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SURFACE PERMANENT MAGNET SYNCHRONOUS MOTOR DRIVE USING A GOPINATH OBSERVER

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Abstract - The paper describes a full mechanical sensorless speed digital control system for surface permanent magnet synchronous motors (SPMSM). A minimum order state observer (Gopinath) is used for the mechanical state estimation of the machine. The observer was developed based on non-linear model of the synchronous motor, that employs a *d-q* rotating reference frame attached to the rotor. The control system includes a rotor frame vector current controller with feedforward decoupling circuit and an integral+integral proportional (I+PI) speed controller.

Key words: AC machines, Permanent magnet motors, Modeling, Variable speed drives, Field oriented control, Sensorless control, Estimation techniques, Real time simulation.

1. INTRODUCTION

PMSM are becoming a very attractive solution for drive applications because of their high torque to inertia ratio, ease of control, superior power density and high efficiency. The control schemes developed for high performance variable speed drives working on synchronous machines are based on control of the current space vector in a rotor frame of reference. This solution requires knowledge of the rotor shaft position for coordinate transformations and the necessary information on speed. In many applications, the rotor position signal is obtained from a mechanical sensor, such as an optical encoder or a resolver, that may reduce system reliability and add significantly to the drive costs. Consequently a strong interest arises in the alternative PMSM mechanical sensorless control, using only stator voltage and current measurement, based on state observers. State observers are usually implemented to reconstruct the inaccessible states of the controlled process. They are especially useful for full-state feedback control, developed on state-space theory, in that the combined observer-controller system can easily designed to meet specific qualitative and quantitative requirements. Despite the close association of observers with the state feedback controllers, the observers can be independently used to estimate the state of the system. An example could be the one described in [2, 3], where observers are employed as software transducers meant to provide the feedback data required by the control system.

Numerous papers is available on the use of complete order observers (Luenberger) for PMSM control [2, 3, 8, 9].

The present paper proposes implementation of the minimum order state observer (Gopinath) a new type of state observers, easily applied in large order systems, that result consequently to accurate modeling of SPMSM or asynchronous machines. The main characteristic of this state observer is that the number of estimated states is lower than the order of the observed system. The main idea when building the Gopinath observer is to reduce the number of estimated variables, that is to diminish the observer dimension by using the measured output data provided by the system.

Fig. 1 shows the simplified block diagram of the observer-based control system. The control system is a multi-sampled time system and consists in a fast inner loop current controller and observer, and a slower outer-loop angular velocity controller. The rotor-frame vector current controller combines feedforward compensation and a linear proportional-integral (PI) controller to control the torque of the machine. The I+PI speed controller gives the reference of the quadrature component of the stator current (i_q^*). This paper considers the constant torque regime only. The state observer generates the estimated values of the actual rotor angular velocity ω and position θ, using the measured values of the currents and voltages from the motor terminals.

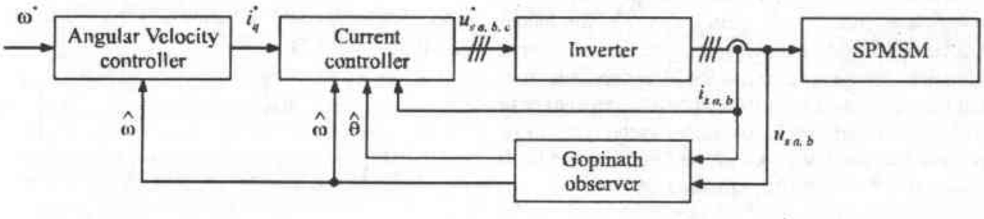


Fig. 1.

2. MOTOR MODEL

The studied SPMSM is supposed to have a symmetrical three-phase, wye-connected, isolated neutral winding. The model is developed according to some simplifying hypotheses. Thus, saturation and iron losses (hysteresis)are not considered. The induced electromotive force is supposed to have a sine form, while eddy currents are neglected. Since excitation is provided by permanent magnets, there is no variation of field currents and there is no rotor cage.

