Arbitrage Opportunities between Energy, Bilateral Contracts and Ancillary Service Markets

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Abstract

A competitive generation company (GENCO) can maximize its profit by discovering arbitrage opportunities in electricity markets. This paper formulates a GENCO's arbitrage problem using price-based unit commitment (PBUC). The GENCO could consider arbitrage between local generation and purchase from energy market for its bilateral contracts, as well as simultaneous trades with spot markets for energy and ancillary services. In this work, arbitrage opportunities among energy, bilateral contract, and ancillary service are discussed based on PBUC that solved by dynamic programming. Given forecasted hourly market prices, a single unit is considered for the analysis of the arbitrage problem. Based on the case studies, the effect of arbitraging for profit maximization is investigated and verified by computer simulations.

NOMENCLATURE

$B(t)$	Power purchase of the unit at time t
$C(.)$	Cost function of the unit, $C(x) = a + bx + cx^2$
$F(t)$	Profit of the unit at time t
$f(P_0(t))$	Profit from bilateral contract of the unit at time t
$I(t)$	Commitment state of the unit at time t, (1:ON), (0:OFF)
$P(t)$	Generation of the unit at time t
$P_{g\min}$	Minimum generation of the unit
$P_{g\max}$	Maximum generation of the unit
$P_0(t)$	Bilateral contract of the unit at time t
$R(t)$	Spinning reserve of the unit at time t
$R_{\min}$	Minimum spinning reserve of the unit
$R_{\max}$	Maximum spinning reserve of the unit
$S(t)$	Start-up cost of the unit at time t
$\rho_m(t)$	Forecasted market price for energy at time t
$\rho_r(t)$	Forecasted market price for spinning reserve at time t
ρ_b	Bilateral contract price of the unit
t	Hour index

I. INTRODUCTION

In energy markets, arbitrage refers to making profit by a simultaneous purchase and sale of the same or equivalent commodity with net zero investment and without any risk [1]. Arbitrage also refers to any activity that attempts to buy a relatively under-priced commodity and to sell a similar and relatively over-priced commodity for profit. There are two types of arbitrage that are considered in a power market: same-commodity arbitrage and cross-commodity arbitrage. When arbitrage is aimed at the same product (e.g., electricity), it is called same-commodity arbitrage which includes arbitrage of bilateral contract and arbitrage between energy and ancillary service markets. Arbitrage that is aimed at different products within a market or different markets is called cross-commodity arbitrage (e.g. arbitrage between fuel market and electricity market).

Arbitrage strategies can play a very important role in maximizing a generation company's (GENCO's) profit. The arbitrage opportunities among energy, bilateral contract, ancillary services, fuel, and emission allowance markets were explored individually using the Lagrangian relaxation (LR) approach in [1]. The simultaneous optimization of arbitrage opportunities in various markets for given market prices was considered in [2] based on a deterministic price-based unit commitment (PBUC)[3]. An arbitrage model which minimizes risk due to market price uncertainties was also proposed in [4].

In this work same-commodity arbitrage opportunities (i.e., between energy, bilateral contracts, and ancillary service) are discussed for a GENCO with one thermal unit. Cross-commodity arbitrage is beyond the scope of this work. Based on the forecasted market prices, the arbitrage problem is formulated as a mixed integer problem using the PBUC model. The problem is solved using incremental cost (i.e., marginal cost) methodology for each state and the optimum commitment schedule is found using dynamic programming. Then, the impacts of the considered arbitrage opportunities to the GENCO's profit are analyzed comparatively for various cases.

The paper is organized as follows: the formulation of the arbitrage problem is given in Section II. Section III gives the numerical examples for various arbitrage opportunity cases. The conclusions are provided in Section IV.

II. Problem Formulation

A. Objective Function for a GENCO

In restructured power markets, a GENCO intends to maximize its profit (i.e., revenue minus cost) which is given for a single unit for time t in (1).

$$\begin{aligned} F(t) = \{ & \rho_m(t)[P(t) - P_0(t)] + \rho_r(t)R(t) \\ & - C(P(t)+R(t)) - S(t) + f(P_0(t)) \} I(t) \\ & + \{ -\rho_m(t)B(t) + f(P_0(t)) \} (1 - I(t)) \end{aligned} \quad (1)$$

The list of symbols is given in the Nomenclature. In this formulation, GENCO with single thermal unit is considered. For the sake of simplicity, among the ancillary services only the spinning reserve is included, and it is assumed that all spinning reserve is called upon in real time by the Independent System Operator (ISO).

