SPECTRUM ESTIMATION USING 2-D ORTHOGONAL LATTICE STRUCTURES

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ABSTRACT

Developed approaches to spectral estimation can be grouped as parametric and non-parametric methods. Non-parametric methods are classical methods which are based on periodogram and they use Fourier transform. In this study, a parametric method is investigated. The two-dimensional orthogonal lattice structures are applied to spectrum estimation and the results are compared with the classical periodogram.

I. INTRODUCTION

The multi-dimensional digital signal processing has been developing rapidly due to the fact that it has many applications in various fields[1]. In the studies on image processing, parametric modeling of the signal is a very widespread approach.

1-D lattice structures found applications in diverse areas such as system identification, spectral estimation, channel equalization, noise cancellation, analysis and synthesis of speech. Lattice structures has many priorities. The property of orthogonality allows the filter to be updated in order, without recalculation of the previous lower order filter coefficients. Also lattice algorithms have a modular structure that makes them attractive candidates for VLSI implementations.

Because of the success of 1-D lattice structures, in recent years lots of researches are made directed to the development of lattice structures. First proposed 2-D lattice structure is three parameter lattice filter[2]. This structure lacks the property of orthogonality so that the cascading stages may not lead to an optimum filter. However, its simplicity is attractive and it has found applications in many fields such as coding , data compression. This lattice structure is then generalised to asymmetric half-plane case and it is applied to the solution of the digital filter design problem[3].

A complete solution for the 2-D lattice structures is presented in [4]. In this study, 2-D orthogonal lattice filters are developed as a natural extension of the 1-D lattice filter theory[5]. The method offers a complete solution for the Levinson type algorithm to compute the prediction error filter coefficients using lattice parameters from the given 2-D augmented normal equation. Burg's matrix formulation is used to derive the Levinson type recursions. The method is applicable for both the quarter-plane and asymmetric half-plane models.

This theory is investigated by using sample autocorrelation values of the original data fields which is known as Yule-Walker approach and the method is applied to spectrum estimation[6].The same theory is investigated by using Burg's method. It was shown that Burg's method has superiority over Yule-Walker method for short data records since Burg's method directly works on original data fields without using sample autocorrelations[7].

In this study, the theory investigated by using Burg's method is applied to spectrum estimation.

II. 2-D ORTHOGONAL LATTICE FILTERS

In 2-D linear modeling, a stationary random field $y(k_1,k_2)$ is predicted by a linear combination of its neighboring samples. 2-D linear prediction can be expressed as follows:

$$\hat{y}(k_1,k_2) = \sum_{(n_1,n_2)\in\bar{S}} a(n_1,n_2)\, y(k_1-n_1,k_2-n_2) \quad (1)$$

where $a(n_1,n_2)$ s are prediction coefficients, $\bar{S}$ is the prediction region mask not including the point $(0,0)$ and (k_1,k_2) are axes for the data field. The procedure starts with generating the AR data field according to the prediction error mask which may be quarter-plane or asymmetric half-plane. The 2-D AR data field can be considered as a one dimensional array by indexing the elements in the prediction support region appropriately. Depending on the indexing specified, for instance the first quadrant backward prediction error filter corresponds to the forward prediction error filters in the fourth quadrant. Unlike the quarter-plane case, the prediction of the last element in the support does not correspond to any other type of asymmetric half-plane models. The indexing schemes used in this study are given in Figure 1 for the quarter-plane case and in Figure 2 for the asymmetric half-plane case. In the figures, N stands for the order of the predictor.

