Maximum A Posteriori Channel Estimation for Space-Frequency Block Coded OFDM Systems

Erdal Panayırcı†, Hakan A. Cirpan‡ and Hakan Dogan‡

†Department of Electronics Engineering, Işık University, Maslak 80670, Istanbul, Turkey

‡Department of Electrical Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey

***Abstract*— Incorporating subchannel grouping, space-frequency coding for transmit diversity orthogonal frequency division multiplexing (OFDM) systems has been proposed recently to achieve maximum diversity gain. Focusing on space-frequency transmit diversity OFDM transmission through frequency selective channels, this paper proposes a computationally efficient, non-data-aided maximum a posteriori(MAP) channel estimation algorithm. The algorithm requires a convenient representation of the discrete multipath fading channel based on the Karhunen-Loeve orthogonal expansion and estimates the complex channel parameters of each subcarriers iteratively using the Expectation Maximization(EM) method, which converges to the true MAP estimation of the unknown channel. An analytical expression is derived for the Modified Cramer-Rao lower bound of the proposed MAP channel estimator.**

I. Introduction

The overwhelming growth of broadband wireless services usage together with the scarcity of bandwidth resources, motivate intense focus of research toward developing efficient coding and modulation schemes that improve the quality and bandwidth efficiency of wireless systems. One approach that shows real promise to overcome the limitations imposed by fading channel, and provide reliable transmission and high spectrum efficiency is to combine two powerful technologies in the physical layer: space-time coding (STC) and OFDM modulation [1],[2].

STC has been proved effective in combating fading and increasing channel capacity without necessarily sacrificing bandwidth efficiency [3],[4],[5]. There is in fact a diversity gain that results from multiple paths between base station and user terminal, and a coding gain that results from how symbols are correlated across transmit antennas. Unfortunately, most existing space-time coding schemes have been developed for flat fading channels initially. Therefore, their successful implementation over broad-band frequency selective channel requires the development of sophisticated signal processing algorithms for channel estimation, and joint equalization/decoding. This task is quite challenging with multiple transmit antennas due to the long delay spread of broad-band channels, which increases the number of channel parameters to be estimated and the number of states in joint equalization/decoding. This, in turn, places significant additional computational load which motivates a more practical reduced-complexity space-time coded OFDM (ST-OFDM) structure [1]. OFDM is chosen over a single-carrier solution due to lower complexity of equalizers for long delay spread channels [6]. In OFDM, a broadband signal is broken down into multiple narrowband carriers, where each carrier is more robust to multipath. OFDM can be implemented efficiently by using fast Fourier transforms (FFTs) at the transmitter and receiver. At the receiver, FFT reduces the channel response into a multiplication constant on each tone. The combined application of OFDM modulation and space-time coding allows us to avoid the complexity of space-time equalizers and therefore yields a unique reduced-complexity physical layer capabilities [1].

The use of OFDM in transmitter diversity systems also offers the possibility of coding in a form of space-frequency OFDM (SF-OFDM) [3]. In [2], it was shown that the SF-OFDM system has the same performance as a previously reported ST-OFDM scheme in slow fading environments but shows better performance in the more difficult fast fading environments. This paper therefore focuses on channel estimation approach for SF-OFDM systems. In this paper, a computationally efficient, non-data-aided maximum a posteriori(MAP) channel estimation algorithm is proposed for orthogonal frequency division multiplexing (OFDM) systems with transmitter diversity using space-frequency block coding. In the development of the MAP channel estimation algorithm, the channel taps are assumed to be random processes. Moreover, orthogonal series expansion based on the Karhunen-Loeve expansion of a random process is applied which makes the expansion coefficient r.v.'s uncorrelated. Thus, the algorithm estimates the uncorrelated complex expansion coefficients iteratively using the Expectation Maximization(EM) method.

II. Alamouti's Transmit Diversity Scheme for OFDM Systems

In this paper, we consider a transmitter diversity scheme in conjunction with OFDM modulation shown in Fig. 1. Many transmit diversity schemes have been proposed in the literature offering different complexity vs. performance trade-offs. We choose Alamouti's space-time block code (STBC) scheme due to its simple implementation and good performance. The Alamouti's scheme imposes an orthogonal spatio-temporal structure

This work was supported in part by the Turkish Scientific and Tehnical Research Institute (TUBITAK) under Grant 100E006.

on the transmitted symbols that guarantees full (i.e., order 2) spatial diversity. In addition to spatial, to realize multipath diversity gains over frequency selective channels, the Alamouti STBC scheme is implemented at a block level in frequency domain. Fig. 1. depicts the Alamouti STBC system with 2