The equations of the SPMSM dynamic model are strongly simplified when expressed in a rotating reference frame attached to the rotor, with d-axis oriented to the north-pole of the permanent magnet. In this case the state space model is described by the matrix equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} i_d \\ i_q \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & 0 & 0 \\ 0 & \frac{R_s}{L_s} & -p\frac{\Psi_e}{L_s} & 0 \\ 0 & \frac{k_m}{J} & -\frac{D}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} p\omega i_q \\ -p\omega i_d \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \\ 0 & 0 & -\frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_d \\ u_q \\ m_l \end{bmatrix} \quad (1)$$

$$k_m = \frac{3}{2}p\Psi_e \quad (2)$$

where: (i_d and i_q) and (u_d and u_q) are the direct and quadrature components of the current and voltage respectively, with respect to the rotor frame; m_l - load torque; p - pole pair number; R_s, L_s - stator resistance and inductance, respectively; Ψ_e - linkage flux of the PM; J - inertia; D - damping factor.

The motor parameters used in the real time simulation are: $p = 2$, $R_s = 0.98\ \Omega$, $L_s = 15.1$ mH, $\Psi_e = 0.174$ Wb, $J = 0.0086$ kg m^2.

The state-space model, described by equation (1), contains non-linearities in form of cross-product of two state variables such as $\omega \cdot i_d$ and $\omega \cdot i_q$. This model cannot be described using the standard form of linear systems with state variables, so the linear observer theory cannot be applied directly. A possible procedure to control and estimate such non-linear systems could be the piece-wise linearisation, but the design would be in this case laborious and time consuming.

Considering the structure of the Gopinath observer, in the paper a model which uses measured currents in order to obtain a global linearisation is proposed. The non-linear system is at first transformed into a linear, time-varying one, in that the state variables vector is split so that the process variables to be effectively observed, are highlighted:

$$x = [\omega\ \theta \mid i_d\ i_q]^{\mathrm{T}} = [x_e^{\mathrm{T}}\ y^{\mathrm{T}}]^{\mathrm{T}} \quad (3)$$

$$x_e = [\omega\ \theta]^{\mathrm{T}} \quad (4) \qquad y = [i_d\ i_q]^{\mathrm{T}} \quad (5)$$

Adopting the vector of the input variables:

$$u = [u_d\ u_q\ m_l]^{\mathrm{T}} \quad (6)$$

the linearised model can be written:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = Ax + Bu \quad (7)$$

$$y = Cx \quad (8)$$

with:

$$A = \begin{bmatrix} -\frac{D}{J} & 0 & 0 & \frac{k_m}{J} \\ 1 & 0 & 0 & 0 \\ p\tilde{i}_q & 0 & -\frac{R_s}{L_s} & 0 \\ -\frac{p}{L_s}(\Psi_e + L_s\tilde{i}_d) & 0 & 0 & -\frac{R_s}{L_s} \end{bmatrix} \quad (9)$$

$$B = \begin{bmatrix} 0 & 0 & -\frac{1}{J} \\ 0 & 0 & 0 \\ \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \end{bmatrix} \quad (10)$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

where $\tilde{i}_d$ and $\tilde{i}_q$ are the values of currents obtained by measuring and computing in dependence on angle θ.

3. STATE OBSERVER DESIGN

In order to develop the state observer structure, first auxiliary matrices:

$$A_{11} = \begin{bmatrix} -\frac{D}{J} & 0 \\ 1 & 0 \end{bmatrix}; \qquad A_{12} = \begin{bmatrix} 0 & \frac{k_m}{J} \\ 0 & 0 \end{bmatrix} \quad (12)$$

$$A_{21} \begin{bmatrix} p\tilde{i}_q & 0 \\ -\frac{p}{L_s}\left(\Psi_e + L_s\tilde{i}_d\right) & 0 \end{bmatrix} \tag{13}$$

$$A_{22} = \begin{bmatrix} \frac{R_s}{L_s} & 0 \\ 0 & \frac{R_s}{L_s} \end{bmatrix} \tag{14}$$

$${}_1 = \begin{bmatrix} 0 & 0 & -\frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix}; \quad B_2 = \begin{bmatrix} \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \end{bmatrix} \tag{15}$$

are written, obtained by splitting of matrices A and B of the linearised model, in the way imposed by the splitting of state variables vector x. The general equations of the Gopinath observer are [5, 7]:

$$\frac{dz}{dt} = Fx + Gu + Hy \tag{16}$$

$$x_e = z + Ly \tag{17}$$

$$z = [z_1 \; z_2]^T \tag{18}$$

where z is the state variables vector of the Gopinath observer, and

$$F = A_{11} - L \cdot A_{21} \tag{19}$$

$$G = B_1 - L \cdot B_2 \tag{20}$$

$$H = A_{12} - L \cdot A_{22} + F \cdot L \tag{21}$$

$$L = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix} \tag{22}$$

The unknown parameters of matrix L will be determined using the method of poles placement for matrix F, that influences the dynamic behaviour of the observer.
Matrix F of the state observer is obtained by replacing relations (12 a), (13) and (22) in equation (19):

