HARMONICS AND RESONANCE CONDITIONS IN THYRISTOR CONTROLLED REACTOR CIRCUITS

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ABSTRACT

In recent years, there has been a rapid increase in the number of static VAR control systems consist of capacitor banks in parallel thyristor controlled reactors used in industrial and utility systems for dynamic power factor correction and terminal voltage stabilization. Although there are a lot of advantages using for thyristor controlled reactor in power systems widely, it produces harmonics in power system. One of the effects of harmonics is resonance. In the paper the resonance conditions caused from harmonics are searched by Fourier matrix model and state variable methods. It was given a numerical application and its results by using the two methods. By the application, it was ensured whether there was high harmonics affecting the system, and if it is the detection and the control of the ones.

1. INTRODUCTION

In a fixed capacitor-thyristor controlled reactor (FC-TCR), while on the hand fixed capacitor produce reactive power, on the other hand TCR consumes reactive power. As the reactive power production of a capacitor group in a determined voltage level is fixed value, the reactive power production of the system is ensured by the change of the firing angles of the thyristor. The changing at the firing angles will control fundamental component of the reactor current and thus amplitude of the reactive power. Sometimes because of selecting not suitable for it leads to disadvantages in the determined values of an active harmonic production of the TCR in the matter resonance conditions.

From the point of view, the operating of a system there is need to accurately model the harmonics that they introduce and to understand the resonance problems they can cause. Different methods have evolved to study these problems. They can be put into three different categories. One looks at the time dependent functions by solving the equations through digital integration techniques. In another, the individual harmonics are looked at in the harmonic phasor space, treating the time functions as complex algebraic equations. In the third, the state equations are formulated and then investigated using linear system theory.

2. THYRISTOR CONTROLLED REACTOR AND HARMONICS

The basic static VAR system (SVS) consists of a static switch in series with an inductor. This is normally called a phase controlled reactor or thyristor controlled reactor (TCR). This basic TCR is illustrated in Figure 1. The thyristor controlled reactor consists of reactor in series with two parallel inverse thyristors. The two inverse parallel thyristors are gated symmetrically. They control the time for which the reactor conducts and thus control the fundamental component of the current. Each thyristor is conduction in each half cycle from the zero crossing of the voltage according to α firing angle σ conduction angle. Full conduction is obtained by 90° of firing angle. In that condition, the current is reactive and sinusoidal as real. There is a partial conduction between 90° and 180° as to be shown in figure 2a and 2b [9]. It is not allowed the firing angle between 0° and 90° causing asymmetrical currents including DC components.

Harmonics are taken place because of the phase control in a TCR. The voltage of the reactor includes various harmonics then fundamental component of the reactor current depend on chosen α or σ. While the firing angle α is increasing from 90° to 180°, the waveform of the current goes away from the original sinusoidal form. In the condition of balanced loading TCR produces odd harmonics. If it is done delta connection in TCR, 3[rd] harmonic and its times is not given to network, and these are eliminated in the delta connection. However, TCR circuits should not be operated on the points related to resonance conditions doing effective harmonic production.

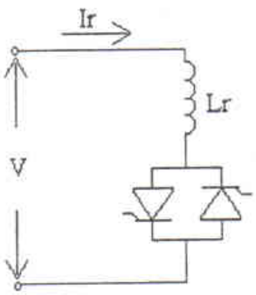

Figure 1. The main elements of thyristor controlled reactor

If the firing angle (α) or the conduction angle (σ) of the thyristors shown in figure 1 is selected appropriately, the effective value of the current $I_r(t)$ can be adjusted in convenience interval. If the voltage of TCR is assumed as cosine function:

$$V(t) = V_m \cos\omega t \tag{1}$$

In that condition, instantaneous current is given by:

$$I_r = \begin{cases} \dfrac{V_m}{\omega L_r}(\sin(\omega t) - \sin(\dfrac{\pi-\sigma}{2})), & \dfrac{\pi-\sigma}{2} \le \omega t \le \dfrac{\pi+\sigma}{2} \\ 0, & \dfrac{\pi+\sigma}{2} \le \omega t \le \dfrac{3\pi-\sigma}{2} \end{cases} \tag{2}$$

Similarly, the effective voltage of TCR is given by:

$$V = \frac{V_m}{\sqrt{2}}\sqrt{\frac{\sigma - \sin(\dfrac{\pi-\sigma}{2})}{\pi}} \tag{3}$$

It will be seen that the reactor current for the symmetric firing angles has odd harmonic components in a single phase system.

$$I_r(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t) \tag{4}$$

where b_n is the amplitude of nth harmonic. Fourier analysis of the current waveform gives the equation related to σ fundamental component.

$$I_{r1} = V_m \frac{\sigma - \sin\sigma}{X_r \pi} \tag{5}$$

where X_r is the reactance of the fundamental frequency of the reactor.