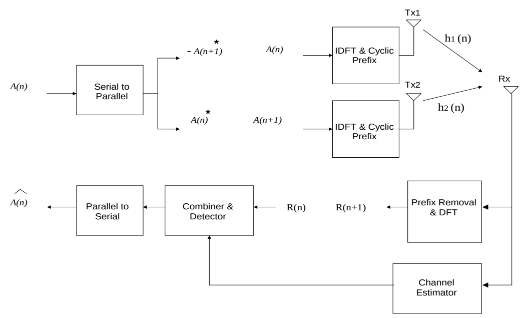

Fig. 1. Space-Time OFDM

transmit antennas and 1 receive antenna, where utilizing N_c subcarriers is employed per antenna transmissions. The fading channel between the μth transmit antenna and the receive antenna is assumed to be frequency selective but time-flat and is described by the discrete-time baseband equivalent impulse response $\boldsymbol{h}_\mu(n) = [h_{\mu,0}(n), \cdots, h_{\mu,L}(n)]$, with L standing for the channel order.

Let $A_{k,\mu}(n)$ be the data symbol transmitted on the kth subcarrier frequency (frequency bin) from the μth transmit antenna during the nth OFDM symbol interval. As defined, the symbols $\{A_{k,\mu}(n), \mu = 1, 2, \;\; k = 0, 1, \cdots, N_c - 1\}$ are transmitted in parallel on N_c subcarriers by 2 transmit antennas.

At the receiver, the antenna receives a noisy superposition of the multiantenna transmissions through the fading channels. We assume ideal carrier synchronization, timing and perfect symbol-rate sampling. We also assume that a cyclic prefix (CP) of length L has been inserted per OFDM symbol and is removed at the receiver end. After FFT processing, the received data sample $R_k(n)$ at the receive antenna can be expressed as

$$R_k(n) = \sum_{\mu=1}^{2} H_{k,\mu}(n) A_{k,\mu}(n) + W_k(n) \tag{1}$$

where $H_{k,\mu}(n)$ is the subchannel gain from the μth transmit antenna to the receive antenna evaluated at the kth subcarier

$$H_{k,\mu}(n) = \sum_{l=0}^{L} h_{\mu,l}(n) e^{-j(2\pi k/N_c)l} \tag{2}$$

and the additive noise $W_k(n)$ is circularly symmetric, zero-mean, complex Gaussian with variance σ^2 that is also assumed to be statistically independent with respect to n and k.

Equation (1) represents a general model for transmit diversity OFDM systems. However, the generation of $A_{k,\mu}(n)$ from the information symbols lead to corresponding transmit diversity OFDM scheme. In our system, the generation of $A_{k,\mu}(n)$ is performed via space-frequency coding, which was first suggested in [3].

A. Space-Frequency Coding

We consider a strategy which basically consists of coding across OFDM tones and is therefore called space-frequency coding. Since an OFDM communication system can be considered as a block transmission system, the serial input data symbols is converted into a data vector $\boldsymbol{A}(n) = [A_0(n), A_1(n) \cdots, A_{N_c-1}(n)]^T$. The space-frequency encoder then codes data symbol vector into two vectors $\boldsymbol{A}_1(n)$ and $\boldsymbol{A}_2(n)$ as

$$\begin{aligned} \boldsymbol{A}_1(n) &= [A_0(n), -A_1^*(n), \cdots, A_{N_c-2}(n), -A_{N_c-1}^*(n)]^T \\ \boldsymbol{A}_2(n) &= [A_1(n), -A_0^*(n), \cdots A_{N_c-1}(n), -A_{N_c-2}^*(n)]^T \end{aligned} \tag{3}$$

In space-frequency Alamouti scheme, $\boldsymbol{A}_1(n)$ and $\boldsymbol{A}_2(n)$ are transmitted through the first and second antenna element respectively during the block instant n. The operations of the