$$F = \begin{bmatrix} -\frac{D}{J} - l_{11}p + l_{12}\frac{p}{L_s}\left(\Psi_e + L_s\tilde{i}_d\right) & 0 \\ 1 - l_{21}p + l_{22}\frac{p}{L_s}\left(\Psi_e + L_s\tilde{i}_d\right) & 0 \end{bmatrix} \tag{23}$$

Observer poles allocation is extremely simple because the characteristic polynom:

$$P_0(s) = \det(s\,I_2 - F) =$$
$$= s\left[s + \frac{D}{J} + l_{11}p\tilde{i}_q - l_{12}\frac{p}{L_s}\left(\Psi_e + L_s\tilde{i}_d\right)\right] \tag{24}$$

is a second order polynom with a nil root. Selecting a convenient value for non-zero pole p_1 and $l_{11} = 0$, there results:

$$l_{12} = -\left(p_1 - \frac{D}{J}\right)\frac{L_s}{p\left(\Psi_e + L_s\tilde{i}_d\right)} \tag{25}$$

The other coefficients of matrix L were selected randomly - for example zero - because they do not influence the observer dynamics: $l_{21} = l_{22} = 0$.
Replacing relations (15) in equation (20) and relations (12 b), (14), (22), (23) in equation (21) respectively and taking into account the above mentioned values of the coefficients of matrix L, following relations are obtained:

$$G = \begin{bmatrix} 0 & -\frac{l_{12}}{J} & -\frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \tag{26}$$

$$= \begin{bmatrix} 0 & \frac{k_m}{J} + l_{12}\left(\frac{R_s}{L_s} - \frac{D}{J} + \frac{l_{12}p}{L_s}\left(\Psi_e + L_s\tilde{i}_d\right)\right) \\ 0 & l_{12} \end{bmatrix} \tag{27}$$

Thus we obtain the final form of the Gopinath observer equations for estimating the angular velocity and the angular position for an SPMSM:

$$\frac{dz_1}{dt} = \left[-\frac{D}{J} + l_{12}\frac{p}{L_s}\left(\Psi_e + L_s\tilde{i}_d\right)\right] z_1$$
$$- \frac{l_{12}}{L_s}\tilde{u}_q - \frac{m_l}{J} + \left(\frac{k_m}{J} + l_{12}\frac{R_s}{L_s}\right)\tilde{i}_q + \tag{28}$$
$$+ l_{12}\left[-\frac{D}{J} + l_{12}\frac{p}{L_s}\left(\Psi_e + L_s\tilde{i}_d\right)\right]\tilde{i}_q$$

$$\frac{dz_2}{dt} = z_1 + l_{12}\tilde{i}_q \tag{29}$$

$$\hat{\omega} = z_1 + l_{12}\tilde{i}_q \tag{30}$$

$$\hat{\theta} = z_2 \tag{31}$$

where $\tilde{u}_d$ and $\tilde{u}_q$ are the voltage values, measured and subsequently transformed in the rotor reference frame using the observed position.

4. STATOR CURRENT CONTROLLER

To achieve a fast response with high performances, a control strategy of the currents in the two-phase rotor frame (*d-q*) with feedforward decoupling was used [4]. Fig. 2 shows the block diagram of the multi-variable current controller, where $G_{ri}(s)$ is the transfer function of the PI type individual controllers.

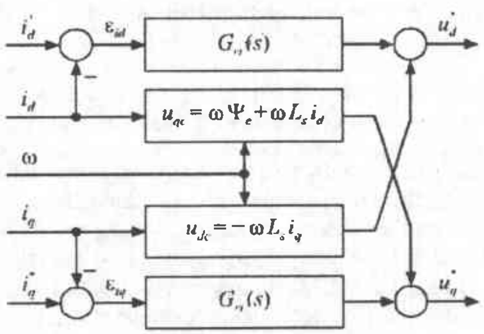


Fig. 2.