As a result of increasing the firing angle α (decreasing of the conduction angle σ), the current of the fundamental component (I_{r1}) will decrease.

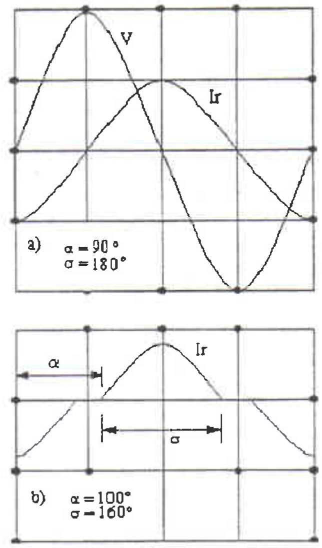

Figure 2. Voltage and Current Waveforms in a TCR

The relation between firing angle and conduction angle as follows:

$$\sigma = 2(\pi - \alpha) \tag{6}$$

When the fundamental frequency component of the current is considered, TCR is controllable susceptance. At the fundamental frequency effective susceptance is given the function of σ as follows [3]:

$$B(\sigma) = \frac{I_{r1}}{V_m} = \frac{\sigma - \sin\sigma}{X_r \pi} \tag{7}$$

The variation is shown in the B (σ) sigma plots for the fundamental harmonic component in Figure 3. The maximum value of effective susceptance happens in the condition of full conduction of α=90° or σ=180° and its value is $1/X_r$. Minimum value of it is zero in the condition of α=180° or σ=0°.

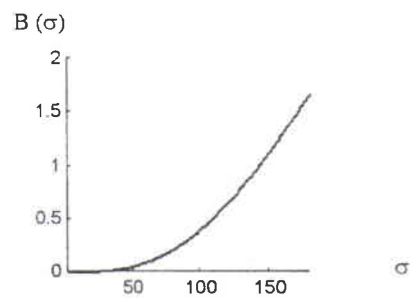

Figure 3. Susceptance versus Conduction Angle

3. TCR AND REZONANCE CONDITIONS

The high harmonic currents happened in the system in the special conditions can cause to be resonance the capacitors used for compensation with other components' inductances in the circuit. In the paper, firstly, if there are resonance points as a result of harmonics, they are detected and then, system analysis can be done.

In the method, after it is entered data of a system including TCR, it is determined lower value of natural frequency (ω_{ol}) and upper value of natural frequency (ω_{ou}) for the values of conduction angle σ as 0° and 180°. Natural frequency is defined by the following equation:

$$\omega_0 = \sqrt{X_{cn} \frac{X_{sn} + X_m}{X_{sn} . X_m}} \quad (8)$$

where X_{cn}, X_{sn} and X_m are nth harmonic component of capacitor reactance, nth harmonic one of system reactance and nth harmonic one of TCR reactance, respectively.

It is looked whether the values include odd harmonics and if it is, it is determined what the values of conduction angle σ put the system into resonance between 0° and 180°. Resonance condition for nth harmonic component is given by:

$$\frac{X_{sn} . X_m}{X_{sn} + X_m} = X_{cn} \quad (9)$$

4. ANALYSIS OF A THYRISTOR CONTROLLED REACTOR

4.1. FOURİER MATRİX ANALYSIS

To develop a more equation frequency plane model, the circuit in figure 4 was analyzed in detail. The represent a FC-TCR static VAR compensator connected to an AC system configured as a system impedance, L_s, and a voltage source, V.

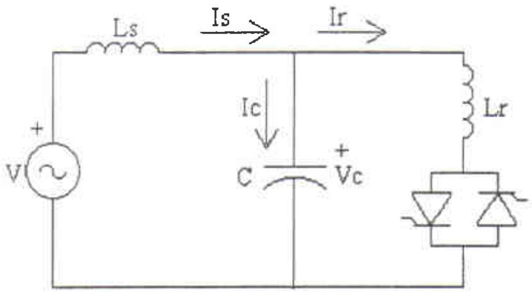

Figure 4. Fixed Capacitor-Thyristor Controlled Reactor Static VAR compensator.

In the circuit I_s, I_r and I_c currents can be defined as follows:

$$I_s = \frac{1}{L_s} \int (V - V_c) dt \quad (10)$$

$$I_r = \frac{1}{L_r} \int V_c H(t) dt \quad (11)$$

$$I_c = C \frac{dV_c}{dt} \quad (12)$$

where H(t) is a switching function and it has a magnitude of unity whenever a thyristor is on and value of zero whenever both thyristors are off (the assumption is being made that the thyristors act as perfect switches, turn on and turn off behavior is being ignored) [4].