The first part of (1) [multiplied by *I(t)*] represents the profit when the unit is ON. Optimizing *P(t)* and *R(t)* (which corresponds to arbitrage between energy and spinning reserve) the GENCO can maximize its profit. The second part of (1) [multiplied by (1-*I(t)*)] represents the profit when the unit is OFF. In this state the GENCO does not generate *P(t)* and *R(t)*, it only satisfies the bilateral contract by purchasing energy *B(t)* from the market (which corresponds to arbitrage of bilateral contract). In the scheduling horizon, the total profit is the sum of the profits at each time t and the PBUC problem is formulated as follows:

$$\text{maximize} \sum_t F(t) \quad (2)$$

subject to the constraints discussed below.

A unit has minimum and maximum generation limits when it is ON such that;

$$P_{g\min} \le P(t) \quad (3)$$

$$P(t) + R(t) \le P_{g\max} \quad (4)$$

If there are any bilateral contracts, the GENCO may satisfy them either by local generation or by purchases from the market;

$$P(t)I(t) + B(t) \ge P_0(t) \quad (5)$$

$$0 \le B(t) \le P_0(t) \quad (6)$$

The limits of the spinning reserve generation are given as follows:

$$0 \le R(t) \le R_{\max} \quad (7)$$

where $R_{\max}$ is the maximum spinning reserve capacity of the unit (which represents the synchronized generation that can ramp up in 10 minutes). Start-up time and shut-down time constraints are ignored in this srudy to simplify the problem. After the arbitrage problem is formulated applying PBUC, the solution method will be discussed in the next part.

B. PBUC Solution Method

In order to solve the PBUC problem, first incremental cost (i.e., marginal cost) methodology is used for each state (ON and OFF) and for each time period, and then the optimum commitment schedule is found using dynamic programming. The original maximization objective is equivalent to the minimization of a revised objective function which is specified as follows:

$$\text{Minimize} \sum_t -F(t) \quad (8)$$

When the unit is ON, using (1) and (8) the objective function becomes;

$$\begin{aligned} -F(t) = & -\rho_m(t)[P(t) - P_0(t)] - \rho_r(t)R(t) \\ & + C(P(t)+R(t)) + S(t) - f(P_0(t)) \end{aligned} \quad (9)$$

Since the start-up cost depends on the previous unit commitment state, it is excluded at this stage, as to be added in the dynamic programming part. The income from bilateral contract is constant; therefore, it does not affect the optimal solution point and it is also excluded from the objective function. Consequently, the objective function becomes

$$\text{minimize} \quad \begin{aligned} & -\rho_m(t)[P(t) - P_0(t)] - \rho_r(t)R(t) \\ & + C(P(t)+R(t)) \end{aligned} \quad (10)$$

The necessary conditions for the optimal solution point are

$$\frac{\partial F(t)}{\partial R(t)} = 0 \text{ and } \frac{\partial F(t)}{\partial P(t)} = 0 \quad (11)$$

Since spinning reserve market price is higher than the energy market price, selling spinning reserve is more advantageous than selling energy; therefore, first the spinning reserve is optimized as follows:

$$\frac{\partial F(t)}{\partial R(t)} = \rho_r(t) - b - 2c(P(t)+R(t)) = 0 \quad (12)$$

where *b* and *c* are cost function parameters (given in Nomenclature). To maximize profit, the upper limit of the higher price commodity (*R(t)*) in [*P(t)*+*R(t)*] should be maximized. Therefore, *P(t)* should be set to minimum $P(t) = P_{min}$. Then the optimum *R(t)* becomes;

$$R_{opt} = \frac{\rho_r(t) - b}{2c} - P_{\min}. \quad (13)$$

If the obtained R_{opt} value is below or above the limits of the constraint, it is confined to R_{min} or R_{max}, respectively. After spinning reserve *R(t)* is optimized, using this optimum R_{opt} value, the lower price commodity (energy, *P(t)*) is optimized as follows:

$$\frac{\partial F(t)}{\partial P(t)} = \rho_m(t) - b - 2c(P(t) + R_{opt}) = 0 \tag{14}$$

$$P_{opt} = \frac{\rho_m(t) - b}{2c} - R_{opt} \tag{15}$$

Similarly, if the obtained P_{opt} is below P_{gmin} or above (P_{gmax}-R_{opt}), it is confined to P_{gmin} or (P_{gmax}-R_{opt}), respectively.

