)^{r2}}}{1+G_{C1}G_{P1}\dfrac{p_{2A}(s)}{(\lambda_2 s+1)^{r2}}} \qquad (3.11)$$

and the process model of the outer loop can be given as

$$G_1(s) = G_{p1}\frac{p_{2A}(s)}{(\lambda_2 s+1)^{r2}} \qquad (3.12)$$

If the primary process model includes invertible and non-invertible portions in the form

$$G_{P1}(s) = p_{1m}(s)p_{1A}(s) \qquad (3.13)$$

and the primary desired closed-loop response is chosen as

$$\frac{C_1}{R_1} = \frac{p_{1A}(s)}{(\lambda_1 s+1)^{r1}} \qquad (3.14)$$

the transfer function of the controller can be given as

$$G_{c1}(s) = \frac{q_1}{1-G_1 q_1} = \frac{p_{1m}^{-1}(s)(\lambda_2 s+1)^{r2}}{(\lambda_1 s+1)^{r1} - p_{1A}(s)} \qquad (3.15)$$

With the same approximation made for the secondary loop controller, G_{c1} can be also written as a PID controller.

3.2 Tuning Method Example for FOPDT Systems

Generally, first-order plus dead time (FOPDT) models are used as approximate models for the process models in industry, so if the models of processes are chosen as

$$G_{p2}(s) = \frac{K_2 e^{-\theta_2 s}}{\tau_2 s + 1} \tag{3.16.1}$$

$$G_{p1}(s) = \frac{K_1 e^{-\theta_1 s}}{\tau_1 s + 1} \tag{3.16.2}$$

it can be seen that G_{p2} can be decomposed as

$$G_{p2}(s) = \frac{K_2}{\tau_2 s + 1} e^{-\theta_2 s} = p_{2m}(s) p_{2A}(s) \tag{3.17}$$

Also, the desired closed-loop response of the secondary system can be specified as

$$\frac{C_2}{R_2} = \frac{p_{2A}(s)}{(\lambda_2 s + 1)^{r2}} = \frac{e^{-\theta_2 s}}{\lambda_2 s + 1} \tag{3.18}$$

According to these equations, ideal controller can be given as

$$G_{c2}(s) = \frac{p_{2m}^{-1}(s)}{(\lambda_2 s + 1)^{r2} - p_{2A}(s)} = \frac{\tau_2 s + 1}{K_2(\lambda_2 s + 1 - e^{-\theta_2 s})} \tag{3.19}$$

This ideal controller of the inner loop can be converted to a PID controller with a Maclaurin series approximation and the parameters of PID controller can be evaluated from calculation made during the conversion period.

The detailed calculations made for these period are given at Appendix D. At the end of the conversion period the PID parameters of the inner loop are obtained as

$$K_{c2} = \frac{\tau_2 + \dfrac{\theta_2^2}{2(\lambda_2 + \theta_2)}}{K_2(\lambda_2 + \theta_2)} \tag{3.20.1}$$

$$\tau_{I2} = \tau_2 + \frac{\theta_2^2}{2(\lambda_2 + \theta_2)} \tag{3.20.2}$$

$$\tau_{D2} = \frac{\theta_2^2}{6(\lambda_2 + \theta_2)}\left[3 - \frac{\theta_2}{\tau_2 + \dfrac{\theta_2^2}{2(\lambda_2 + \theta_2)}}\right] \tag{3.20.3}$$

Moreover, the model of outer loop is given as

$$G_1(s) = G_{p1}\frac{p_{2A}(s)}{(\lambda_2 s + 1)^{r2}} = \frac{K_1 e^{-\theta_1 s}}{\tau_1 s + 1}\frac{e^{-\theta_2 s}}{\lambda_2 s + 1} \tag{3.21}$$

Eq. 3.21 can be decomposed as

$$G_1(s) = p_{1m}(s)p_{1A}(s) = \frac{K_1}{(\tau_1 s + 1)(\lambda_2 s + 1)}e^{-(\theta_1 + \theta_2)s} \tag{3.22}$$

When the desired closed-loop response is specified as

$$\frac{C_1}{R_1} = \frac{p_{1A}(s)}{(\lambda_1 s + 1)^r} = \frac{e^{-(\theta_1 + \theta_2)s}}{(\lambda_1 s + 1)} \tag{3.23}$$

the ideal controller can be given as

$$G_{c1}(s) = \frac{p_{1m}^{-1}(s)(\lambda_2 s+1)^{r2}}{(\lambda_1 s+1)^{r1} - p_{1A}(s)} = \frac{(\tau_1 s+1)(\lambda_2 s+1)}{K_1(\lambda_1 s+1-e^{-(\theta_1+\theta_2)s})} \tag{3.24}$$

Similarly; when the approximation is made for the outer loop's ideal controller, a PID controller and its parameters can be evaluated as given below. The detailed calculations for this period are given in Appendix E.

$$K_{c1} = \frac{\tau_{I1}}{K_1(\lambda_1+\theta_1+\theta_2)} \tag{3.25.1}$$

$$\tau_{I1} = \tau_1 + \lambda_2 + \frac{(\theta_1+\theta_2)^2}{2(\lambda_1+\theta_1+\theta_2)} \tag{3.25.2}$$

$$\tau_{D1} = \frac{\lambda_2\tau_1 - \dfrac{(\theta_1+\theta_2)^3}{6(\lambda_1+\theta_1+\theta_2)}}{\tau_{I1}} + \frac{(\theta_1+\theta_2)^2}{2(\lambda_1+\theta_1+\theta_2)} \tag{3.25.3}$$

Table 1: Results of the Tuning Rules for FOPDT and SOPDT Systems

process	process model	reference trajectory
FOPDT (inner loop)	$G_{p2}(s)=\dfrac{K_2 e^{-\theta_2 s}}{\tau_2 s+1}$	$\dfrac{C_2}{R_2}=\dfrac{e^{-\theta_2 s}}{\lambda_2 s+1}$
SOPDT (outer loop)	$G_{p2}(s)=\dfrac{K_2 e^{-\theta_2 s}}{\tau^2 s^2+2\xi\tau s+1}$	$\dfrac{C_2}{R_2}=\dfrac{e^{-\theta_2 s}}{\lambda_2 s+1}$
FOPDT (inner loop)	$G_{p1}(s)=\dfrac{K_1 e^{-\theta_1 s}}{\tau_1 s+1}$	$\dfrac{C_1}{R_1}=\dfrac{e^{-(\theta_1+\theta_2)s}}{(\lambda_1 s+1)}$
SOPDT (outer loop)	$G_{p1}(s)=\dfrac{K_1 e^{-\theta_1 s}}{\tau^2 s^2+2\xi\tau s+1}$	$\dfrac{C_1}{R_1}=\dfrac{e^{-(\theta_1+\theta_2)s}}{(\lambda_1 s+1)}$

	Kc	τ I
FOPDT (inner loop)	$\dfrac{\tau_I}{K_2(\lambda_2+\theta_2)}$	$\tau_2+\dfrac{\theta_2^2}{2(\lambda_2+\theta_2)}$
SOPDT (outer loop)	$\dfrac{\tau_I}{K_2(\lambda_2+\theta_2)}$	$2\xi\tau+\dfrac{\theta_2^2}{2(\lambda_2+\theta_2)}$
FOPDT (inner loop	$\dfrac{\tau_I}{K_1(\lambda_1+\theta_1+\theta_2)}$	$\tau_1+\lambda_2+\dfrac{(\theta_1+\theta_2)^2}{2(\lambda_1+\theta_1+\theta_2)}$
SOPDT (outer loop)	$\dfrac{\tau_I}{K_1(\lambda_1+\theta_1+\theta_2)}$	$2\tau\xi+\lambda_2+\dfrac{(\theta_1+\theta_2)^2}{2(\lambda_1+\theta_1+\theta_2)}$

