ORDER SELECTION IN AUTOREGRESSIVE POWER SPECTRUM ESTIMATION OF SLEEP EEG

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Abstract

Order selection is a general problem in autoregressive (AR) power spectrum estimation of sleep EEG. In this paper we tried to determine a common AR model order for sleep EEG. Order selection criterias are applied and the most fitting orders are selected. The effect of sampling frequency on AR modeling is investigated with experiments on EEG and sinusoidals.

I. INTRODUCTION

The selection of an optimal model order is acommon problem in AR modeling. It's seen that sampling frequency of the data is also related to AR modeling and order delection criterias.

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II. THEORY

In autoregressive (AR) power spectrum estimation, we assume that our sequence x(n) is the output of a system driven by white noise w(n). The system has an all pole transfer function as specified below [3]:

$$H(z)=\frac{1}{1+c_1z^{-1}+c_2z^{-2}+....+c_pz^{-p}} \qquad (1)$$

The generation of the p'th order AR(p) process of x(n) may be expressed by the difference equation [2]:

$$x(n)+c_1x(n-1)+....+c_px(n-p)=w(n) \qquad (2)$$

The power spectrum takes the form:

$$S_x(e^{jw})=\frac{\sigma_w^{\ 2}}{\left|1+c_1e^{-jw}+....+c_pe^{-jwp}\right|^2} \qquad (3)$$

So, to estimate the power spectrum estimation we require the estimation of the AR(p) model parameters (c_1, c_2, …c_p), and the variance of the noise (σ_w^2). The model parameters can be estimated using Yule-Walker method. In this approach w(n) is thought as the estimation error, so from Eq. (1), estimate of x(n) can be found [2].

$$x(n)-\hat{x}(n)=e(n) \qquad (4)$$

$$\hat{x}(n)=-\sum_{i=1}^{p}c_ix(n-i) \qquad (5)$$

The least squares predictor by finding the set of coefficients c_1, c_2,…c_p, that minimises the total squared error can be formed:

$$E_T=\sum_n e^2(n)=\sum\left[x(n)-\hat{x}(n)\right]^2$$

$$=\sum_n\left[x(n)+\sum_{i=1}^{p}c_ix(n-i)\right]^2 \qquad (6)$$

Minimising E_T with respect to coefficients c_i , leads to Yule-Walker equations:

$$\sum_{k=1}^{p}c_kR_{xx}(k-i)=-R_{xx}(i) \qquad (7)$$

or,

$$\mathbf{R}.\mathbf{c}=-\mathbf{g} \qquad (8)$$

where **R** is the autocorrelation matrix of x(n) with entries $\mathbf{R}(i,j)=R_{xx}(|i-j|)$, $\mathbf{g}=[R_{xx}(1)\ R_{xx}(2)\ldots.\ R_{xx}(p)]^T$ and $\mathbf{c}=[c_1, c_2, \ldots c_p]^T$.

We may estimate the autocorrelation values using the biased estimator [2]:

$$R_{xx}(k)=\frac{1}{N}\sum_{n=0}^{N-p-1}x(n)x(n+k), \quad 0\le k\le p. \qquad (9)$$

We may calculate the variance of the generating sequence σ_w^2, by multiplying both sides of (2) by x(n) and taking the expectation:

$$\sigma_w^2=R_{xx}(0)+c_1R_{xx}(1)+...+c_pR_{xx}(p). \qquad (10)$$

Since we have found all the required parameters in (3), the AR power spectrum estimation of x(n) can be calculated.

The model order is not known apriory, so to

minimise the prediction error, an adequate order should be selected. The model order can also be estimated using the Akaike information criterion (AIC) which minimises the information entropy of the signal identified as follows [5]:

$$AIC(p) = \log\left[\sigma_e^2(p)\right] + \frac{2}{N}p \qquad (11)$$

where $\sigma_e^2(p)$ is the estimated variance at order p, and N is the number of samples in the signal.

Another order selection criterion is defined by Rissanen [5] as follows:

$$MDL(p) = \log\left[\sigma_e^2(p)\right] + \frac{\log(N)}{N}p \qquad (12)$$

The Final Prediction Error (FPE) is also a commonly used order detection criterion [2]:

$$FPE(p) = \sigma_e^2\left(\frac{N+p-1}{N-p-1}\right) \qquad (13)$$

Both AIC and MDL are based upon minimising noise variance, σ_e^2. The second term in Eqs. (11) and (12) represents the penalty for higher orders.

Inverse filtering the signal x(n) with the determined coefficients gives the error signal e(n). The more e(n) is Gaussian, the more the model is true. So basic tests can be done to verify how well the model describes the signal [2]:

The probability of a sign change in each sample, in a Gaussian signal is ½. Therefore the number of successive samples with the same sign is exponentially distributed [1].