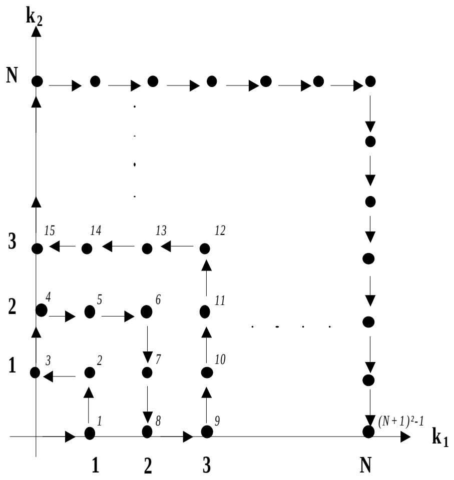

Figure 1 The indexing scheme for the quarter-plane case

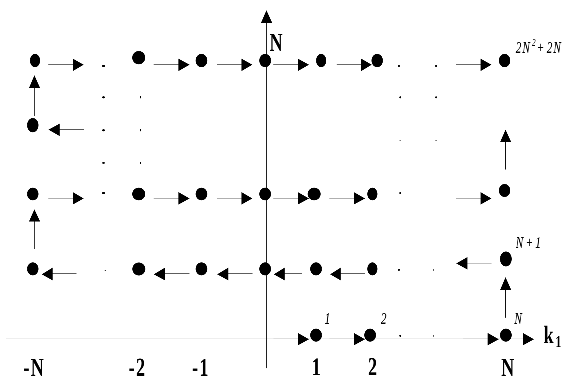

Figure 2 The indexing scheme for the asymmetric half-plane case

The 2-D AR data in the indexed form can be shown as:

$$y_{\mathrm{p,q}}(k_1,k_2)=[y((k_1,k_2)-p) \quad y((k_1,k_2)-p-1) \quad \dots \quad y((k_1,k_2)-q)]^{\mathrm{T}} \tag{2}$$

The notation $y((k_1,k_2)-i)$ denotes the *i*th element before $y(k_1,k_2)$ and the subscripts p and q denote the first and last elements in the array where $p<q$.

The forward prediction error associated with the prediction of the *zero*th sample from the previous m samples within the prediction region can be defined as

$$f_0^{(m)}(k_1,k_2)=\mathbf{a}_0^{(m)^T}\mathbf{y}_{0,m}(k_1,k_2) \tag{3}$$

where

$$\mathbf{a}_0^{(m)}=\left[1 \quad a_0^{(m)}(1) \quad \dots \quad a_0^{(m)}(m)\right]^T \tag{4}$$

and

$$\mathrm{y}_{0,m}(k_1,k_2)=\left[y((k_1,k_2)-0) \quad y((k_1,k_2)-1) \quad \dots \quad y((k_1,k_2)-m)\right]^T \tag{5}$$

The backward prediction error associated with the prediction of the mth sample(last element), from the m samples prior to it in the prediction region can be defined as

$$b_m^{(m)}(k_1,k_2)=\mathbf{g}_m^{(m)^T}\mathbf{y}_{0,m}(k_1,k_2) \tag{6}$$

where

$$\mathbf{g}_m^{(m)}=\left[g_m^{(m)}(m) \quad g_m^{(m)}(m-1) \quad \dots \quad g_m^{(m)}(1) \quad 1\right] \tag{7}$$

Let the m-by-1 vectors $\mathbf{a}_0^{(m-1)}$ and $\mathbf{g}_m^{(m-1)}$ denote the tap weight vector of the corresponding forward and backward prediction error filters of order (m-1), respectively. The tap weight vectors of 2-D forward and backward prediction error filters may be order updated as follows:

$$\mathbf{a}_0^{(m)}=\begin{bmatrix}\mathbf{a}_0^{(m-1)}\\ 0\end{bmatrix}+\Gamma_{b_m}^{(m)}\begin{bmatrix}0\\ \mathbf{g}_m^{(m-1)}\end{bmatrix} \tag{8}$$

and

$$\mathbf{g}_m^{(m)}=\begin{bmatrix}0\\ \mathbf{g}_m^{(m-1)}\end{bmatrix}+\Gamma_{f_0}^{(m)}\begin{bmatrix}\mathbf{a}_0^{(m-1)}\\ 0\end{bmatrix} \tag{9}$$

where $\Gamma_{f_0}^{(m)}$ and $\Gamma_{b_m}^{(m)}$ are the *m*th order reflection coefficients for the forward and backward predictors.