The recurrent algorithm of this control structure is obtained using the Euler's discrete integrating method:

$$\varepsilon_{id}(n+1) = i_d^*(n+1) - i_d(n+1) \tag{32}$$

$$\varepsilon_{iq}(n+1) = i_q^*(n+1) - i_q(n+1) \tag{33}$$

$$x_d(n+1) = x_d(n) + \frac{T_{ei}\, k_{ri}\, \varepsilon_{id}(n)}{T_{ri}} \tag{34}$$

$$x_q(n+1) = x_q(n) + \frac{T_{ei}\, k_{ri}\, \varepsilon_{iq}(n)}{T_{ri}} \tag{35}$$

$$\begin{aligned} u_d^*(n+1) &= x_d(n+1) + k_{ri}\, \varepsilon_{id}(n+1) - \\ &L_s\, \omega(n+1)\, i_q(n+1) \end{aligned} \tag{36}$$

$$\begin{aligned} u_q^*(n+1) &= x_q(n+1) + k_{ri}\, \varepsilon_{iq}(n+1) + \\ &+ \Psi_e\, \omega(n+1) + L_s\, \omega(n+1)\, i_d(n+1) \end{aligned} \tag{37}$$

where $x_{d,q}$ are the auxiliary variables associated to the integrator, T_{ei} is the sampling period of the stator currents control loops, n - the index of the sampling period T_{ei}, k_{ri} and T_{ri} are the tuning parameters of the two current controllers.
The reference quantity for the quadrature component of the stator current (i_q^*) is obtained directly from the output of the rotor speed controller. The reference quantity for the direct component of the stator current (i_d^*) will be adopted such as to maximise the electromagnetic torque - absorbed current ratio, what implies in this case $i_d^* = 0$.

5. ANGULAR VELOCITY CONTROLLER

The angular velocity controller has an important influence on the performances of the entire control system during both transient and steady state regimes. Good performances could be achieved using an I type control law for the angular velocity reference quantity ω^*, and a PI control law for the angular velocity feedback ω respectively. The I+PI controller will operate with both these laws. The main advantage of an I+PI controller is the achievement of good performances in transient regime and, at the same time, due to the unitary discrete pole, a nil stationary error with respect to the perturbation of the load torque (including the friction torque of the machine). The block diagram of this controller – using *z*-transfer functions - is presented in fig. 3.

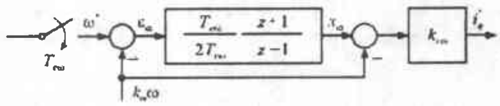


Fig. 3.

It can be noticed that the integration used the trapeze method. The controller algorithm, obtained applying the inverse *Z* transform, is described by recurrent equations:

$$\varepsilon_\omega(l+1) = \omega^*(l+1) - k_\omega\, \omega(l+1) \tag{38}$$

$$(l+1) = x_\omega(l) + \frac{T_{e\omega}}{2\,T_{r\omega}}\, [\varepsilon_\omega(l+1) - \varepsilon_\omega(l) \tag{39}$$

$$i_q^*(l+1) = k_{r\omega}\, [x_\omega(l+1) - k_\omega\, \omega\, (l+1)] \tag{40}$$

where: ε_ω, x_ω - the auxiliary variables associated to angular velocity loop error, and to the integrator respectively; $k_{r\omega}$, $T_{r\omega}$ - the tuning parameters of the angular velocity controller; k_ω - the transfer factor of the angular velocity transducer, l - the index of the sampling period $T_{e\omega}$.

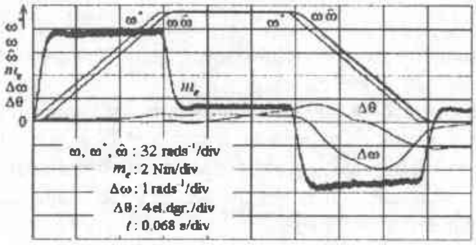
6. OBSERVER BASED CONTROLLER DESIGN

This section integrates the observer with the current and angular velocity controller. The internal current loops and the observer use a sampling period of $T_{ei} = 0.5$ ms, whereas the outer-loop angular velocity controller operates with a sampling period $T_{e\omega} = 3$ ms.
The actual values of the stator current components are obtained using the measured values of the phase currents, then they are transformed from the three phase system into a two phase stationary reference frame, and then into a rotor two-phase frame using the estimated rotor angle $\hat{\theta}$. For this reason in equations (32), (33), (36) and (37), i_d and i_q are replaced by $\hat{i}_d$ and $\hat{i}_q$. In a similar way - but with the transformation made in reverse order - the values of the reference quantities of