The autocorrelation of e(n), $R_e(k)$ is Gaussian distributed with zero mean and variance 1/N [1].

III. METHODS

In this study a 75-second EEG signal sampled at 100Hz is used. AR models are computed for 5-second segments. In order to select an appropriate order, the Akaike Order Criterions (Eq. 11) for these AR models are calculated for orders from 1 to 25. Figure 1 illustrates order criterion for one of the 5-sec. segments calculated by the methods AIC and MDL. The minimums for both AIC and MDL are observed at order 10.

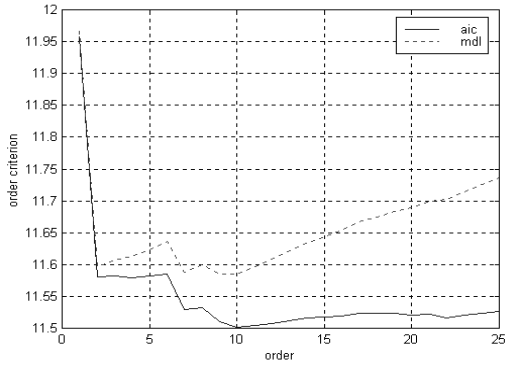

Figure 1. AIC and MDL for a 5 second long sleep EEG.

By the same method, the whole data (15 segments) is analyzed. The mean of the orders at which the minimum occurs are found to be 10. But this order cannot be an approprate order for sleep EEG because the variance of the orders at which the minimum occurs is 15. Instead of the true minimum, it's better to determine a limiting minimum, since the variation of the criterions are little at higher orders. The limiting minimum is determined as the lowest order at which the AIC is deviated less than 0.3% from the true minimum. The mean of the limiting minimums is determined at order 10 and the variance is decreased to 3.4 as expected. It's not a good idea to determine an order valid for sleep EEG. Because at some segments overestimation occurs and in some segments the order can be low to estimate the data.

It's interesting that the AR modelling is dependent on the sampling frequency. An experiment is done to investigate this property. First a 5-sec. segment is chosen and the AIC is applied to this data. As mentioned before, the sampling frequency is 100Hz. The Akaike Information Criterions for orders 1 to 25 are shown in Figure 2. The true minimum is at order 7, and the limiting minimum is at order 6. Then the same segment is down sampled to 50Hz by taking one of the samples from each of the two and AIC's is plotted in Figure 3. The true minimum and the limiting minimums are occurred at order 4. So AR power spectrum estimation can give better results for lower sampling rates.

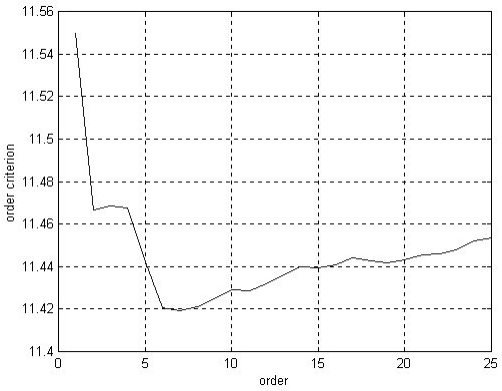

Figure 2. Illustration of AR model estimation using the criteria AIC. The data is a 5 sec. sleep EEG sampled at 100Hz.

As in the case of AIC, the FPE has two terms acting in opposite senses, σ_p^2 that decreases as p increases and the rest of the expression, which increases with p., Again the model order selected is the value that minimises the FPE. Again the same segment which was used for AIC is analysed. Again the minimum are shifted to left as the sampling frequency is decreased.

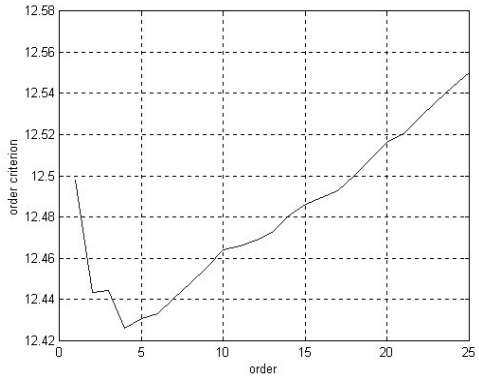

Figure 3. Illustration of AR model estimation using the criteria AIC. The data is the same with Figure 3 except that the sampling frequency is 50Hz. The minimums are shifted to left as shown.