In the Levinson order-update recursions, one can find general expressions for lattice parameters, forward and backward prediction error fields and error powers in more compact form. For p=1,2,..,m and n=1,2,..p,lattice parameters;

$$\Gamma^{(n)}_{f_{p-n}} = -\frac{\Delta^{(n-1)}_{b_p}}{E^{(n-1)}_{f_{p-n}}} ; \Gamma^{(n)}_{b_p} = -\frac{\Delta^{(n-1)}_{f_{p-n}}}{E^{(n-1)}_{b_p}} \qquad (10)$$

The error propagation equations or the general form of the orthogonal 2-D lattice filters is given by:

$$\begin{bmatrix} \mathrm{f}^{(n)}_{p-n}(\mathrm{k}_1,\mathrm{k}_2) \\ \mathrm{b}^{(n)}_{p}(\mathrm{k}_1,\mathrm{k}_2) \end{bmatrix} = \begin{bmatrix} 1 & \Gamma^{(n)}_{\mathrm{b}_p} \\ \Gamma^{(n)}_{\mathrm{f}_{p-n}} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{f}^{(n-1)}_{p-n}(k_1,k_2) \\ \mathrm{b}^{(n-1)}_{p}(k_1,k_2) \end{bmatrix} \qquad (11)$$

where p=1,2,…,m; n=1,2,…,p and starting with

$$f^{(0)}_p(k_1,k_2) = b^{(0)}_p(k_1,k_2) = y((k_1,k_2)-p) \qquad (12)$$

for p=0,1,…,m, algorithm starts from the *0*th order and continues up to the *m*th order.

The algorithm for the 2-D orthogonal lattice filter is given in Figure 3. M is the number of elements in the 2-D prediction support region.

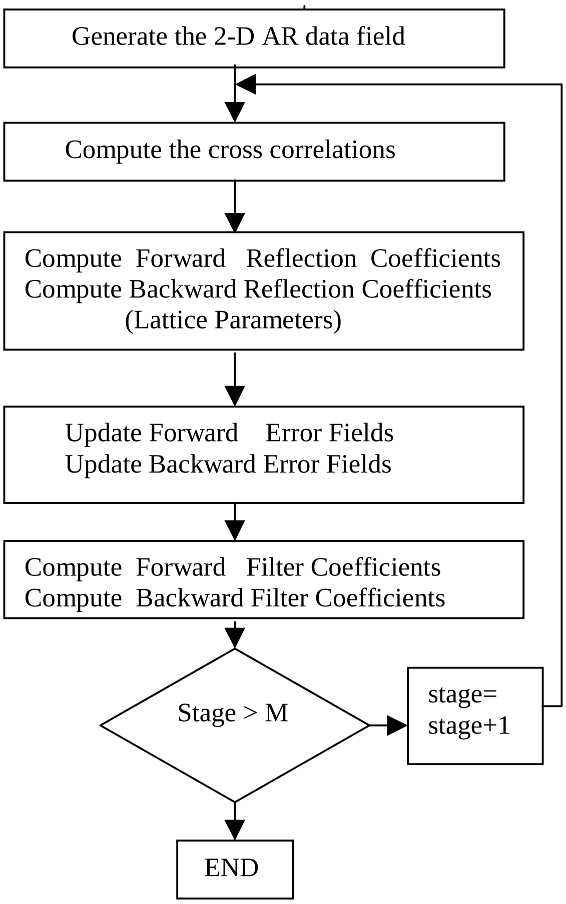

Figure 3 The algorithm for the 2-D orthogonal lattice filter.

III. SPECTRUM ESTIMATION

Our goal is to estimate the power spectral density of a random data field. Developed approaches to spectral estimation can be grouped as parametric and non-parametric methods. Non-parametric methods are classical methods which are based on periodogram and they use Fourier transform. According to the classical method called periodogram spectrum estimation is computed with the following equation;

$$\hat{P}_y(w_1,w_2) = \frac{1}{N}\left|Y(w_1,w_2)\right|^2 \qquad (13)$$

where N is the number of elements in the data field and $Y(w_1,w_2)$ is the Fourier transform of the data which is given as $y(k_1,k_2)$.

Parametric methods assume a model for the expression of the problem and coefficients of the model are predicted from the infinite number of observations of the process. The predicted power spectral density $\hat{P}_x(w_1,w_2)$ from the process $x(n_1,n_2)$ can be expressed as follows

$$\hat{P}_x(w_1,w_2) = \frac{\sigma^2_w}{\left|1 + \sum\sum_{(k_1,k_2)\in A} a(k_1,k_2) e^{-jw_1n_1} e^{-jw_2n_2}\right|^2} \qquad (14)$$

where constants $a(k_1,k_2)$ s are coefficients of the AR model.