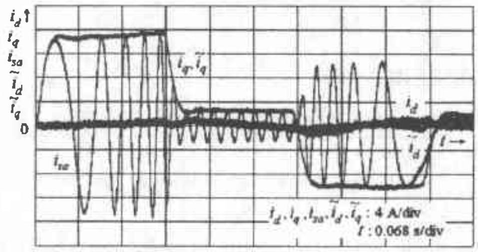
the phase stator voltages could be obtained.
Voltages u_d^* and u_q^* are calculated with relations (36) and (37), by replacing ω by the estimated value $\hat{\omega}$. The slower outer-loop angular velocity controller requires, also replacement of ω by the estimated angular velocity $\hat{\omega}$ in its control law (38)-(40).
For a good behaviour of the control system, an optimum tuning of the current and rotor angular velocity controllers parameters is required.
The selection of the parameters for current controllers is carried out similarly to continuous systems – considering the two control loops linear and decoupled - due to the feedforward compensation and to the small sampling period of the current loops.
The model for the controlled process of the rotor angular velocity loop is obtained using the discrete form of the closed-loop quadrature current transfer function, that is approximated by a first order delay element. The selection of parameters for the angular velocity controller is achieved using the discrete version of the extended modulus criterion. The load torque and the damping factor (D = 0) are neglected in the controller design. The application way of this design technique and the equations for controller parameters are presented in detail in [7].

7. ANALYSIS OF THE CONTROL SYSTEM

The analysis of the control system was carried out using real time simulation in the conditions mentioned above. The angular velocity reference quantity has a trapeze variation shape specific to incremental motion servo-drives. The load and the viscous friction torque are taken into The mechanical and electrical waveforms are presented in fig. 4 *a*, *b*. They were obtained assuming the mechanical and electrical parameters of the electrical drive known and constant. Also, an exact initialization of the observer position is considered. Under these circumstances, it is noticed that the estimation errors of the rotor angular velocity $\Delta\omega = \omega - \hat{\omega}$ and rotor position $\Delta\theta = \theta - \theta$^ are acceptable.
The variation of the machine electrical parameters influences only in a relatively small proportion the performances of the observer-based control system. consideration in the simulated machine model (m_l = 1 Nm and D = 0.002 Nms/rad). The synchronous machine is supplied from a 12 kHz PWM converter.
This aspect can be noticed in fig. 5 *a*, *b*, that present the same waveforms as in fig. 4 *a*, *b* but in this situation stator resistance is twice as big as the rated ($R_s' = 2R_s$) and inductance 30% smaller than the rated values.
In these situations the observer-based control system becomes unstable. The simulations show that the proposed structure is not capable of eliminating the incorrect position initializations, even at small values of the initial estimation error. For this reason it is necessary to determine the initial rotor position with an alternative method, like the one presented in [6].

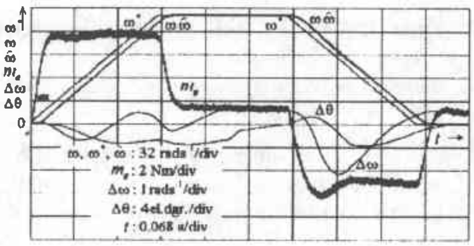


a)

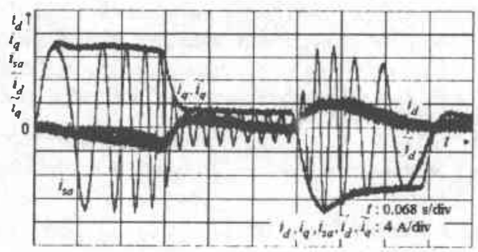


b)

Fig. 4.



a)



b)

Fig. 5.

8. CONCLUSIONS

The paper develops a mechanical sensorless digital control system of the rotor speed for a SPMSM. Mechanical state estimation is achieved by using a minimum order observer (Gopinath). The paper studies the interaction between the observer and the closed loop

The validation of the proposed control system is achieved by real time simulation in C++, under the natural operation circumstances, using the estimated quantities as feedback signals for the control system.
The simulation shows that the dynamic behaviour of the observer based control system is acceptable, even if the electrical parameters of the machine are different from the rated values used for the design of the controllers and observer.
The control system is very sensitive to the incorrect initialization of the estimated position and to mechanical parameter variation (inertia, load torque).

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