After finding the final optimum R_{opt} and P_{opt} values, the minimum cost is obtained using (10) for ON state for given forecasted market prices.

When the unit is OFF, similarly, the income from bilateral contract is constant; therefore, it is excluded from the objective function, and since

$$B(t) = P_0(t) \tag{16}$$

using (1) and (8), the objective function becomes,

$$\text{minimize } \rho_m(t)P_0(t). \tag{17}$$

After the minimum costs are found for ON and OFF state at each time period, the minimum cost unit commitment schedule is obtained using dynamic programming. This process is illustrated in Fig. 1.

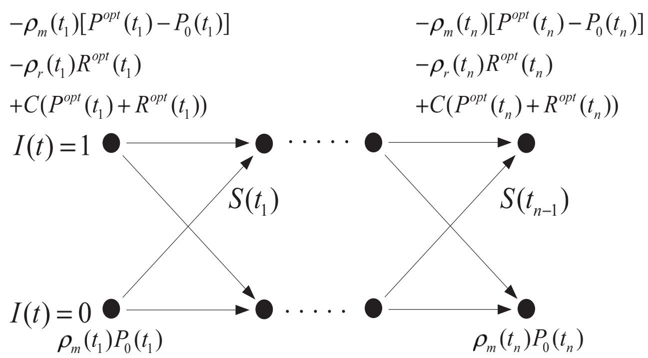

Fig. 1. Dynamic programming problem.

Finally, the maximum total profit is found by subtracting the minimum total cost (obtained by dynamic programming) from the bilateral contract income as follows:

$$\text{Total profit} = \sum_t \rho_b P_0(t) - (\text{Total cost}) \tag{18}$$

The flow chart of the problem solution is given in Fig. 2.

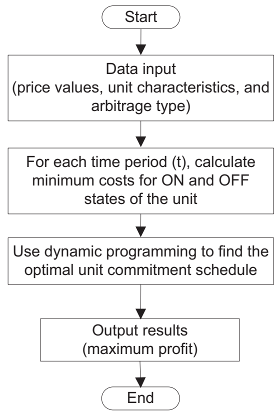

Fig. 2. The flow chart of the arbitrage problem solution.

III. Case Studies

In this section, the considered arbitrage opportunities are studied for a generic single unit. The data on the considered generator unit and bilateral contract are given in Table I. The hourly market prices for the energy and the spinning reserve are given in Table II. The spinning reserve prices are assumed to be 1$/MWh higher than the energy prices at all hours.

TABLE I
The Unit Characteristics and Bilateral Contract

P_{gmax}	1000 MW
P_{gmin}	100 MW
R_{min}	0
R_{max}	400 MW
C(x)	$310 + 6.85x + 0.00384x^2$ \$/h
P_0	100 MW
ρ_b	9 $/MWh
S(t)	100 $

Case 1) No Arbitrage

In this case it is considered that the unit participates only in energy market (i.e., *R(t)*=0 all the time) and the bilateral contract is fulfilled by only local generation (i.e., *B(t)*=0 all the time). Therefore, the GENCO has no arbitrage opportunity in this case.

Case 2) Arbitrage of Bilateral Contract

In this case it is considered that the unit participates only in energy market (i.e., *R(t)*=0 all the time), but the bilateral contract is fulfilled either by local generation or energy purchases from the market. When the market price is high, the GENCO can satisfy bilateral contract by local generation, and when the market price is low, the GENCO can prefer energy purchases from the market, based on its start-up time & shut-down time constraints.