	τ D
FOPDT (inner loop)	$\dfrac{\theta_2^2}{6(\lambda_2+\theta_2)}\left[3-\dfrac{\theta_2}{\tau_I}\right]$
SOPDT (outer loop)	$\dfrac{\tau^2-\dfrac{\theta_2^3}{6(\lambda_2+\theta_2)}}{\tau_I}+\dfrac{\theta_2^2}{2(\lambda_2+\theta_2)}$
FOPDT (inner loop	$\dfrac{\lambda_2\tau_1-\dfrac{(\theta_1+\theta_2)^3}{6(\lambda_1+\theta_1+\theta_2)}}{\tau_I}+\dfrac{(\theta_1+\theta_2)^2}{2(\lambda_1+\theta_1+\theta_2)}$
SOPDT (outer loop)	$\dfrac{\tau^2+2\tau\xi\lambda_2-\dfrac{(\theta_1+\theta_2)^3}{6(\lambda_1+\theta_1+\theta_2)}}{\tau_I}+\dfrac{(\theta_1+\theta_2)^2}{2(\lambda_1+\theta_1+\theta_2)}$

The results of the tuning method are summarized for FOPDT and SOPDT models in Table 1. These controllers can be used for any cascade mode. For instance, when proportional controller is tuned, only the Kc parameter is used and the others are not used. This is an advantage of the tuning method.

3.3 Simulation Studies

In this part, various simulations are made in order to analyze the advantages of cascade control systems. For the simulation study, the system model of the inner loop is chosen as the system model represented in Eq. 2.17.2 and the system of the outer loop is chosen as the system represented in Eq. 2.17.1, but the parameter tuning of the outer system is made for the approximate model of the outer system as represented in Eq. 2.18. These two systems are tried to be controlled both with cascade and single-feedback control systems. During the simulation period, different types of controllers and different λ values are tried. The consequent results are compared with each other. Figure 3.2 represents the simulation diagram used for comparision.

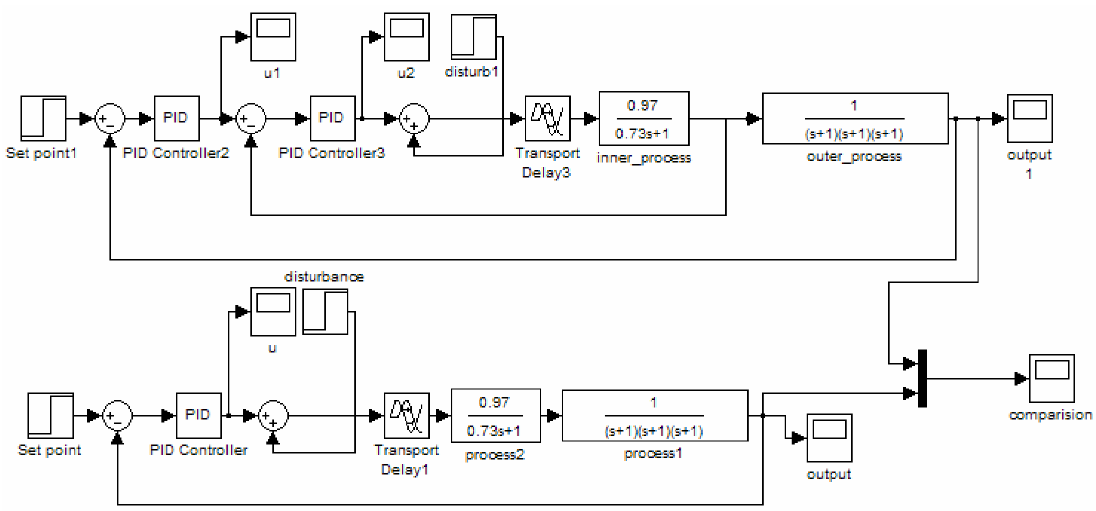

Figure 3.2: Simulation Block Diagram of the Single-Feedback and Cascade Control Systems

In Figure 3.2, the simulation block diagrams of the single-feedback and cascade control systems are shown with a unit step input as reference values and step inputs with a magnitude of 0.5 as disturbances. Various simulations are made by modifying the values of the desired-closed loop time constant for the single-feedback system (λ), the inner loop (λ_2) and the outer loop (λ_1). Also, controller types are modified for single-feedback and cascade control systems. In order not to make the systems have steady state error, single-feedback controller is chosen as PID or PI and outer loop controller of the cascade control system is chosen as PID or PI controller. Moreover, tables are given below the unit step responses of the systems to represent the percent overshoot, settling time and ITSE values that belongs to each control system.

a) Single-feedback controller is chosen as PID, λ=0.9, λ_2=0.5 and λ_1 =2.5

(i) Cascade Control System: When outer loop controller is PID and inner loop controller is PI controller for cascade control system

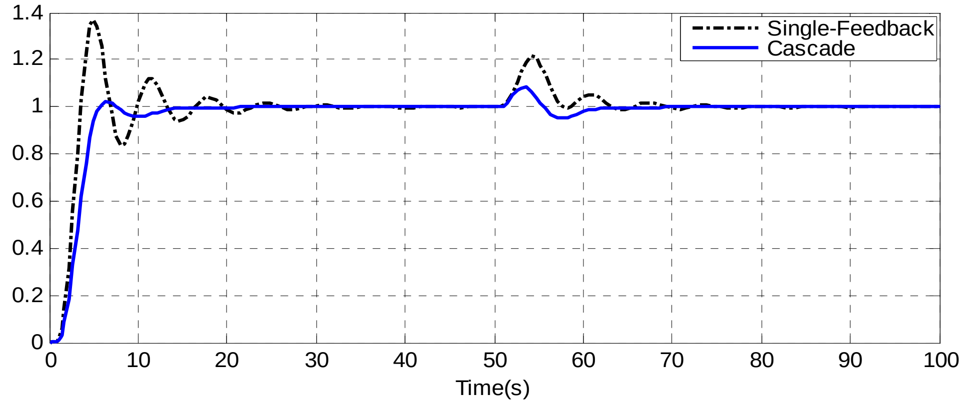

Figure 3.3: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PID-PI)

Table 2: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%2	12.5s	3.9	0.1018
Single-Feedback Contr.	%37.2	21.6s	4.437	0.5814

(ii) Cascade Control System: When outer loop controller is PID and inner loop controller is PID controller for cascade control system

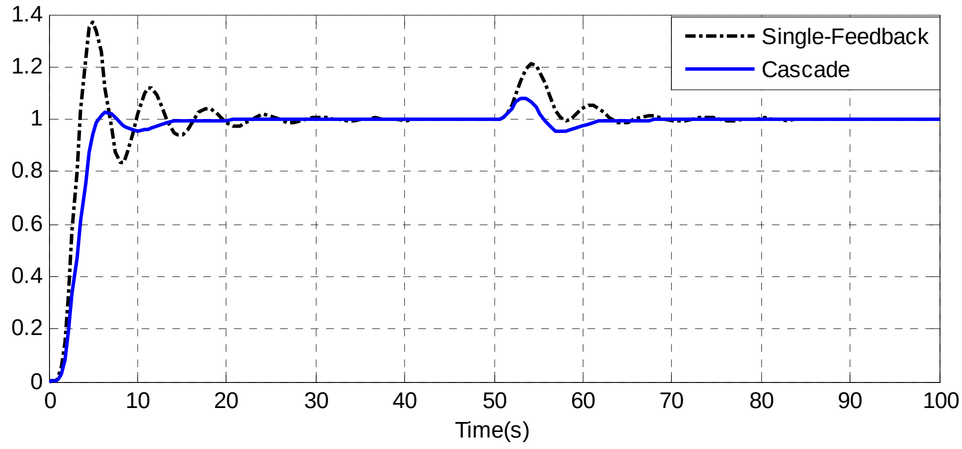

Figure 3.4: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PID-PID)

Table 3: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%2.7	12.62s	3.939	0.1037
Single-Feedback Contr.	%37.3	21.65s	4.446	0.5823

(iii) Cascade Control System: When outer loop controller is PI and inner loop controller is PI controller for cascade control system