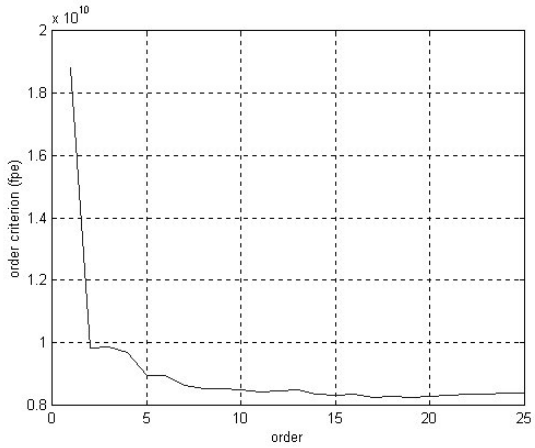

Figure 4. Illustration of AR model estimation using the criteria FPE. The data is a 5-sec. sleep EEG sampled at 100Hz.

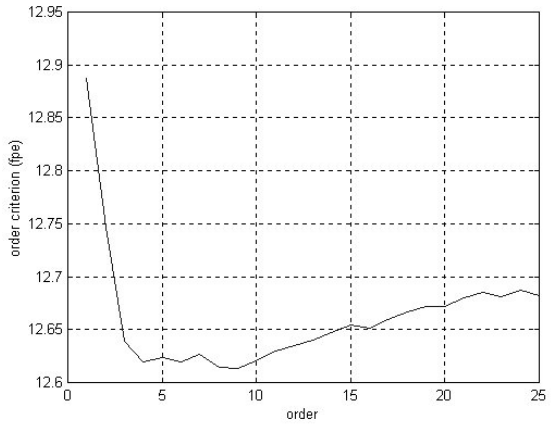

Figure 5. Illustration of AR model estimation using the criteria FPE. The data is the same with Figure 4 except that the sampling frequency is 50Hz. The minimums are shifted to left as shown.

The order estimation for the data with lower sampling frequency results in lower orders. The power spectrums are calculated for 5 segments with the data sampled both at 100 Hz and 20 Hz to see the difference. The spectrum of the data sampled at 20Hz contains components at approximately 4 Hz and 6 Hz but these frequency components cannot be seen at the spectrum of the 100Hz sampled data. Both of the power spectral densities are calculated with 10^{th} order AR modelling. These spectra's are represented in Figure 4 and Figure 5.

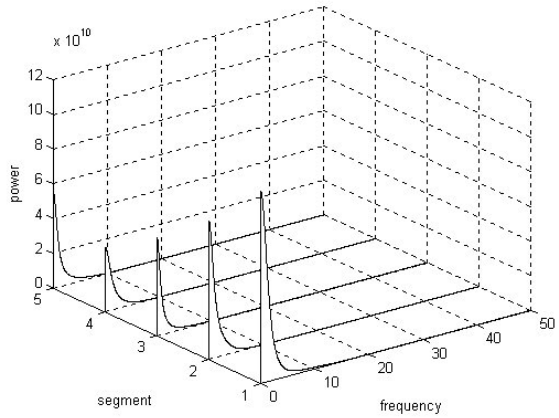

Figure 6. The power spectrum estimations of sleep EEG for 5 segments. These are 5 second long nonoverlapping segments. The sampling frequency of the data is 100Hz.

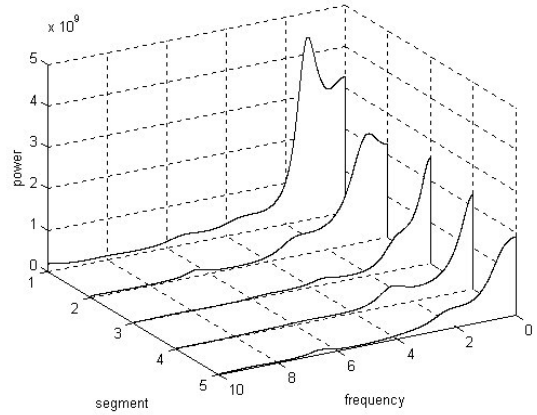

Figure 7. The power spectrum estimation of the same EEG segments in Figure 4 but now the sampling frequency is 20 Hz.

IV. CONCLUSION

For the sleep EEG analysed, the mean of the orders, which minimise the order criterions, is found to be 10 but highly varying in different segments. So it's better to select different orders for different segments to avoid overestimation or inadequate estimation. In some segments lower orders can be sufficient for estimation. Selecting a higher order for such a data sequence causes high frequency peaks, which are not desired.

For higher sampling rates the adequate orders are higher than the ones for lower sampling rates. If the order is fixed, the spectrum estimation of the data with

lower sampling rate will be better. It's known that better results are obtained as the data length increases. Increasing sampling rate also increases the data length but the result is a worse estimation.

V. REFERENCES

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