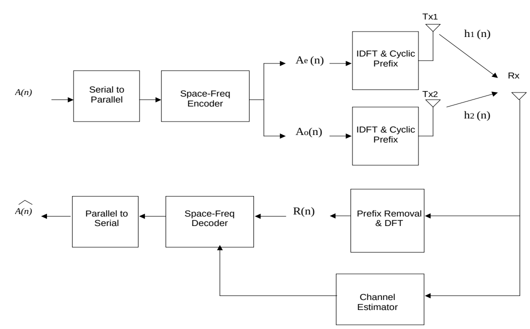

Fig. 2. Space Time OFDM

space-frequency encoder can best be described in terms of even and odd polyphase component vectors. If we denote even and odd component vectors of $\boldsymbol{A}(n)$ as

$$\begin{aligned} \boldsymbol{A}_e(n) &= [A_0(n), A_2(n), \cdots, A_{N_c-4}(n), A_{N_c-2}(n)]^T \\ \boldsymbol{A}_o(n) &= [A_1(n), A_3(n), \cdots, A_{N_c-3}(n), A_{N_c-1}(n)]^T \end{aligned} \tag{4}$$

then the space-frequency block code transmission matrix is

$$\begin{array}{cc} & space \rightarrow \\ frequency \downarrow & \left[\begin{array}{cc} \boldsymbol{A}_e(n) & \boldsymbol{A}_o(n) \\ -\boldsymbol{A}_o^*(n) & \boldsymbol{A}_e^*(n) \end{array} \right] \end{array} . \tag{5}$$

B. Vector Signal Model

If the received signal sequence is parsed in even and odd blocks of N_c tones, $\boldsymbol{R}_e(n) = [R_0(n), R_2(n), \cdots, R_{N_c-2}(n)]^T$ and $\boldsymbol{R}_o(n) = [R_1(n), R_3(n), \cdots, R_{N_c-1}(n)]^T$, the received signal can be expressed in vector form as

$$\begin{aligned} \boldsymbol{R}_e(n) &= \mathcal{A}_e(n)\boldsymbol{H}_{1,e} + \mathcal{A}_o(n)\boldsymbol{H}_{2,e} + \boldsymbol{W}_e(n) \\ \boldsymbol{R}_o(n) &= -\mathcal{A}_o^*(n)\boldsymbol{H}_{1,o} + \mathcal{A}_e^*(n)\boldsymbol{H}_{2,o} + \boldsymbol{W}_o(n) \end{aligned} \tag{6}$$

where $\mathcal{A}_e(n)$ and $\mathcal{A}_o(n)$ are an $N_c/2 \times N_c/2$ diagonal matrices with $\mathrm{diag}\mathcal{A}_e(n) = \boldsymbol{A}_e$ and $\mathrm{diag}\mathcal{A}_o(n) = \boldsymbol{A}_o$ respectively. $\boldsymbol{H}_{\mu,e}(n) = [H_{0,\mu}(n), H_{2,\mu}(n), \cdots, H_{N_c-2,\mu}(n)]^T$ and $\boldsymbol{H}_{\mu,o}(n) = [H_{0,\mu}(n), H_{1,\mu}(n) \cdots, H_{N_c-1,\mu}(n)]^T$ be $N_c/2$ length vectors denoting the even and odd component vectors of the channel attenuations between the lth transmitter and the receiver. Finally, $\boldsymbol{W}_e(n)$ and $\boldsymbol{W}_o(n)$ are an $N_c/2 \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive noise in the N_c tones.

Equation (6) shows that the information symbols $\mathcal{A}_e(n)$ and $\mathcal{A}_o(n)$ are transmitted twice in two consecutive adjacent subchannel groups through two different channels. In order to estimate the channels and decode $\boldsymbol{A}$ with the embedded diversity gain through the repeated transmission, for each n, we define, $\boldsymbol{R} = [\boldsymbol{R}_e^T(n) \;\; \boldsymbol{R}_o^T(n)]^T$ and write (6) into a matrix form[1]

$$\boldsymbol{R} = \boldsymbol{A}\,\boldsymbol{H} + \boldsymbol{W} \tag{7}$$

where $\boldsymbol{H} = [\boldsymbol{H}_{1,e}^T \;\; \boldsymbol{H}_{2,e}^T]^T$, $\boldsymbol{W} = [\boldsymbol{W}_e^T(n) \;\; \boldsymbol{W}_o^T(n)]^T$ and

$$\boldsymbol{A} = \begin{bmatrix} \mathcal{A}_e(n) & \mathcal{A}_o(n) \\ -\mathcal{A}_o^*(n) & \mathcal{A}_e^*(n) \end{bmatrix} . \tag{8}$$

Obviously, channel estimation is very essential for decoding space-frequency codes. In the absence of channel state information, decoder must estimate the channel states and there has been extensive affords in the direction of channel parameter estimation. In this paper a novel channel estimation algorithm is presented by representing the discrete multipath channel based on the Karhunen-Loeve orthogonal representation and make use of the Expectation Maximization technique.