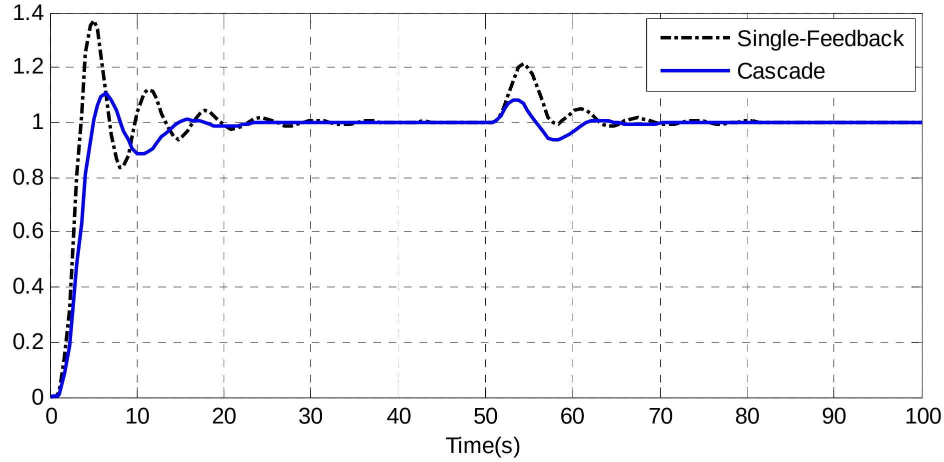

Figure 3.5: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PI-PI)

Table 4: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%10.5	13.84s	4.293	0.151
Single-Feedback Contr.	%37.1	21.65s	4.432	0.5811

(iv) Cascade Control System: When outer loop controller is PI and inner loop controller is PID controller for cascade control system

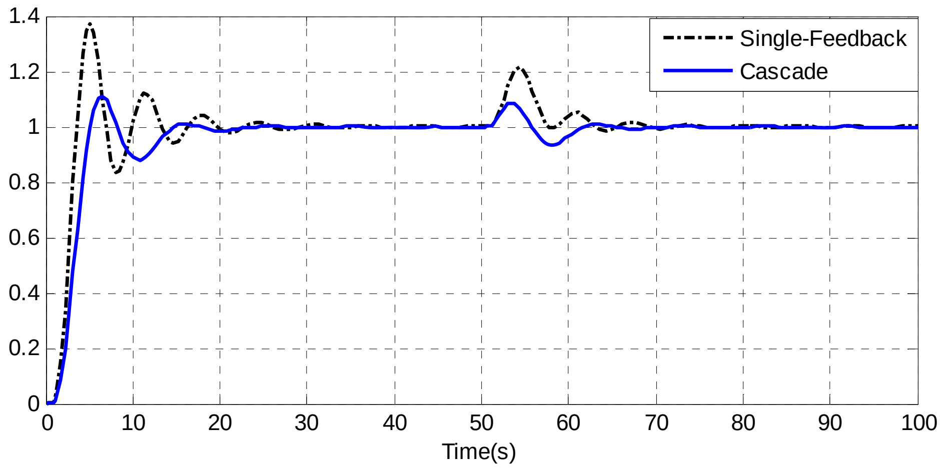

Figure 3.6: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PI-PID)

Table 5: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%11.1	13.905s	4.363	0.1539
Single-Feedback Contr.	%37.3	21.65s	4.45	0.5817

b) **Single-feedback controller is chosen as PID, λ=2.5, λ_2=0.5 and λ_1 =2.5**

(i) **Cascade Control System: When outer loop controller is PID and inner loop controller is PI controller for cascade control system**

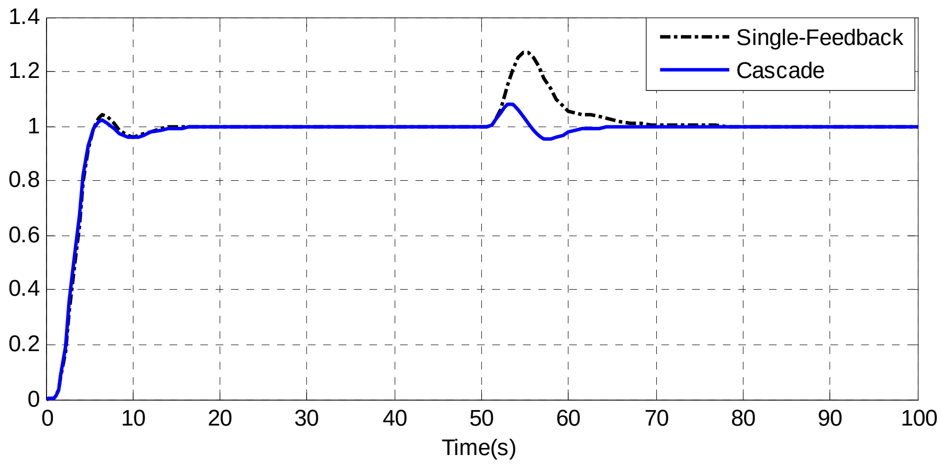

Figure 3.7: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PID-PI)

Table 6: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling time**	**Step Response**	**Disturbance**
Cascade Cont.	%2	12.5s	3.897	0.1018
Single-Feedback Contr.	%4	12s	4.176	1.604

(ii) Cascade Control System: When outer loop controller is PID and inner loop controller is PID controller for cascade control system

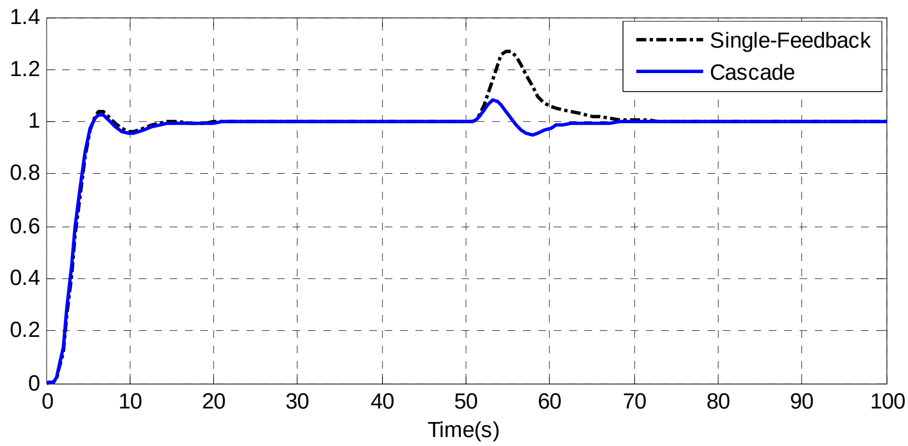

Figure 3.8: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PID-PI)

Table 7: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%2.7	12.5s	3.935	0.1038
Single-Feedback Contr.	%3.9	12s	4.175	1.605

(iii) Cascade Control System: When outer loop controller is PI and inner loop controller is PI controller for cascade control system

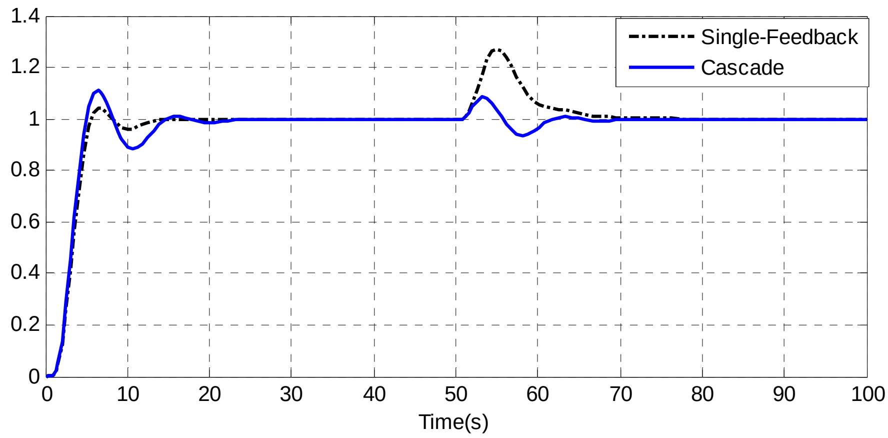

Figure 3.9: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PI-PI)

