Comparative Review of Multi-Phase Apparent Power Definitions

M. E. Balci[1] and M. H. Hocaoglu[2]

Department of Electronics Engineering, Gebze Institute of Technology, 41400 Kocaeli, TURKEY
[1]m.balci@gyte.edu.tr, [2]hocaoglu@gyte.edu.tr

Abstract

In this paper, the widely recognised apparent power definitions are rigorously reviewed and their ability on the measurement of the system efficiency is investigated by using the apparent power definition, which is calculated in terms of the total line loss of the system with compensation and without compensation. Therefore, in a test system, the results are obtained for minimum rms current compensation strategy by taking into account the unbalance among the resistances of neutral and phase lines.

1. Background

By the proliferation of a.c. in the transmission and distribution systems, the classical apparent power is defined as the product of rms values of voltage and current, and also it is used as a tool for the sizing of system's equipments. Furthermore, the rms value of the current is divided into two orthogonal components: These are active current, which transports the net energy from the source to the load, and reactive current, which is not capable to transport any energy. Therefore, the apparent power is expressed as the vector sum of ''active power'' and ''reactive power'', which flows due to active and reactive currents, respectively. The classical apparent power has also been used for the measurement and also for the improvement of power transfer efficiency of the system. On the other hand, conventionally, apparent power is calculated for three-phase systems by treating each phase individually.

According to this approach, Arithmetic and Vector apparent powers are proposed in the literature [1], [2]. Vector apparent power is the vector sum of active and reactive powers of each phase:

$$S_V = \sqrt{\left(\sum_{m=a,b,c} P_m\right)^2 + \left(\sum_{m=a,b,c} Q_m\right)^2} \qquad (1)$$

where the active and reactive powers of m^{th} phase are $P_m = V_m I_m \cos\theta_m$ and $Q_m = V_m I_m \sin\theta_m$, respectively. Other definition is the Arithmetic apparent power that is the arithmetic sum of each phase's apparent power:

$$S_{Ar} = \sum_{m=a,b,c} S_m \qquad (2)$$

where the apparent power of m^{th} phase is calculated with the product of m^{th} phase voltage and current rms values as $S_m = V_m I_m$. Vector and Arithmetic apparent powers give the same numerical values for balanced and sinusoidal systems.

On the other hand, for unbalanced conditions, Buchollz pointed out that apparent power could not be calculated by treating each phase individually, therefore; in three-phase and three-line systems he proposed an apparent power, which threats the system as a single unit by using ''collective voltage'' and ''collective current'' values [3]. In a follow up study [4], Buchollz expanded his power definition to m-phase systems. At the present time, German standard DIN 40110 [5] encouraged the usage of Buchollz's apparent power, which is derived into power components based on the current relations presented by FBD (Fryze-Buchollz-Depenbrock) theory [6]. This power resolution clearly figures out that Buchollz's apparent power as maximal active power, which can be transmitted for the given voltage waveform and the given current rms value. On the other hand, the power factor calculated with respect to Bucholl'z apparent power can be expressed in terms of the minimum and the actual line losses where the system has identical supply line resistances. However, this is not case for the systems with different supply line resistances. Thus, Mayordomo & Usaola and S.J. Jeon redefined Bucholl'z apparent power by keeping this property for such systems [7], [8].

In addition to the studies mentioned above, IEEE std. 1459 working group describes apparent power as ''the maximal active power, which can be transmitted under sinusoidal and balanced conditions with the same rms voltage and current values'' [2].

In this study, firstly the literature on the apparent power definitions is summarized in section II. Secondly, the outlines of the widely recognized apparent power definitions are given in section III. Finally, in the unbalanced and nonsinusoidal test system, the ability of the apparent power definitions on the measurement of the system efficiency is investigated by taking into account the unbalance among the resistances of neutral and phase lines.

2. Literature Summary

The philosophy of apparent power was explained and interpreted in many engineering publications. Accessible important studies are summarized here to show the evolution of apparent power theory.