C. Karhunen-Loeve Representation of the Multipath Channel

The Karhunen-Loeve expansion methodology has been used for efficient simulation of multipath fading environments [10]. An exception to this approach, we model discrete frequency response vector of the wireless channel gain vector, $\boldsymbol{H}_{\mu,e}(n)$ based on the Karhunen-Loeve expansion [7], [9] since it makes the expansion coefficient random variable's uncorrelated. Thus, correlated channel gains, in frequency, of a Gaussian process can be expressed as

$$\boldsymbol{H}_{\mu,e}(n) = \Psi\, \boldsymbol{G}_{\mu,e}(n) \tag{9}$$

where $\boldsymbol{G}_{\mu,e}(n)$ is an $N_c/2 \times 1$ zero-mean i.i.d. Gaussian vector whose covariance matrix is $\boldsymbol{\Lambda} = diag(\lambda_0, \lambda_1, \cdots, \lambda_{N-1})$. The variances of the components of $\boldsymbol{G}_{\mu,e}(n)$, arranged in decreasing order, are equal to the eigenvalues λ_j of the Karhunen Loeve(KL) transformation with the orthogonalized eigenfunctions $\boldsymbol{\Psi} = [\boldsymbol{\psi}_0, \boldsymbol{\psi}_1 \cdots, \boldsymbol{\psi}_{N_c-1}]$ of the discrete channel autocorrelation matrix $\boldsymbol{r}_\mu$ defined by $\boldsymbol{r}_\mu = E\{\boldsymbol{H}_\mu(n)\boldsymbol{H}_\mu^\dagger(n)\}$ which satisfies $\boldsymbol{r}_\mu \boldsymbol{\psi}_j = \lambda_j \boldsymbol{\psi}_j$ where † denotes conjugate transpose.

[1] We assume that the complex channel gains between adjacent subcarriers are approximately constant, i.e., $\boldsymbol{H}_{1,e} \approx \boldsymbol{H}_{1,o}$ and $\boldsymbol{H}_{2,e} \approx \boldsymbol{H}_{2,o}$.

III. EM-BASED MAP CHANNEL ESTIMATION

In the MAP estimation approach we choose $\widehat{\boldsymbol{G}}$ to maximize the posterior PDF or

$$\widehat{\boldsymbol{G}} = \arg\max_{\boldsymbol{G}} p(\boldsymbol{G}|\boldsymbol{R}) . \tag{10}$$

To find MAP estimator, we must equivalently maximize $p(\boldsymbol{R}|\boldsymbol{G})p(\boldsymbol{G})$. The prior PDF of the Karhunen-Loeve expansion coefficient r.v.'s of the fading channel can be expressed as

$$p(\boldsymbol{G}) \sim \exp(-\boldsymbol{G}^\dagger \widetilde{\boldsymbol{\Lambda}}^{-1} \boldsymbol{G}) , \tag{11}$$

where $\boldsymbol{G} = [\boldsymbol{G}_{1,e}^T, \boldsymbol{G}_{2,e}^T]^T$ and $\widetilde{\boldsymbol{\Lambda}} = diag(\boldsymbol{\Lambda} \;\; \boldsymbol{\Lambda})$.

Given the transmitted signals $\mathbf{A}$ as coded according to space-frequency transmit diversity scheme and the discrete channel orthonormal series expansion representation coefficients $\boldsymbol{G}$ and taking into account the independence of the noise components, the conditional probability density function of the received signal $\mathbf{R}$ can be expressed as,

$$p(\boldsymbol{R}|\boldsymbol{A},\boldsymbol{G}) \sim \exp\left[-(\boldsymbol{R} - \boldsymbol{A}\widetilde{\boldsymbol{\Psi}}\boldsymbol{G})^\dagger \widetilde{\boldsymbol{\Sigma}}^{-1} (\boldsymbol{R} - \boldsymbol{A}\widetilde{\boldsymbol{\Psi}}\boldsymbol{G})\right] \tag{12}$$

where $\widetilde{\boldsymbol{\Sigma}} = diag(\boldsymbol{\Sigma} \;\; \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma}$ is an $N \times N$ diagonal matrix with $\boldsymbol{\Sigma}[k,k] = \sigma^2$, for $k = 0, 1, \cdots, N-1$ and $\widetilde{\boldsymbol{\Psi}} = diag(\boldsymbol{\Psi}\;\boldsymbol{\Psi})$.

Direct maximization of (10) is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. This algorithm inductively reestimate $\mathbf{G}$ so that a monotonic increase in the *a posteriori* conditional pdf in (7) is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q(\mathbf{G}|\mathbf{G}^{(i)}) = \sum_{\mathbf{A}} p(\mathbf{R}, \mathbf{A}, \mathbf{G}) \log p(\mathbf{R}, \mathbf{A}, \mathbf{G}^{(i)}) \tag{13}$$

where $\mathbf{G}^{(i)}$ is the estimation of $\mathbf{G}$ at the ith iteration.