Table 8: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%11.1	13.88s	4.29	0.1509
Single-Feedback Contr.	%4	12s	4.175	1.603

(iv) Cascade Control System: When outer loop controller is PI and inner loop controller is PID controller for cascade control system

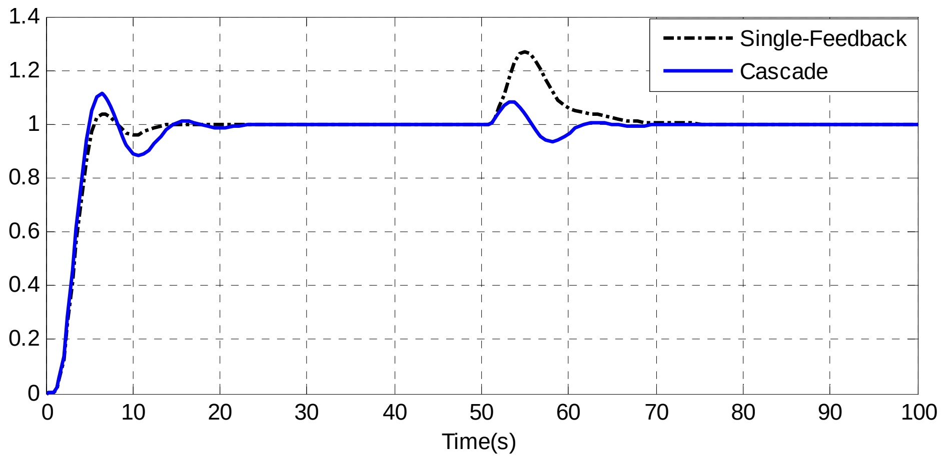

Figure 3.10: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PI-PID)

Table 9: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%11.15	13.88s	4.358	0.1538
Single-Feedback Contr.	%4	12s	4.174	1.605

c) **Single-feedback controller is chosen as PI, λ=2.5, λ_2=0.5 and λ_1 =2.5**

(i) Cascade Control System: When outer loop controller is PID and inner loop controller is PI controller for cascade control system

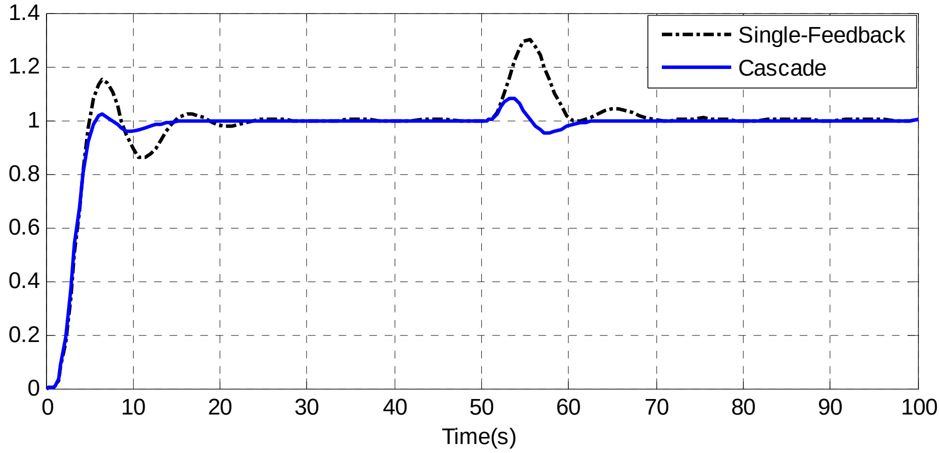

Figure 3.11: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PID-PI)

Table 10: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%2.1	12.4s	3.898	0.1018
Single-Feedback Contr.	%15.5	14.3s	4.984	1.826

(ii) Cascade Control System: When outer loop controller is PID and inner loop controller is PID controller for cascade control system

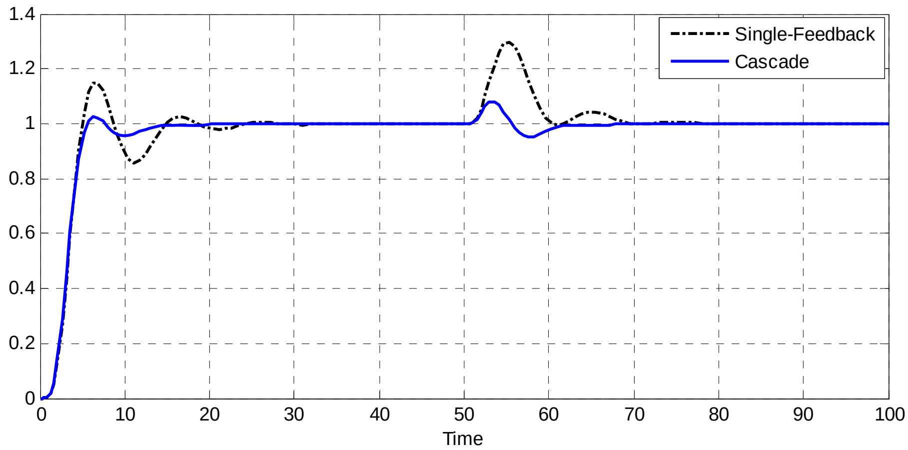

Figure 3.12: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PID-PID)

Table 11: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%2.7	12.57s	3.936	0.1038
Single-Feedback Contr.	%14.95	14.3s	4.983	1.826

(iii) Cascade Control System: When outer loop controller is PI and inner loop controller is PI controller for cascade control system

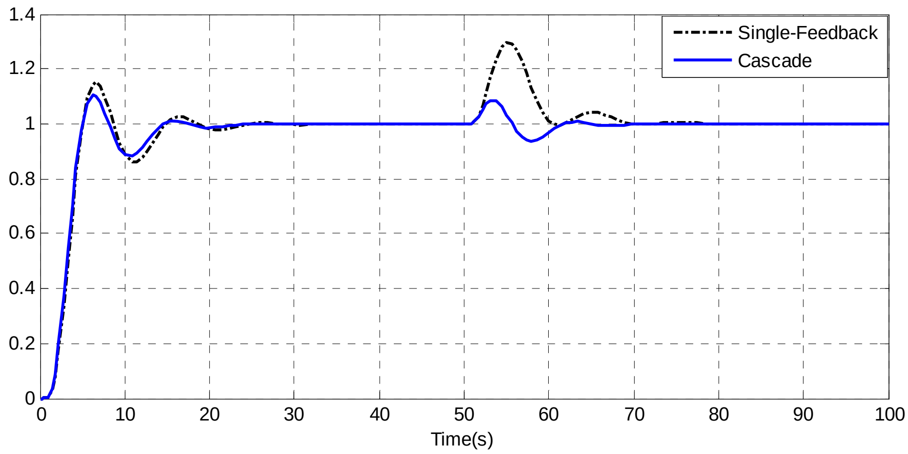

Figure 3.13: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PI-PI)

Table 12: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%10.46	13.83s	4.289	0.1509
Single-Feedback Contr.	%15.3	14.31s	4.982	1.827

(iv) Cascade Control System: When outer loop controller is PI and inner loop controller is PID controller for cascade control system