One of these studies [9] presents that the resolution of Bucholl'z apparent power, which contains active and nonactive powers calculated by using symmetrical components, for a sinusoidal but unbalanced poly-phase system. Thus, it is shown that the negative and zero sequence powers cause additional power loss in the network and they should be viewed as useless powers unless they are generated purposely with the goal of cancelling these powers of another load. Therefore, author concluded that power factor should be defined as the ratio of positive-sequence active and Bucholl'z apparent powers.

In another study [10], author concludes that system power loss is not a linear function of the square of Arithmetic and Vector Apparent Powers. Only the apparent power defined by Buchholz holds this property for all possible situations, which cover balanced, unbalanced, linear and nonlinear load sides.

On the other hand, Williems [11] makes clear that the characterizations of the transmission efficiency and the instantaneous power's oscillation are the most important reasons to define the apparent power. Thus, the definitions bare the aim of efficiency improvement or minimization of oscillation.

According to the first, transmission efficiency concept, apparent power is a function of the rms values of voltage and current. However, in the latter one, apparent power is a quantity, which measures the oscillation of instantaneous power. It is also concluded that the power factor calculated with latter definition indicates power transfer efficiency for only sinusoidal single-phase systems.

Williems et al. [12], [13] discussed the relationships and differences between two apparent power definitions that took place in DIN and IEEE standards. It has been shown in the studies that two definitions give different results only if zero sequence voltage exists in the system.

Pajic and Emanuel also compare these two major apparent power definitions in [14]. Authors pointed out that DIN std. definition uses a pure theoreticall approach to obtain unity power factor where negative and zero-sequence currents may be present. And also, IEEE std.'s definition uses a practical approach that unity power factor implies a perfectly balanced system with pure positive-sequence voltages and currents. In addition, the quantitative analysis, presented in this paper, show that for practical power networks, where the differences among the supplying lines resistances are small, and the zero-sequence voltage is kept low, the pure theoretical (DIN std. definition) and the practical (IEEE std. definition) approaches yield results that are nearly overlapping.

Finally, a recent study [15] summarizes the power theory, based on frequency and time domain approaches, and shows that none of the approaches, available in the literature, can be used to solve all of the power system's problems, i.e. compensation, metering and billing, in total.

Above literature summary shows that apparent power is still a controversial topic in the systems that consists of unbalanced and nonlinear loads despite the fact that it is the core for design and operation of power systems. Therefore, the studies on the analysis of the apparent powers defined in the literature should continue to understand their capabilities and limitations.

3. Apparent Power Definitions

In this section, the apparent power definitions a part from the arithmetic apparent power, which is given in the background, and S. J. Jeon's apparent power, which is identical with Mayordomo & Usaola's apparent power, are briefly summarized:

3.1. The Vector Apparent Power Proposed By Budeanu-Curtis-Silsbee

This is one of the oldest [1] and probably the most common definition [16]. According to the theory, the powers are measured individually for each one of the three phases a, b and c:

Active powers,

$$P_m = \sum_h V_{mh} I_{mh} \cos(\theta_{mh}) \tag{3}$$

Reactive powers,

$$Q_m = \sum_h V_{mh} I_{mh} \sin(\theta_{mh}) \tag{4}$$

Apparent powers,

$$S_m = \sqrt{\left(\sum_h V_{mh}^2\right)\left(\sum_h I_{mh}^2\right)} \quad , \qquad m = a, b, c \tag{5}$$

and the calculated distortion powers,

$$D_m = \sqrt{S_m^2 - P_m^2 - Q_m^2} \tag{6}$$

giving the vector apparent power

$$S_V = \sqrt{\left(\sum_{m=a,b,c} P_m\right)^2 + \left(\sum_{m=a,b,c} Q_m\right)^2 + \left(\sum_{m=a,b,c} D_m\right)^2} \tag{7}$$

Since each phase is treated as an independent single phase system, the apparent power proposes the compensation of the each phase separately.