Note that $p(\boldsymbol{R},\boldsymbol{A},\boldsymbol{G}) \sim p(\boldsymbol{R}|\boldsymbol{A},\boldsymbol{G})p(\boldsymbol{G})$ since the data symbols $\boldsymbol{A} = \{A_{k,\mu}(n)\}$ are assumed to be independent of each other and identically distributed and the fact that $\boldsymbol{A}$ is independent of $\boldsymbol{G}$. Therefore, (13) can be evaluated by means of the expressions (10) and (12).

Given the received signal $\boldsymbol{R}$, the EM algorithm starts with an initial value $\boldsymbol{G}^0$ of the unknown channel parameters $\mathbf{G}$. The $(i+1)$th estimate of $\boldsymbol{G}$ is obtained by the maximization step described by $\boldsymbol{G}^{(i+1)} = \arg\max_{\mathbf{G}} Q(\boldsymbol{G}|\boldsymbol{G}^{(i)})$.

A. Initialization

To choose good initial values for the unknown channel parameters, the N_{PS} data symbols $\{A_{k,\mu}(n)\}$ for $k \in S_{PS}$, in each OFDM frame are generally used as pilot symbols known by the receiver. To interpolate the channel estimates, initially, there exist a minimum subcarrier spacing, l_{SC}, between pilots given by $l_{SC} < 1/\tau_{max}$, where τ_{max} is the maximum delay spread of the channel in the frequency domain. Therefore for

PSK modulated alphabet set, the initial value of the channel parameters $\boldsymbol{G}^{(0)}_{\mu,e}(n)$ $\mu = 1,2$, can be selected according to the following data-aided scheme.

Let for $\mu = 1,2$, $\boldsymbol{H}^{p}_{\mu,e}(n)$ denote an $N_{PS} \times 1$ vector with $\boldsymbol{H}^{p}_{\mu,e}(n)[k] = H_{\mu,k}(n)$, resulting the channel gains at frequencies $k \in S_{PS}$. Using $2N_{PS}$ pilot data symbols in the subcarrier groups, the linear minimum mean-square error (LMMSE) estimate of $\widehat{\boldsymbol{H}}^{p}_{\mu,e}(n)$ is given by [8]

$$\widehat{\boldsymbol{H}}^{p}_{\mu,e}(n) = \boldsymbol{\Psi}^{p}\boldsymbol{\Delta}^{p}\boldsymbol{\Psi}^{p\dagger}\widehat{\boldsymbol{H}}^{p}_{\mu,e,ls}(n) \tag{14}$$

where $\widehat{\boldsymbol{H}}^{p}_{\mu,e,ls}(n)$ is the least-square estimate of $\boldsymbol{H}^{p}_{\mu,e}(n)$ as defined in ([8], page 932), $\boldsymbol{\Psi}^{p}$ is an unitary matrix containing the eigenvectors of the $N_{PS} \times N_{PS}$ dimensional channel covariance matrix $\boldsymbol{r}^{p}_{\mu}$ with $\boldsymbol{r}^{p}_{\mu}[k,k'] = r_{\mu}(k,k'), k, k' \in S_{PS}$. $\boldsymbol{\Delta}^{p}_{\mu}$ is an diagonal matrix with entries $\delta_{k,\mu} = 1/(1+\sigma^2/\lambda_{k,\mu})$ where, $\lambda_{k,\mu}$'s are the eigenvalues of $\mathbf{r}^{\mathbf{p}}_{\mu}$. Then, given $2N_{PS}$ channel estimated samples $\widehat{\boldsymbol{H}}^{p}_{\mu,e}(n)[k]$, $k \in S_{PS}$ and $\mu = 1,2$, the complete initial channel gains $\boldsymbol{H}^{0}_{\mu,e}[k], k = 0,1,\cdots,N_c-1$ can easily be determined using an interpolation technique, i.e., Lagrange interpolation algorithm. Finally the initial values of $\boldsymbol{G}^{(0)}_{\mu,e}(n)$ can be determined as $\boldsymbol{G}^{(0)}_{\mu,e}(n) = \boldsymbol{\Psi}^{\dagger}\boldsymbol{H}^{(0)}_{\mu,e}(n)$.