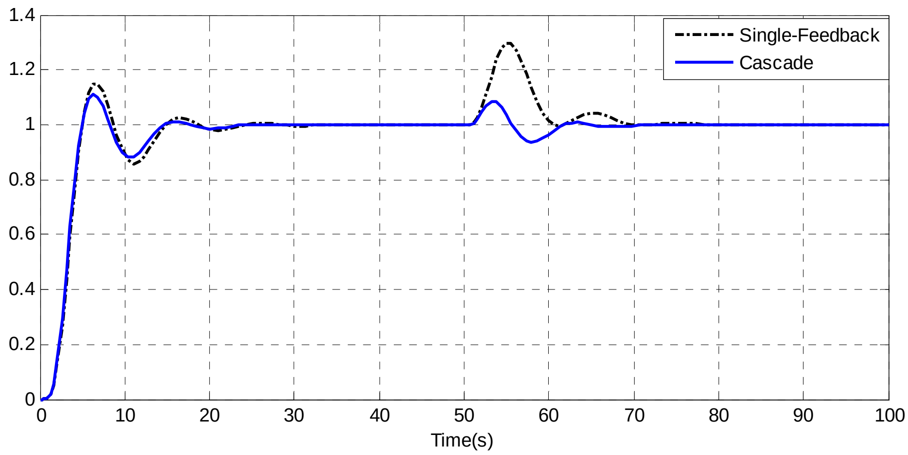

Figure 3.14: Unit Step Responses of the Single-Feedback and Cascade Control Systems (PI-PID)

Table 13: Percent Overshoot, Settling Time and ITSE Values for the System Responses

			ITSE	
	Overshoot	**Settling Time**	**Step Response**	**Disturbance**
Cascade Cont.	%11.1	13.9s	4.358	0.1539
Single-Feedback Contr.	%15	14.3s	4.984	1.827

3.4 Comments on Cascade Controller Tuning Method

As it is seen from the simulations presented in the previous section, in single-feedback control; when λ value is increased, the temporary system response improves but the disturbance effect on system response increases. When λ value is decreased, the disturbance effect on system response decreases but the temporary system response becomes worse. For example, it can be observed that temporary system response in Figure 3.7 is better than the temporary system response in Figure 3.3 and the rejection of disturbance in Figure 3.3 is better compared to Figure 3.7.

In cascade control, it is possible to select λ_1 and λ_2 to improve temporary system response and decrease the disturbance effect simultaneously. For this case, the unit step system responses in Figure 3.13 can be given as examples that indicate the better performance of cascade control compared to single-feedback control. Since λ value of the single feedback control system is given as 2.5, better performance is expected in temporary system response of the single feedback control system. However; as it can be seen from Figure 3.13, the disturbance rejection and the temporary system response performance of cascade controller is better compared to single feedback controller. The aim of the inner loop controller is to reject the disturbance effect, so the system in the inner loop should trace its reference as fast as possible. In order to make the inner loop faster λ_2 is usually chosen as a small value. This small value makes K_{c2} greater. Therefore, it is possible for the outer system to have overshoot or to make oscillation. This state is generally tried to be rejected by selecting λ_1 close to τ_1 so that K_{c1} becomes smaller.

Therefore, λ_1 and λ_2 are selected as 2.5 and 0.5 respectively in simulations. Outer loop controllers are selected as PID or PI, inner loop controllers are selected PID or PI and single-feedback controllers are selected as PID or PI controllers to observe the various system outputs. Outer loop controller and single-feedback controller are chosen in the form that includes integrator, not to cause steady state error in system outputs. Also, inner loop controller types are selected as to make the inner system trace its reference faster.

In consequence, the separate ITSE values for disturbance and step response in simulation study part represents that cascade control makes progress for disturbance effect and it generally improves temporary system response.

4. Application

4.1 Derivation of Digital Controller

The transfer of PID controller in z-domain can be given as

$$G_{PID}(z) = K + \frac{KT_s}{T_i}.\frac{1}{1-z^{-1}} + \frac{KT_D}{T_s}.(1-z^{-1}) \tag{4.1}$$

Here, backward integration method is used for the conversion of PID controller's integral term.

If the error signal (e) is the input of the controller and u(t) is the control signal, the difference equation of the controller is evaluated as

$$u_p(k) = K.e(k) \tag{4.2}$$

$$u_I(k) = \frac{KT_s}{T_i}.e(k) + u_I(k-1) \tag{4.3}$$

$$u_D(k) = \frac{KT_D}{T_s}.(u(k) - u(k-1)) \tag{4.4}$$

$$u(k) = u_P(k) + u_I(k) + u_D(k) \tag{4.5}$$

After obtaining the transfer function of the digital controller, the controller can be realized with a computer or PLC. An important constituent of digital PID controller is the sampling period. As a matter of fact, a proper selection of sampling period for a discrete time control system is very important. The sampling period does not only determine the time interval in which the controller is active but it also alters the controller parameters. The sampling period in these simulations is selected as 50ms because this value makes the controller sufficiently fast not to cause information lost. In order to calculate the digital controller parameters of cascade control system, a program is prepared in MATLAB. This program is represented in Appendix F. Also, Appendix C includes digital controller parameter calculation for single-feedback control system.

4.2 Simulation Study in z-domain

In this section, using the models of the systems that will be used for real time application, simulations have been performed.

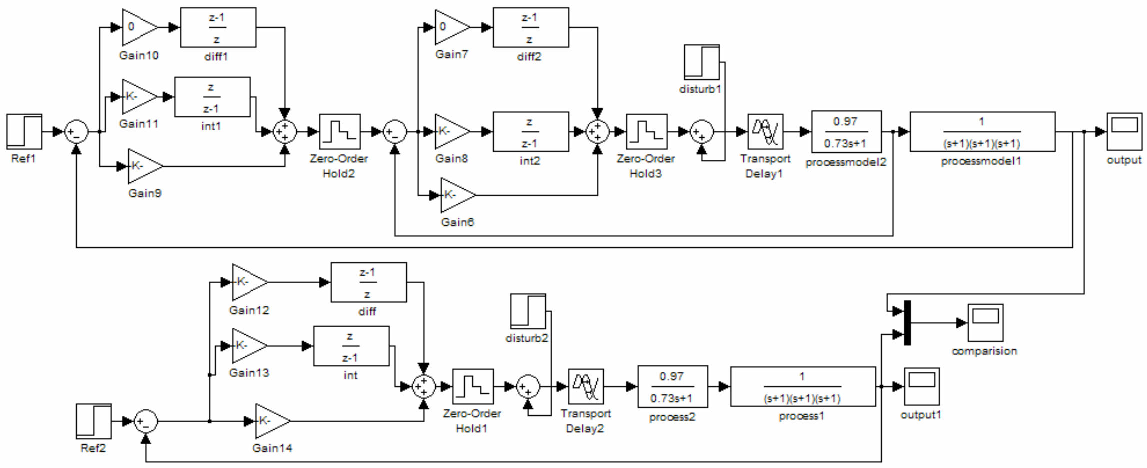

Figure 4.1: Simulation Block Diagram for Digital Control

In Figure 4.1, simulation diagram for digital cascade and single-feedback control systems are represented. In cascade control system, the controllers of inner and outer loops are chosen as PI controllers because this combination of controllers makes progress despite its simple structure when compared to PID-PID structure. In these simulations λ_1 and λ_2 are selected as 2.5 and 0.5 respectively. In single-feedback control system controller is first chosen as PI and then PID controller. The following results are obtained.