TABLE II
MARKET PRICES FOR ENERGY AND SPINNING RESERVE FOR 24 HOURS

Hour	Energy Price ($/MWh)	Spinning Reserve Price ($/MWh)
1	12	13
2	8.6	9.6
3	8	9
4	5.9	6.9
5	4	5
6	6	7
7	8.4	9.4
8	9.1	10.1
9	10.3	11.3
10	8	9
11	11.7	12.7
12	16.3	17.3
13	19	20
14	21.6	22.6
15	24	25
16	24.2	25.2
17	19.4	20.4
18	14	15
19	12.5	13.5
20	9.6	10.6
21	9.1	10.1
22	8.45	9.45
23	6	7
24	4.5	5.5

The following four cases are analyzed in this work:

Case 3) Arbitrage between Energy and Ancillary Service

In this case it is considered that the unit participates both in energy and ancillary service markets, but the bilateral contract is fulfilled by only local generation (i.e., $B(t)$=0 all the time). If the price of spinning reserve is higher than that of energy, the GENCO may reduce the sale of energy and decide to sell more spinning reserve.

Case 4) Arbitrage among Energy, Bilateral Contract, and Ancillary Service

In this case it is considered that the unit participates both in energy and ancillary service markets, and the bilateral contract is fulfilled either by local generation or energy purchases from the market. Therefore, the GENCO can consider arbitrage opportunities among energy, bilateral contract, and spinning reserve simultaneously.

The case studies are simulated using MATLAB ™ [5]. In the following, the results for all cases described above are discussed comparatively. Table III illustrates the total profits for the considered cases. The profit in Case 1 where there is no arbitrage opportunity is the lowest of all (56764$).

In Case 2, the profit is increased compared to Case 1 by 3182$ (5.6% increase) by arbitraging between local power generation and power purchase from the market depending on the energy price. Case 2 shows how the arbitrage between local generation and purchase from the market can provide more profit than the local generation alone.

In Case 3, the profit is also increased compared to Case 1 by 5992$ (10% increase), when the GENCO participates both in energy and ancillary service markets. Case 3 illustrates that the arbitrage between energy and ancillary service can improve the profitability compared to participating only energy market.

In Case 4 the profit is the highest (65301$) when the GENCO arbitrages simultaneously between energy, bilateral contract, and ancillary service (15% increase compared to Case 1). From the above analysis, it can be concluded that depending on arbitrage opportunities, the GENCO can increase its profit.

TABLE III
TOTAL PROFITS FOR THE CONSIDERED CASES

Case 1	56764 $
Case 2	59946 $
Case 3	62756 $
Case 4	65301 $

Fig. 3 shows the market price variations that considered in all case studies for energy and spinning reserve over 24 hours. It should be noted that spinning reserve price is assumed to be 1$/MWh higher than that of energy all the time.

In Fig. 4 energy generation of the unit is shown for Case 1 (no arbitrage case). As seen the generation of the unit approximately follows the energy price. As the energy price increases, the unit also increases its generation. At hours 12-17 where price is relatively high, it reaches its maximum generation capacity (1000MW). At hours 4-6 and 23-24, the unit generates at its minimum capacity (100MW), where it only satisfies the bilateral contract and sells no energy to the market. Since it can fulfill the bilateral contract by only local generation, it never shuts down and always remains at ON state. The summary of the schedule for Case 1 is shown in Table IV.

In Fig. 5 energy generation of the unit is shown for Case 2 (arbitrage between energy and bilateral contract). When the energy price is high, the unit is ON and it satisfies the bilateral contract by local generation. Also the generation of the unit at ON states is similar to the Case 1, as expected. However, at hours 2-7, 10 and 22-24 where the energy price is relatively low, the unit remains OFF and purchase energy from the market to fulfill its bilateral contract. The summary of the schedule for Case 2 is shown in Table V. Note that, given ignoring start-up time and shut-down time, the unit is able to be ON and OFF at each hour.

In Fig. 6, energy and spinning reserve generations of the unit are shown for Case 3 (arbitrage between energy and ancillary service). As the market prices increase, the unit first fills its spinning reserve capacity which has higher price than the energy, and similarly, as the market prices fall, the unit first decreases its energy generation which has lower price than the spinning reserve. At hours 4-6 and 23-24, where the market prices are relatively low, it sells no energy and no spinning reserve, it only satisfies the bilateral contract by local generation (since it has no power purchase opportunity in this case). It should be noted that when the spinning reserve is at the maximum (400MW), the maximum energy generation is limited to 600MW since P_{gmax}=1000MW. The summary of the schedule for Case 3 is shown in Table VI.