Taking the pilot symbols into account, after long algebraic manipulations, the expression of the reestimate $\boldsymbol{G}^{(i+1)}_{\mu,e}(n)$ ($\mu = 1,2$) can be obtained as follows:

$$\begin{aligned}
\boldsymbol{G}^{(i+1)}_{1,e} &= (\boldsymbol{I} + \boldsymbol{\Sigma}\boldsymbol{\Lambda}^{-1})^{-1}\boldsymbol{\Psi}^{\dagger}\left[\boldsymbol{V}^{(i)}_{1}\boldsymbol{R}_e(n) - \boldsymbol{V}^{\dagger(i)}_{2}\boldsymbol{R}_o(n)\right] \\
\boldsymbol{G}^{(i+1)}_{2,e} &= (\boldsymbol{I} + \boldsymbol{\Sigma}\boldsymbol{\Lambda}^{-1})^{-1}\boldsymbol{\Psi}^{\dagger}\left[\boldsymbol{V}^{(i)}_{2}\boldsymbol{R}_e(n) - \boldsymbol{V}^{\dagger(i)}_{1}\boldsymbol{R}_o(n)\right]
\end{aligned} \tag{15}$$

where $(\boldsymbol{I} + \boldsymbol{\Sigma}\boldsymbol{\Lambda}^{-1}_{\mu})^{-1} = \mathrm{diag}([(1+\sigma^2/\lambda_{\mu,0})^{-1}, \cdots, (1+\sigma^2/\lambda_{\mu,N_c-2})^{-1}])$ and $\boldsymbol{V}^{(i)}_{l} = \mathrm{diag}[v^{(i)}_{\mu}(0), v^{(i)}_{\mu}(2), \cdots, v^{(i)}_{\mu}(N_c-2)]$ and $v^{(i)}_{\mu}(k)$, is given as

$$v^{(i)}_{1}(k) = \begin{cases} A_{k,1}(n); & \text{if } k \in S_{PS} \\ \Gamma^{(i)}_{1}(k); & \text{if } k \in S^{c}_{PS} \end{cases},$$

$$v^{(i)}_{2}(k) = \begin{cases} A_{k,2}(n); & \text{if } k \in S_{PS} \\ \Gamma^{(i)}_{2}(k); & \text{if } k \in S^{c}_{PS} \end{cases}.$$

Here, for $k \in S^{c}_{PS}$, $\Gamma^{i}_{\mu}(k)$ represents the *a posteriori* probabilities of the data symbols at the ith iteration step and is defined by

$$\Gamma^{(i)}_{\mu}(k) = \sum_{a_1,a_2 \in S_k} a^{*}_{\mu} P(A_{k,1}(n) = a_1, A_{k,2}(n) = a_2 | \boldsymbol{R}, \boldsymbol{G}^{(i)}) \tag{16}$$

and S_k denotes alphabet set taken by the kth OFDM symbol.

B. Computation of $\Gamma^{(i)}_{\mu}(k)$ for QPSK Signaling

Let $a = (\pm 1 \pm j)$ represents independent identically distributed data sequence modulating the QPSK carrier. Since for $\mu = 1,2$ and $k = 0,1,\cdots,N_c-1$, the data sequence $s_{\mu}(k)$ is independent, $\boldsymbol{\Gamma}_{\mu}$ in (15) can be computed as follows:

$$\boldsymbol{\Gamma}^{(i)}_{\mu} = \tanh\left[\frac{2}{\sigma^2}Re(\boldsymbol{Z}^{(i)}_{\mu})\right] - j\tanh\left[\frac{2}{\sigma^2}Im(\boldsymbol{Z}^{(i)}_{\mu})\right] \tag{17}$$

where

$$\begin{aligned}
\boldsymbol{Z}^{(i)}_{1} &= \boldsymbol{R}_{e_d}\boldsymbol{\Psi}^{*}\boldsymbol{G}^{*(i)}_{1,e} + \boldsymbol{R}_{o_d}\boldsymbol{\Psi}^{*}\boldsymbol{G}^{(i)}_{2,e} \\
\boldsymbol{Z}^{(i)}_{2} &= \boldsymbol{R}_{e_d}\boldsymbol{\Psi}^{*}\boldsymbol{G}^{*(i)}_{2,e} - \boldsymbol{R}_{o_d}\boldsymbol{\Psi}^{*}\boldsymbol{G}^{(i)}_{1,e}
\end{aligned}$$

and $\boldsymbol{R}_{e_d} = \mathrm{diag}\boldsymbol{R}_e$ and $\boldsymbol{R}_{o_d} = \mathrm{diag}\boldsymbol{R}_o$.