(i) When cascade controllers are PI-PI controllers and single feedback controller is PID controller with λ=2.5

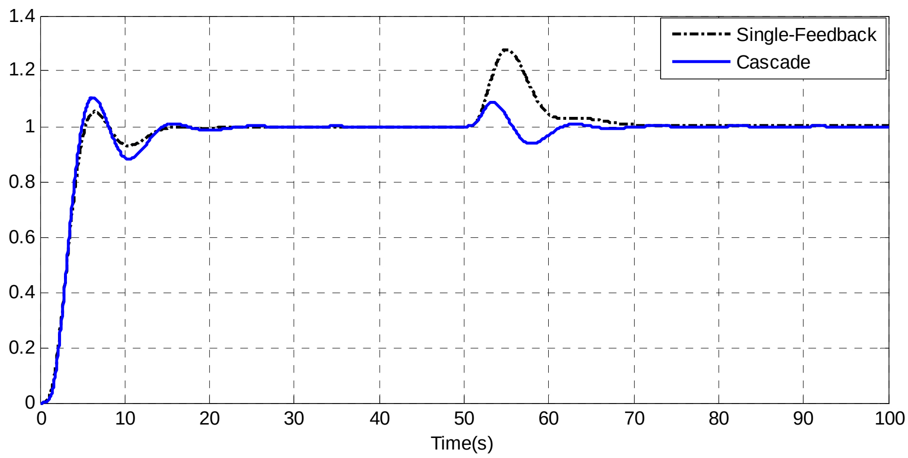

Figure 4.2: Unit Step Responses of the Cascade (PI-PI) and Single-Feedback (PID) Control Systems

Table 14: ITSE Values of the Systems in Figure 4.2 for Step Response and Disturbance

	ITSE	
	Step Response	**Disturbance**
Cascade Cont.	4.135	0.1451
Single-Feedback Contr.	3.549	1.601

(ii) When cascade controllers are PI-PI controllers and single feedback controller is PID controller with λ=0.9

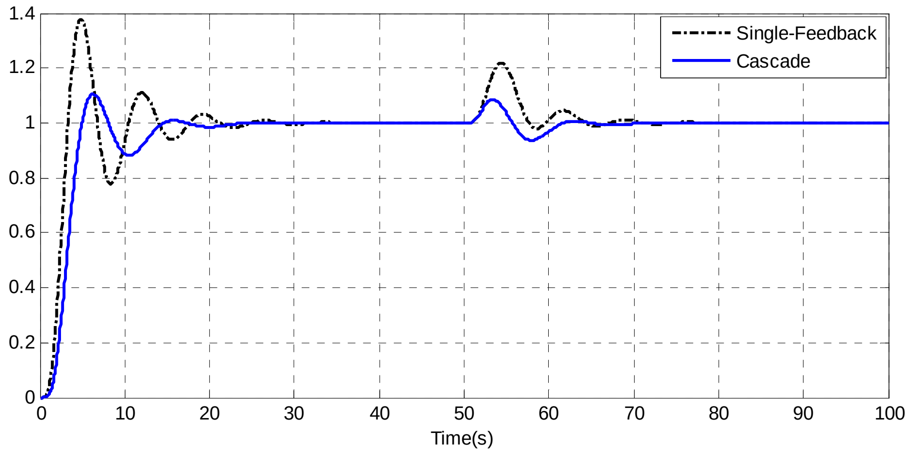

Figure 4.3: Unit Step Responses of the Cascade (PI-PI) and Single-Feedback (PID) Control Systems

Table 15: ITSE Values of the Systems in Figure 4.3 for Step Response and Disturbance

	ITSE	
	Step Response	**Disturbance**
Cascade Cont.	4.135	0.1458
Single-Feedback Contr.	2.945	0.5545

(iii) When cascade controllers are PI-PI controllers and single feedback controller is PI controller with λ=2.5

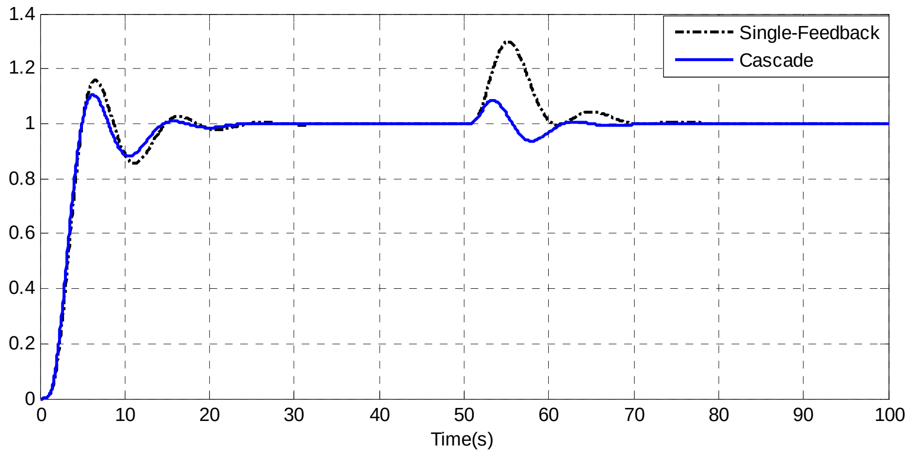

Figure 4.4: Unit Step Responses of the Cascade (PI-PI) and Single-Feedback (PI) Control Systems

Table 16: ITSE Values of the Systems in Figure 4.4 for Step Response and Disturbance

	ITSE	
	Step Response	**Disturbance**
Cascade Cont.	4.2	0.1458
Single-Feedback Contr.	4.912	1.812

Comments on Simulation Study:

When a continuous time controller is converted into a discrete time controller, it is possible to observe overshoot at the system response. Therefore, the small increase observed in overshoots of the systems, when compared to continuous time simulations of the same systems, is normal.

As it can be seen from the simulation results and ITSE values, cascade control makes progress in rejecting the disturbance effect but it is sometimes not successful about the temporary system response when compared to the single-feedback control system. However; if the ITSE criteria for both step response and disturbance are evaluated together, its failure is much less than its success. So, it can be said that cascade control is superior to single-feedback control.

4.3 Realization of the Control Systems

In order to observe the cascade controllers in real world, two systems are serially connected and two feedbacks from the system outputs are taken. One of the systems is Thermal Process Control Set PT326 and the other is the process simulator PCS327. It is not obligatory for cascade control to be applied to two serially connected systems. It can also be applied to a system, whose one state variable at least, can be measured. However, we have the opportunity to connect these systems serially and control them with cascade control system.

4.3.1 Thermal Process Control Set PT326

This system takes the air from the environment with the help of the propeller and the heater of the system increases the temperature of the air taken from outside. The hot air passes through a tube and at the end of the tube, the temperature of the air is measured. When the dimension of the air income gap changes, the temperature of the air passing through the tube changes. The aim of us is to keep the temperature of the air, passing through the tube, at the reference temperature value which is set by the user. In addition, it is necessary to produce the right control signal that makes the heater work in right way.

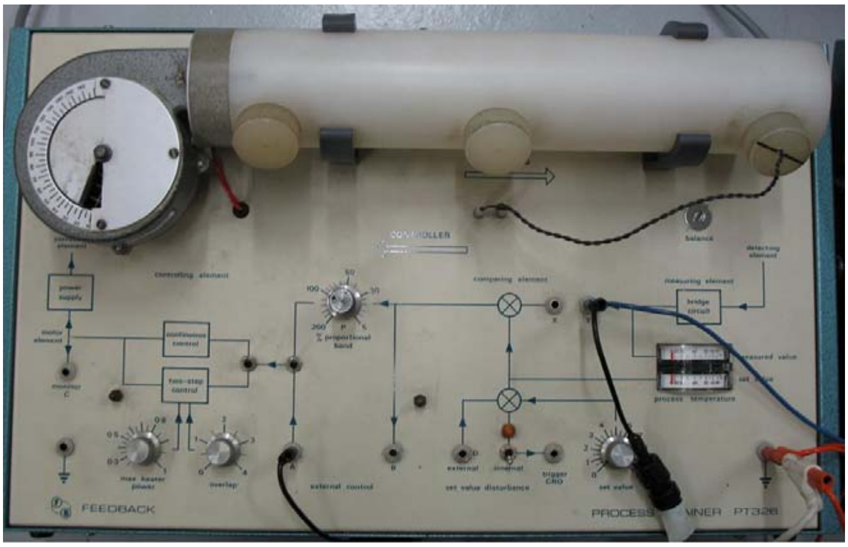

Figure 4.5: Upside View of the PT326

Since the control signal is received by the heater and the measurement is made at the end of the tube not at a near place to the heater, there is delay between the control signal and the system response. So, this system is modeled as a first-order plus dead time (FOPDT) system.

Its model changes with respect to the dimension of the air income gap. Therefore, a fixed size 20° is chosen for the air income gap. The process model is evaluated as

$$G(s) = \frac{0.97}{0.73s + 1} e^{-0.35s} \tag{4.1}$$

4.3.2 Process Simulator PCS327

This system can be separated into four parts.