In Fig. 7 energy and spinning reserve generations of the unit are shown for Case 4 (arbitrage among energy, bilateral contract, and ancillary service). Similar to Case 3, as the market prices increase, the unit first fills its spinning reserve capacity which has higher price than the energy, and while the markets prices are falling, the unit first decreases its power generation which has lower price than the spinning reserve. Different from Case 3, the unit has an arbitrage opportunity between local power generation and power purchase from the market. Therefore, at hours 3-6, 10, and 23-24 where the market prices are relatively low, the unit is OFF and purchases energy from the market to fulfill the bilateral contract. As in Case 3, when the spinning reserve is at the maximum (400MW), the maximum energy generation is limited to 600MW since P_{gmax}=1000MW. The summary of the schedule for Case 4 is shown in Table VII.

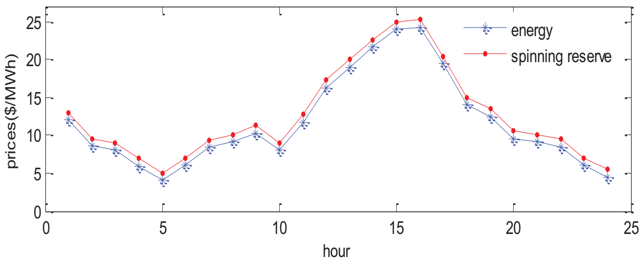

Fig. 3. Hourly market prices for the energy and the spinning reserve.

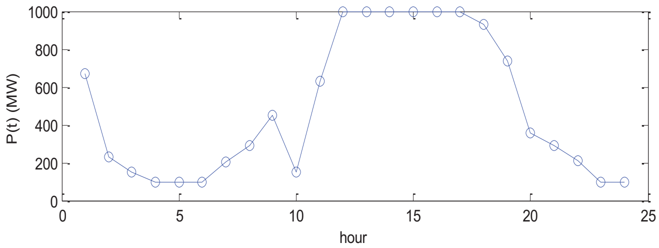

Fig. 4. Power generation P(t) for Case 1.

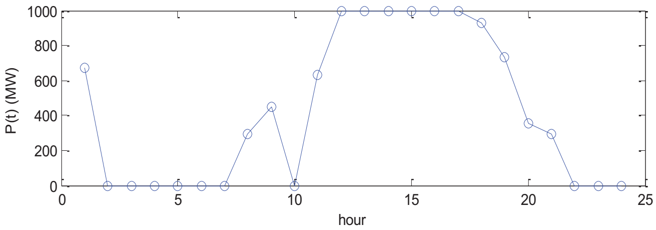

Fig. 5. Power generation P(t) for Case 2.

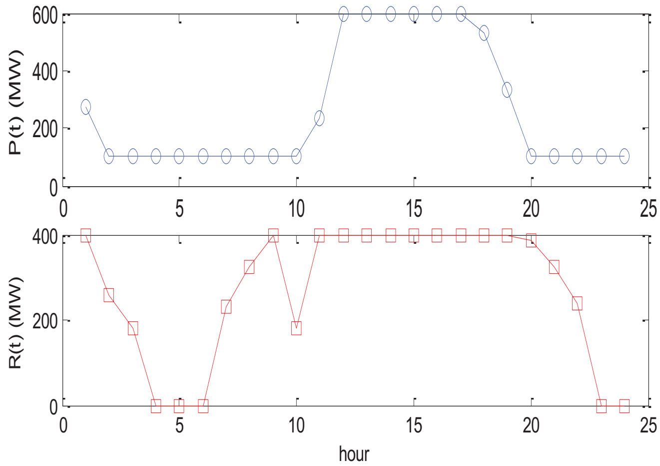

Fig. 6. Power P(t)(blue) and spinning reserve R(t)(red) generation for Case 3.