IV. Modified-Cramer-Rao Bound(MCRB)

Let for $\mu = 1,2$ and $m = 0,1,\cdots,N_c-1$, $\{G_{\mu}(m)\}$'s be the random parameters to be estimated. The (m,n)th element of the Fisher information matrix is defined as

$$J_{\mu}(m,n) = -E\left[\frac{\partial^2 \ln p(\mathbf{R}|\mathbf{A},\mathbf{G}_{\mu})}{\partial G_{\mu}(m)\partial G_{\mu}(n)}\right] + E\left[\frac{\partial^2 \ln p(\mathbf{G}_{\mu})}{\partial G_{\mu}(m)\partial G_{\mu}(n)}\right]$$

where the joint probability density functions $p(\mathbf{G})$ and $p(\mathbf{R}|\mathbf{A},\mathbf{G})$ are given by (11) and (12), respectively and, expectations should be taken over $\mathbf{R}$, $\mathbf{A}$ and $\mathbf{G}$. Performing the the above derivatives and taking into fact that the eigenfunctions $\psi_m(k)$ are orthogonal, it follows that

$$J_{\mu}(m,n) = \begin{cases} 2(\frac{1}{\lambda_m} + \frac{1}{\sigma^2}) & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$$

Therefore

$$MCRB(G_{\mu}(m)) = J^{-1}(m,m)$$

where σ^2 is the noise variance and λ_m are the eigenvalues of the discrete autocorrelation function $r(k,k')$ of the multipath fading channel.

V. Simulations

The simulation results for estimating the channel parameters of OFDM systems with transmitter diversity via EM algorithm are now presented. We consider the scheme with 2 transmit and 1 receive antennas with the fading multipath channels between transmitters and the receiver. $H_{\mu}(k)$'s are with an exponentially decaying power delay profile $\theta(\tau_{\mu}) = C\exp(-\tau_{\mu}/\tau_{rms})$ and delays τ_{μ} that are uniformly and independently distributed over the length of the cyclic prefix. C is a normalizing constant. Note that the normalized discrete channel-correlations for different subcarriers and blocks of this channel model were presented in [3] as follows,

$$r_1(k,k') = \frac{1-\exp\left[-L\left[\frac{1}{\tau_{rms}} + \frac{2\pi j(k-k')}{N}\right)\right]}{\tau_{rms}(1-\exp(\frac{-L}{\tau_{rms}}))\left(\frac{1}{\tau_{rms}} + \frac{j2\pi(k-k')}{N}\right)}$$

$$r(n,n') = J_o(2\pi(n-n'f_d)T_s)$$

where $J_o$is the zeroth-order Bessel function of the first kind and f_d is the Doppler frequency .

The scenario for our ST-OFDM simulation study consists of a wireless QPSK OFDM system operating with a 2MHz bandwidth and is divided into 256 tones with a total period (Ts) of 136 μs,of which 8 μs constitute the cyclix prefix (L=4). The uncoded data rate 3.76 Mbit/s. we assume that the rms width is $\tau_{rms} = 1$ sample (2 μs) for the power-delay profile and the dopler frequencies are $f_d = 5, 10, 100, 200, 300H_z$. For SF-OFDM simulation scenario we fixed data that will be send and by double the number of tones simulation is done.

Fig. 1 demonstrates the average MSE performance of the EM-based channel estimation algorithm as a function of the average SNR and different doppler frequencies.The average SNR was defined as $E[|H_l(k)|^2]E[|A_l(k)|^2/\sigma^2$. Since $E[|A_l(k)|^2 = 1$ for QPSK signaling and $E[|H_l(k)|^2] = 1$ for normalized frequency response of the fading channel, the normalized SNR simply becomes $1/\sigma^2$, where σ^2 is the variance of the complex white Gaussian noise entering the system. Average Mean-square-error(MSE) is defined as the norm of the difference between the vectors $\boldsymbol{G} = [\boldsymbol{G}_{1,e}^T, \boldsymbol{G}_{2,e}^T]$ and $\widehat{\boldsymbol{G}}_{map}$, representing the true and the estimated values of channel parameters, respectively. Namely,

$$MSE = \frac{1}{2N} \|\boldsymbol{G} - \widehat{\boldsymbol{G}}_{map}\|^2.$$

Notice that the modified CRB provides a looser bound which gets closer to MSE as SNR increases. In Figs. 3, the average MSE performance of the EM-based algorithm are presented as a function of the number of iterations for $\tau_{rms} = 4$. It is concluded from these curves that the MSE performance of the EM-based algorithm converges within 2-5 iterations, depending on the average SNR.