(i) Reference and reference disturbance inputs: This part includes reference inputs between ±10V and disturbances that affect the reference.

(ii) Controller: This part includes a comparing element for reference and output; also it includes a continuous controller.

(iii) Non-linear unit: In this part five distinct non-linear elements can be formed ideally.

(iv) Process: This part contains integral, lag, dead time and inverter blocks. The time constants of the integral and lag blocks can be selected as 10ms or 1s.

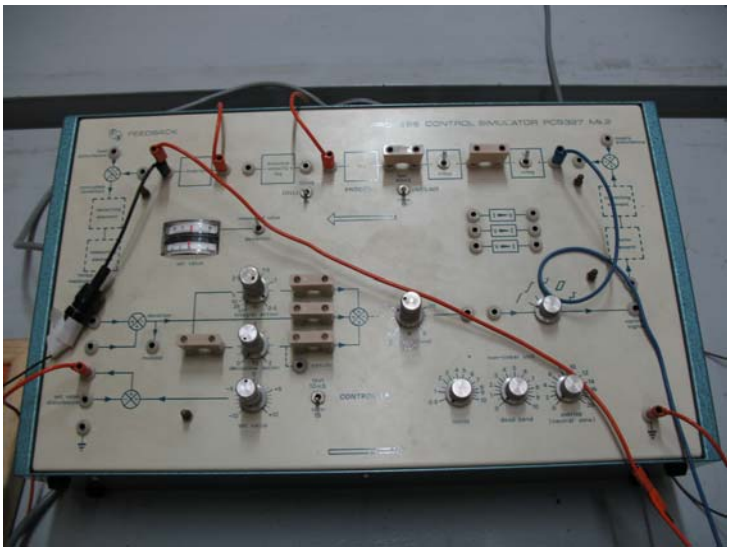

Figure 4.6: Upside View of the PCS327

Only the fourth part of the simulator is used for the application study. Three serially connected lag blocks with time constant 1s is chosen as process. So the real process model is

$$G(s) = \frac{1}{(s+1)^3} \tag{4.2}$$

But to tune the PID parameters, this system is approximated to a FOPDT model (Aström, 1995). Then the approximate process model is evaluated as

$$G(s) = \frac{1}{2.69s+1} e^{-0.589s} \tag{4.3}$$

4.3.3 PLC

In this study, SIEMENS 315 2-DP PLC is used with one digital input, one digital output, one analog input and one analog output module. Technical characteristics of the PLC are shown in Table 17.

Table 17: Characteristics of the PLC Used in Application Study

Power Supply	PS307	2A
PLC CPU	315-2DP	
DI Module	SM321	DI32×DC24V
DO Module	SM322	DO32×DC24V, 0.5A
AI Module	SM331	AI8×12BIT
AO Module	SM332	AO4×12BIT

Figure 4.7: Upside View of the PLC

4.4 Application Process

As it is mentioned above, we have two systems to be serially connected to each other. The system in the inner loop should be faster than the system in the outer so as to eliminate the disturbance effect faster. Since we have two separate systems, we have the opportunity to determine the order of the systems with respect to their unit step responses. In Figure 4.8, simulation diagram of two systems are shown and in Figure 4.9 step responses of the same systems are represented.

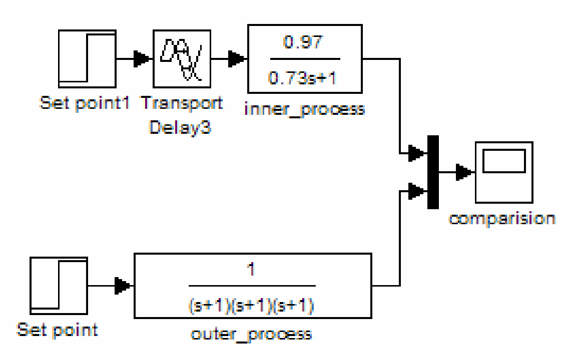

Figure 4.8: Simulation Diagram of the Two Systems

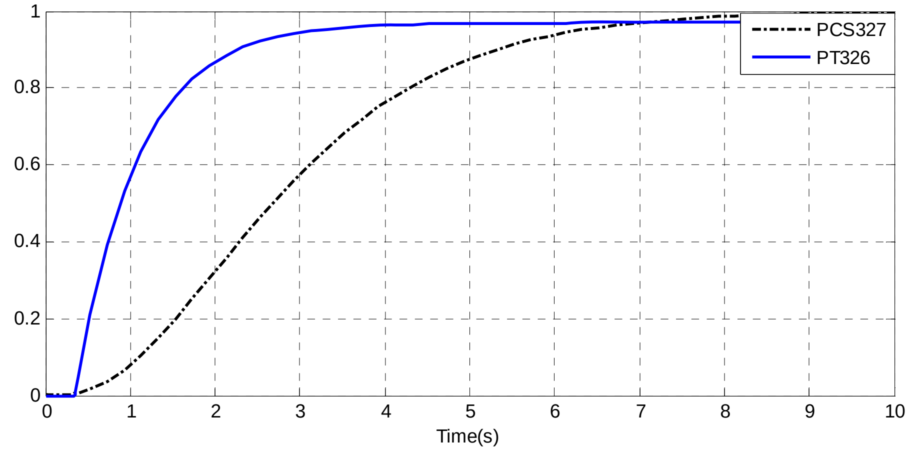

Figure 4.9: Step Responses of the Two Systems

As it can be seen from Figure 4.9, PT326 is faster than PCS327. So, the system taking part in the inner loop is determined as PT326 and the system taking part in the outer loop is determined as PCS327. Connection of the systems is shown in Figure 4.10.

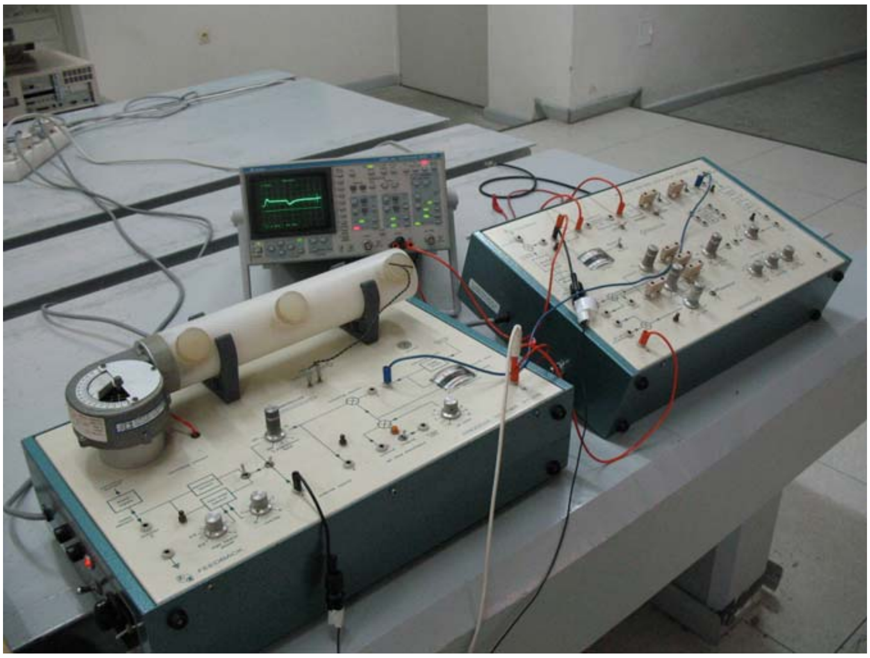
Figure 4.10: Connection of the Systems

Control element of the single-feedback and cascade control systems is selected as PLC and the controller types are determined as PID and PI controllers. PID controller program code is prepared to be used both for PID and PI controllers. This PID program code is represented in Appendix G. The results obtained from the system outputs with respect to controller combinations, are compared with each other. In these system responses, the reference value can be thought as 8950 (3.23 V) and the disturbance can be supposed to be a negative step input with a magnitude of 0.5. With this disturbance, the system response in Figure 4.11 can be observed when the system is controlled with a cascade controller (λ_1=2.5 and λ_2=0.5).