The currents can be summed at the center node.

$$I_s = I_r + I_c \quad (13)$$

The equation for the system:

$$\frac{dI_r}{dt} = \frac{1}{Lr} H(t) V_c \quad (14)$$

$$\frac{d^2 V_c}{dt^2} + \left(\frac{1}{CL_s} + \frac{H(t)}{CL_r} \right) V_c = \frac{V}{CL_s} \quad (15)$$

can be obtained.

In a TCR the current is phase shifted 90° from the voltage and is thus symmetrical about the voltage zero crossing. The dependence of H(t) on the voltage has switched from one of sensitivity to disturbances in the zero crossing to one of sensitivity to the symmetry of the voltage about the zero crossing. Thus, the assumption of the independence of H(t) can be realistically made as long as the changes in the voltage are small.

Using the above equations (14) and (15) are linear differential equations with periodic coefficients [6]. They can be easily transformed into a system of three first order inhomogeneous differential equations with periodic coefficients.

As a conclusion of the analysis and interpretation of the current and voltage waveforms, the physical equivalent of TCR can be obtained as to be seen in figure 5 [2].

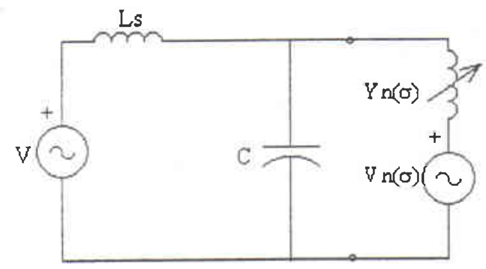

Figure 5. Variable Voltage Source and Admittance Model for a TCR

The existence function H (t) is given by:

$$H(t) = \frac{\sigma}{\pi} + \sum_{1}^{\infty} \frac{2}{n\pi} \cos(n\pi)\sin(n\sigma)\cos(2n\omega t) \quad (16)$$

If the source voltage is:

$$V = V_m \cos\omega t \quad (17)$$

nth harmonic components of the current and voltage can be written as below, respectively.

$$I_{rn} = \frac{4V_m}{n\omega L_r\pi}\left(\frac{\cos(n(\frac{\pi-\sigma}{2}))\sin(\frac{\pi-\sigma}{2})}{n^2-1} - \frac{n\sin(n(\frac{\pi-\sigma}{2}))\cos(\frac{\pi-\sigma}{2})}{n^2-1}\right)\sin(n\omega t) \quad (18)$$

$$V_n = \frac{-4V_m}{\sigma - \frac{\sin(n\sigma)}{n}}\left(\frac{\cos(n(\frac{\pi-\sigma}{2}))\sin(\frac{\pi-\sigma}{2})}{n^2-1} - \frac{n\sin(n(\frac{\pi-\sigma}{2}))\cos(\frac{\pi-\sigma}{2})}{n^2-1}\right) \quad (19)$$

where the current and the voltage consist of only odd harmonics, and the amplitudes are the functions of the conduction angle. As an example, for σ is 180° (α is 90°) each odd harmonics are shown in figure 6.

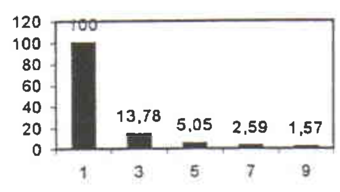

Figure 6. The maximum amplitudes of the odd harmonics in TCR

According to the model the fundamental frequency source has no affect on the harmonic currents because it acts at a different frequency. Using the harmonic voltage (V_n) and the harmonic current (I_m) the admittance (Y_n) of TCR can be easily calculated. Y_n would be:

$$Y_n = \frac{I_n}{V_n} \quad (20)$$

If equations (18) and (19) are locates in (20) equation, Y_n would be as below:

$$Y_n = -j\frac{\sigma - \frac{\sin(n\sigma)}{n}}{n\omega L_r \pi} \quad (21)$$

4.2. THE STATE VARIABLES METHOD

The circuit shown in figure 4 can be solved using state variable methods.

Let $x_1 = V_c$, $x_2 = I_c/C$ and $\dot{x}_1, \dot{x}_2$ be derivative of x_1, x_2 respectively, then state-space equation of the system can be written in the form of matrix.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -(\frac{1}{CL_s} + \frac{H(t)}{CL_r}) & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{CL_s} \end{bmatrix} V \quad (22)$$

It can be written as below:

$$\dot{x}(t) = [A(t)]x(t) + f(t) \quad (23)$$

This is a linear equation with time varying, periodic coefficient. Both the matrix [A(t)] and the vector f(t) are periodic with the same periodic. In general, the state equations for any switching circuit will be linear with periodic coefficient.