VI. Conclusions

In this paper, we proposed an optimum channel estimation algorithm for SF OFDM systems. This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing M-PSK modulation scheme with additive Gaussian noise. The discrete multipath channel was represented in terms of a Karhunen-Loeve expansion which makes full use of frequency-domain correlation of the frequency response of the time-varying dispersive fading channel. we investigated estimation over fast frequency selective channels for SF-OFDM and ST-OFDM systems. We observed that how doppler frequency effect our estimation.

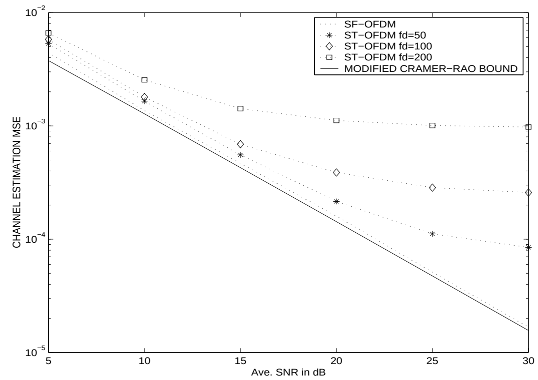

Fig. 3. MSE performance of the EM algorithm as a function of average SNR

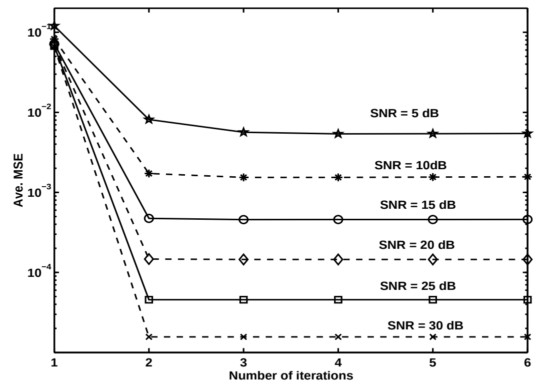

Fig. 4. MSE performance of the EM algorithm as a function of number of iterations ($\tau_{rms} = 4$ sample for the exponentially decaying power delay profile

References

[1] Y. (G.) Li, L. J. Cimini, N. Seshadri and S. Ariyavistakul, “Channel estimation for OFDM systems with transmitter diversity in mobile wireless channels”,*IEEE Journal Selected Areas on Commun.*, vol. 17, no.3, pp. 461-471, July 1999.

[2] K.F. Lee and D.B. Douglas,“A Space-Frequency Transmitter Diversity Technique for OFDM Systems”, *in Proc. IEEE Globecom*, San Francisco, CA, November 2000, pp. 1473-1477.

[3] S. M. Alamouti, “A simple transmitter diversity scheme for wireless communications”,*IEEE J. Selected Areas Commun.*, vol. 16, pp. 1451-1458, October 1998.

[4] V. Tarokh, N. Seshadri, and A. R. Calderbank “Space-time codes for high data rate wireless communications: Performance analysis and code construction,”, *IEEE Trans. Inform. Theory*, vol. 44, pp. 744-765, March 1998.

[5] V. Tarokh, Hamid Jafarkhani, and A. R. Calderbank, “Space-time block Codes from orthogonal designs”,*IEEE Trans. Inform. Theory* , vol. 45, no.5 pp. 1456-1467, July 1999.

[6] R. Van Nee and R. Prasad, “ OFDM Wireless Multimedya Communications”, in *Artech House Publishers*, 2000.

[7] M. Siala and D. Dupontiel,“Maximum A Posteriori Multipath Fading Channel Estimation for CDMA Systems”, *Proc IEEE Vehicular Technology Conference(VTC’99*,1999.

[8] J.-J. van de Beek, O. Edfors, M. Sandell, S.K. Wilson, and P.O. Börjesson, “OFDM channel estimation by singular value decomposition”, *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931-936, July 1998.

[9] E. Panayirci and Hakan A. Cirpan, ”A Non-Data Aided EM -Based Channel Estimation for OFDM Systems with Time-Varying Fading Channels”, 2001 Third International Workshop on Multi-Carrier Spread-Spectrum (MCSS2001) and Related Topics, September 26-28 2001, Oberpfaffenhofen, Germany.

[10] K. Yip and T. Ng, ”Karhunen-Loeve Expansion of the WSSUS Channel Output and its Application to Efficient Simulation,” *IEEE Journal on Selected Areas in Communications*, vol. 15, no 4, pp.640-646, May 1997.

[11] R. Steele, *Mobile Radio Communications*, New York, IEEE Press, 1974.