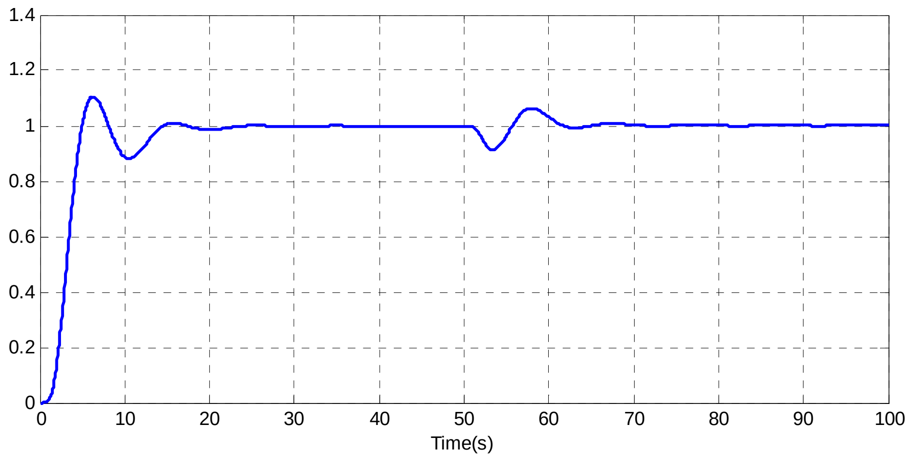

Figure 4.11: Unit Step System Response of a Cascade Control System with Negative Disturbance

(i) Cascade Controllers: PI-PI with λ_1=2.5 and λ_2=0.5 (ref = 8950)

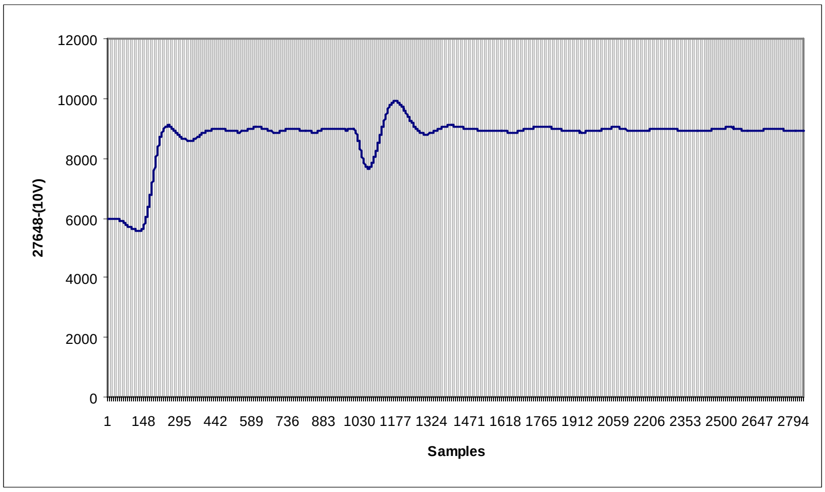

Figure 4.12: Unit Step Response of the Cascade Control System

(ii) Single-Feedback Controller: PID with λ=1.8 (ref = 8950)

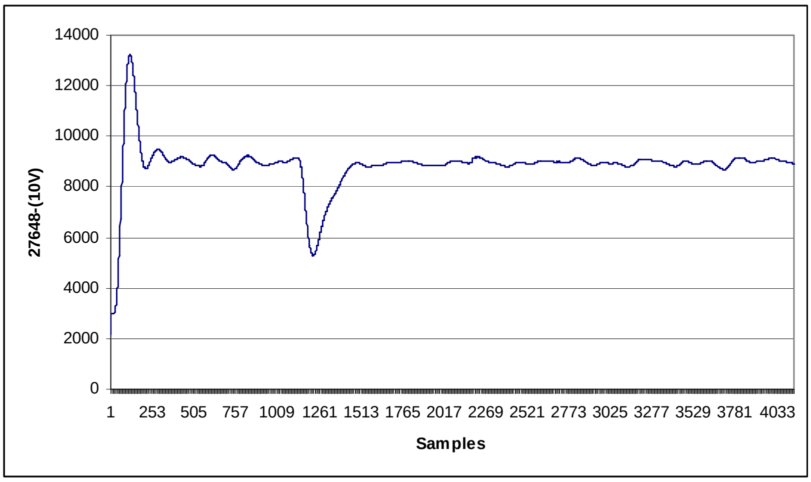

Figure 4.13: Unit Step Response of the Single-Feedback Control System

(iii) Single-Feedback Controller: PID with λ=2.5 (ref = 8950)

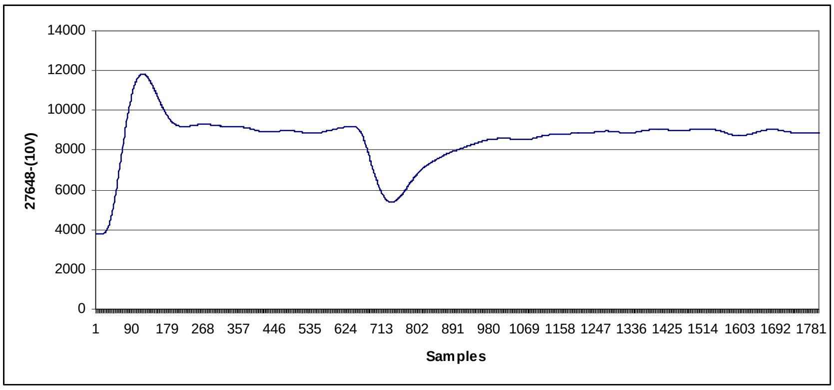

Figure 4.14: Unit Step Response of the Single-Feedback Control System

(iv) Cascade Controllers: PI-PI with λ_1=2.5 and λ_2=0.8 (ref = 8950)

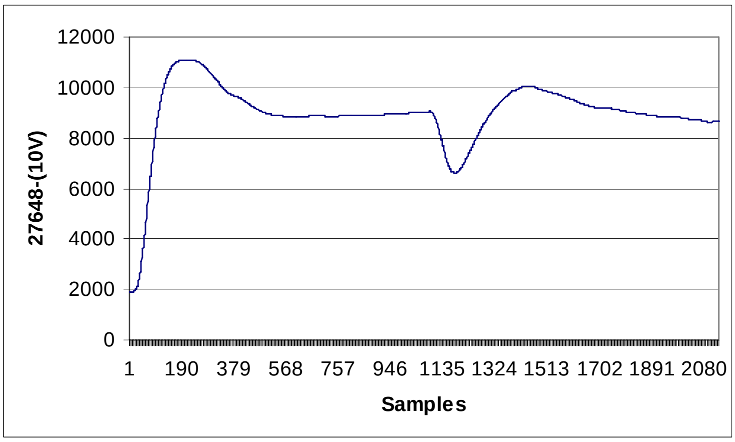

Figure 4.15: Unit Step Response of the Cascade Control System

5. Conclusion

The PID tuning rule is used both for single-feedback and cascade control systems. This tuning rule is based on the process model and the desired closed loop response. The ideal controller which can give the desired closed loop response is found and the ideal controller is obtained by taking the first three terms from Maclaurin series expansion of the ideal controller. Extensive simulation studies illustrate that the application of the tuning rule to cascade control systems gives better performance compared to application of the tuning rule to single – feedback control systems. In addition to this main benefit, the method has several advantages: it is simple and easy to use because the tuning parameters are in analytical form; the tuning of inner and outer controllers can be done simultaneously, and no additional identification step is required even when the secondary controller is retuned because the tuning method is based on the model parameters of the process; the cascade control system can be tuned to meet the specifications of both inner and outer loops because the tuning method has two adjustable parameters.