In spectrum estimation, AR spectrum analysis has the following superiorities over periodogram: If signal-to-noise ratio (SNR) is greater than 0 dB, AR spectrum estimation has a better frequency resolution. The distortion that comes out of the side lobes and naturally observed in periodogram is not observed in AR spectrum estimation. Third, for short data records AR method gives a better prediction result.

In 2-D spectrum estimation, another important point is the mask used. Predictors with single quadrant support region are not sufficient especially when estimating the sinuses in different quadrants. Single quadrant models cause some distortion in the estimated spectrum. One method to reduce this distortion would be to form a combination of the spectral estimates[8]. Let $\hat{P}_{x_1}(w_1,w_2)$ and $\hat{P}_{x_2}(w_1,w_2)$ be spectral estimations for two different quadrants, the improved estimation will be

$$\hat{P}_x(w_1,w_2) = \frac{1}{\dfrac{1}{\hat{P}_{x_1}(w_1,w_2)} + \dfrac{1}{\hat{P}_{x_2}(w_1,w_2)}} \qquad (15)$$

Unlike the quarter-plane case, asymmetric half-plane models does not have such a problem.

IV. SIMULATION EXAMPLES

In this part, simulation examples for spectral estimation are presented. The AR coefficients of the data field consisting sinuses in noise are predicted with the lattice filter and spectrums are obtained with the equation (14).

Two sinuses buried in noise are generated with the following equation

$$y(k_1,k_2) = \sum_{i=1}^{2} \sqrt{2}a_i \cos(w_{i1}k_1 + w_{i2}k_2) + w(k_1,k_2) \qquad (16)$$

where $w(k_1,k_2)$ is white noise with zero mean. Signal-to-noise ratio can be computed with the following equation

$$SNR = \frac{\sum_{i=1}^{2} a_i^2}{\sigma_w^2} \qquad (17)$$

where σ_w^2 corresponds to noise power.

In simulation examples, the amplitudes of all signals are unit.

In Figure 4-a, the spectral estimation of two sinuses in different quadrants by using FFT based classical method is shown. In this figure, frequencies are normalised to 2Π. These sinuses are also estimated by using second order quarter-plane model in Figure 4-b and by using second order asymmetric half-plane model in Figure 4-c. These figures shows that both the second order quarter-plane model and the second order asymmetric half-plane model has a better performance comparing to FFT based classical method when the data dimension is short.

In Figure 5-a, the spectral estimation of two sinuses in the same quadrant by using FFT based classical method is shown. Frequencies are normalised to 2Π in this figure. Figure 5-b shows the sinuses in the same quadrant estimated by using second order quarter-plane model. Figure 5-c shows the same sinuses estimated by using second order asymmetric half-plane model. It can be seen that Figure 5-b and Figure 5-c has a better performance of estimating the sinuses.

Example1

Two sinuses at the points (-0.2,0.2) and (0.3,0.3) are estimated with different methods. Data dimensions are 10X10 and noise variance σ_w^2 is 1.