The solution to the linear equation (23) is given by:

$$x(t) = [\Phi_0(t)]x_0 + \int_{t_0}^{t} [\Phi_0(t)]f(t)dt \quad (24)$$

where $[\phi_0(t)]$ is a fundamental matrix. This matrix can be written as[7]:

$$[\phi_0(t)] = [R(t)]\exp([\Gamma_0]t) \quad (25)$$

Where [R(t)] is a continues, periodic matrix with the same period as [A(t)] and $[\Gamma_0]$ is a constant matrix.

A resonance condition exists if the natural frequency of x(t) is the same as a harmonic of forcing function. This is equivalent to saying that x(t) has a period of T, the period of f(t). By the nature of the system, x(t) also needs to have half wave odd symmetry. A condition for resonance can be found by applying this. To find it look at the homogeneous equation for x(t) [1].

Using half wave symmetrical:

$$\det[I + \exp([\Gamma_0]\frac{T}{2})] = 0 \qquad (26)$$

can be obtained.

5. NUMERICAL APPLICATION

In this paper, the numerical application has been done and studied the resonance analysis in the conditions including TCR by using two different methods. The values in the application as per-unit has been taken X_r = 0.6 pu, X_c = 2.0 pu and fundamental frequency 50 Hz. Firstly, Fourier matrix analysis was used. A computer program was developed for different systems parameters. The results were given in Table 1.

X_s	Effective harmonic order	$\omega_{0l} \le \omega_0 \le \omega_{0u}$	Cunduction angle (σ)
0.025	9	$8.94 \le \omega_0 \le 9.13$	$12^0 \le \sigma \le 102^0$
0.035	-	$7.56 \le \omega_0 \le 7.78$	-
0.042	7	$6.9 \le \omega_0 \le 7.14$	$45^0 \le \sigma \le 90^0$
0.045	-	$6.67 \le \omega_0 \le 6.91$	-
0.055	-	$6.03 \le \omega_0 \le 6.3$	-
0.065	-	$5.55 \le \omega_0 \le 5.84$	-
0.075	-	$5.16 \le \omega_0 \le 5.48$	-
0.082	5	$4.94 \le \omega_0 \le 5.27$	$30^0 \le \sigma \le 38^0$
0.085	5	$4.85 \le \omega_0 \le 5.18$	$62^0 \le \sigma \le 97^0$
0.25	3	$2.83 \le \omega_0 \le 3.37$	$56^0 \le \sigma \le 57^0$

Table 1. The Result of Using Fourier Matrix Analysis

In order to compare the results as a second method it was also developed another computer program used state variable method and the results also were given in Table 2.

X_s	Effective harmonic order	$\omega_{0l} \le \omega_0 \le \omega_{0u}$	Cunduction angle (σ)
0.025	9	$8.94 \le \omega_0 \le 9.13$	$19^0 \le \sigma \le 99^0$
0.035	-	$7.56 \le \omega_0 \le 7.78$	-
0.042	7	$6.9 \le \omega_0 \le 7.14$	$50^0 \le \sigma \le 101^0$
0.045	-	$6.67 \le \omega_0 \le 6.91$	-
0.055	-	$6.03 \le \omega_0 \le 6.3$	-
0.065	-	$5.55 \le \omega_0 \le 5.84$	-
0.075	-	$5.16 \le \omega_0 \le 5.48$	-
0.082	5	$4.94 \le \omega_0 \le 5.27$	$34^0 \le \sigma \le 35^0$
0.085	5	$4.85 \le \omega_0 \le 5.18$	$69^0 \le \sigma \le 105^0$
0.25	3	$2.83 \le \omega_0 \le 3.37$	$52^0 \le \sigma \le 55^0$

Table 2. The Result of Using Linear System Theory.

6. CONCLUSIONS

The system which will not produce high harmonics is designed. At the beginning if there are harmonics in the system, filters must be used to eliminate. One of the negative effects of harmonics lead to the system is resonance. In this study, resonance conditions are determined. It is explained operating conditions to put the system into resonance in the paper for all two methods were obtained close results. Operating point and parameter values of a FC-TCR can readily influence series/parallel resonant frequencies of a system and consequently tune the resonant conditions. Thus, the model used for the firing circuitry should generate actual firing instants. Otherwise, the amplitudes and orders of noncharacteristic harmonics will be noticeably distorted as a result of improper firing instant [8]. These approaches designing for the minimization of effects of system harmonics help us in the operation of the system and as a result they provides steady-state operations of the system.

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