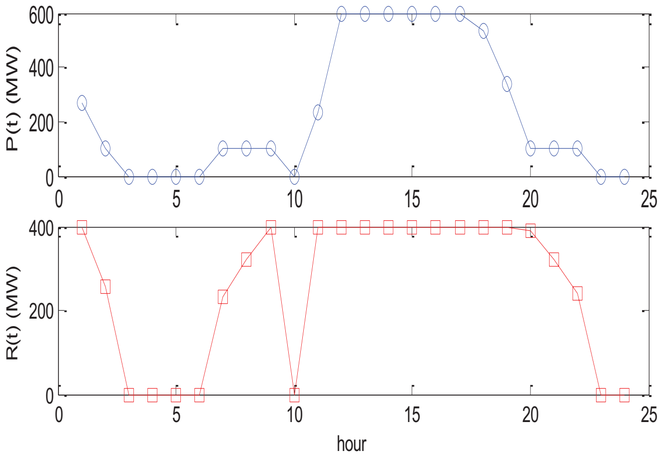

Fig. 7. Power P(t)(blue) and spinning reserve R(t)(red) generation for Case 4.

IV. Conclusions

In restructured power markets, a GENCO could maximize its profit by considering arbitrage opportunities. In this paper, the arbitrage opportunities among energy, bilateral contract, and ancillary services have been formulated applying PBUC and it has been shown that arbitrage could have a significant impact on a GENCO's profit.

In this work, for the sake of simplicity, one single unit has been considered, and non-spinning reserve has not been included. Also all spinning reserve is assumed to be called upon in real time by the ISO. Although the results are consistent and they show the benefit of arbitrage, inclusion of additional arbitrage commodities (such as non-spinning reserve, fuel, etc.) and multiple units increase the complexity of the problem which should then be handled using LR or more effective solution methods.

As a future work, considering GENCOs with multiple units can be considered. Also including non-spinning reserve (which can be sold when unit is OFF) as an ancillary service, and cross-commodity arbitrage (such as between gas market and electricity market) could show the significance of arbitrage more noticeably. It is also very important to consider the risks due to the market price uncertainties.

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[5] http://www.mathworks.com/products/matlab/.

V. APPENDICES

TABLE IV
THE UNIT SCHEDULE FOR CASE 1

Hour	Energy (MW)	Spinning Reserve (MW)	Bilateral (MW)	Purchase (MW)
1	670.57	0	100	0
2	227.86	0	100	0
3	149.74	0	100	0
4-6	100	0	100	0
7	201.82	0	100	0
8	292.97	0	100	0
9	449.22	0	100	0
10	149.74	0	100	0
11	631.51	0	100	0
12-17	1000	0	100	0
18	930.99	0	100	0
19	735.68	0	100	0
20	358.07	0	100	0
21	292.97	0	100	0
22	208.33	0	100	0
23-24	100	0	100	0

TABLE V
THE UNIT SCHEDULE FOR CASE 2

Hour	Energy (MW)	Spinning Reserve (MW)	Bilateral (MW)	Purchase (MW)
1	670.57	0	100	0
2-7	0	0	100	100
8	292.97	0	100	0
9	449.22	0	100	0
10	0	0	100	100
11	631.51	0	100	0
12-17	1000	0	100	0
18	930.99	0	100	0
19	735.68	0	100	0
20	358.07	0	100	0
21	292.97	0	100	0
22-24	0	0	100	100

TABLE VI
THE UNIT SCHEDULE FOR CASE 3

Hour	Energy (MW)	Spinning Reserve (MW)	Bilateral (MW)	Purchase (MW)
1	270.57	400	100	0
2	100	258.07	100	0
3	100	179.95	100	0
4-6	100	0	100	0
7	100	232.03	100	0
8	100	323.18	100	0
9	100	400	100	0
10	100	179.95	100	0
11	231.51	400	100	0
12-17	600	400	100	0
18	530.99	400	100	0
19	335.68	400	100	0
20	100	388.28	100	0
21	100	323.18	100	0
22	100	238.54	100	0
23-24	100	0	100	0

TABLE VII
THE UNIT SCHEDULE FOR CASE 4

Hour	Energy (MW)	Spinning Reserve (MW)	Bilateral (MW)	Purchase (MW)
1	270.57	400	100	0
2	100	258.07	100	0
3-6	0	0	100	100
7	100	232.03	100	0
8	100	323.18	100	0
9	100	400	100	0
10	0	0	100	100
11	231.51	400	100	0
12-17	600	400	100	0
18	530.99	400	100	0
19	335.68	400	100	0
20	100	388.28	100	0
21	100	323.18	100	0
22	100	238.54	100	0
23-24	0	0	100	100