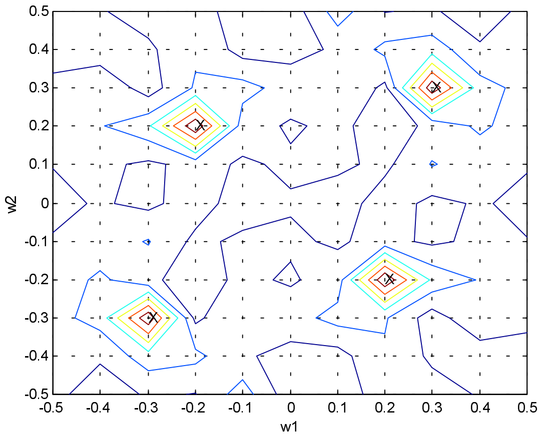

Figure 4 -a FFT based classical method

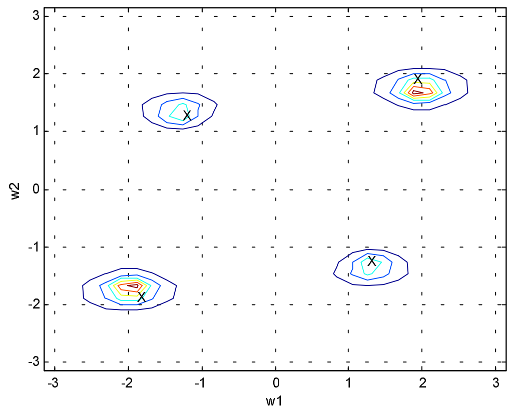

Figure 4-b Second order quarter plane model

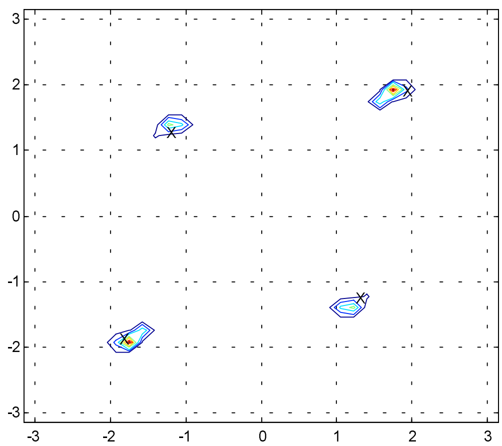

Figure 4-c Second order asymmetric half-plane model

Example2

Two sinuses at the points (0.1,0.3) and (0.3,0.1) are estimated with different methods. Data dimensions are 10X10 and noise variance σ_w^2 is 0.25.

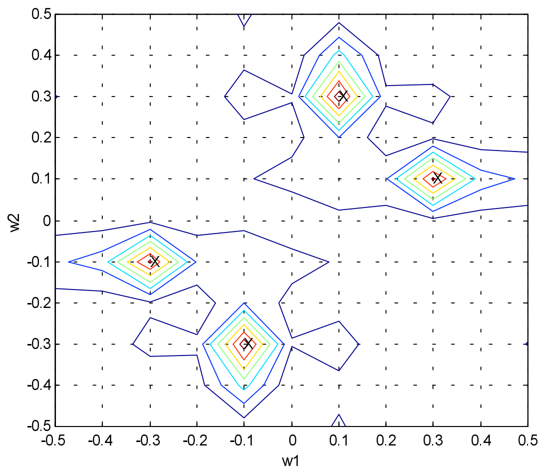

Figure 5-a FFT based classical method

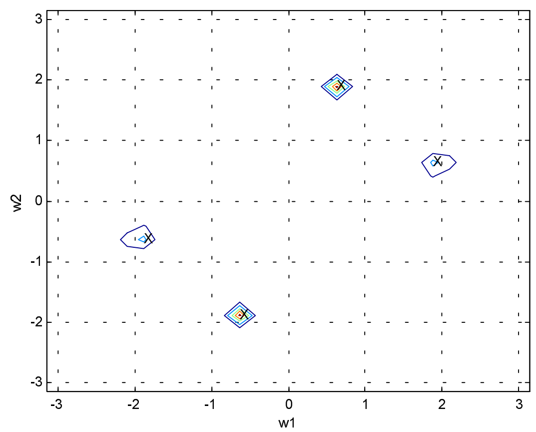

Figure 5-b Second order quarter plane model

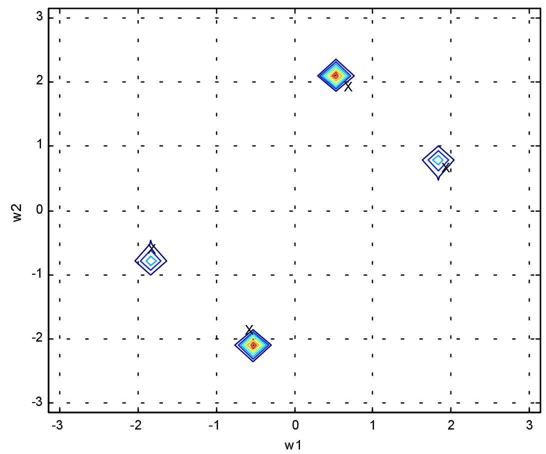

Figure 5-c Second order asymmetric half-plane model

V.CONCLUSION

In this study, the two-dimensional orthogonal lattice structures are applied to spectrum estimation. Sinuses buried in noise are estimated using both quarter-plane and asymmetric half-plane filters. The results show that for short data records, the method which depends on lattice filter modeling has a better performance than the classical periodogram.

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