3.2. Buchollz's (or DIN std. 40110's) Apparent Power Definition

Buchollz [4] collectively formed the voltage and currents of the m[th] line system as;

$$v_\Sigma = \begin{bmatrix} v_{10} \\ v_{20} \\ \vdots \\ v_{m0} \end{bmatrix}, \quad i_\Sigma = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_m \end{bmatrix} \tag{8}$$

where v_{m0} denotes the instantaneous voltage between m[th] line and virtual neutral point that can be calculated as;

$$v_{m0} = v_{mn} - \frac{1}{m}\sum_m v_{mn} \tag{9}$$

Thus, he proposed the apparent power definition relies on collective rms voltage,

$$V_\Sigma = \sqrt{\frac{1}{T}\int_0^T v_\Sigma^{\mathrm{T}} v_\Sigma dt} = \sqrt{\sum_m V_{m0}^2} \tag{10}$$

and collective rms current,

$$I_\Sigma = \sqrt{\frac{1}{T}\int_0^T i_\Sigma^{\mathrm{T}} i_\Sigma dt} = \sqrt{\sum_m I_m^2} \tag{11}$$

giving the apparent power as below:

$$S_\Sigma = V_\Sigma I_\Sigma \tag{12}$$

In DIN std. 40110, by taking into account the current decomposition defined in FBD theory, Buchollz's apparent power is separated into three power components:

P, active power,

$$P = \sum_m P_m, \quad P_m = \frac{1}{T}\int_0^T v_{m0} i_m dt \tag{13}$$

$Q_{tot\Sigma\mathrm{IIu}}$, the nonactive power drawn by the difference between m[th] line conductance, defined as $G_m = P_m/V_{m0}^2$, and equivalent conductance, defined as $G = P/V_\Sigma^2$,

$$Q_{tot\,\Sigma\,\mathrm{IIu}} = V_\Sigma \sqrt{\sum_m (G_m - G)^2 V_{m0}^2} \tag{14}$$

and $Q_{tot\,\Sigma\perp}$, the nonactive power drawn by the current component orthogonal with the voltage,

$$Q_{tot\,\Sigma\perp} = V_\Sigma \sqrt{\sum_m \left[I_m^2 - G_m^2 V_{m0}^2\right]} \tag{15}$$

giving the Buchollz's apparent power,

$$S_\Sigma = \sqrt{P^2 + Q_{tot\,\Sigma\,\mathrm{IIu}}^2 + Q_{tot\,\Sigma\perp}^2} \tag{16}$$

DIN std. 40110 considers the compensated line current, which is a perfect replica of its respective line-to-a virtual neutral voltage. Thus, the neutral wire is treated as a fourth phase, consequently after the compensation some imbalance and distortion may persist in the line currents.

3.3. The IEEE Apparent Power Definition

For the general (three-phase and four-line) case of the power systems, IEEE 1459 standard [2] proposed apparent definition relies on equivalent rms voltage,

$$V_e = \sqrt{\frac{1}{18}\left[3\left(V_a^2 + V_b^2 + V_c^2\right) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2\right]} \tag{17}$$

and equivalent rms current,

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}} \tag{18}$$

giving apparent power as below;

$$S_e = 3V_e I_e \tag{19}$$

The IEEE definition has two main power components: first is fundamental effective apparent power;

$$S_{e1} = 3V_{e1}I_{e1} \quad (20)$$

where V_{e1} and I_{e1} are the fundamental harmonic equivalent rms voltage and current values, which are calculated as only fundamental harmonic voltages and currents substituted in (17) and (18), and second is non-fundamental apparent power;

$$S_{eN}^2 = S_e^2 - S_{e1}^2 \quad (21)$$

Fundamental effective apparent power is decomposed into fundamental harmonic positive-sequence apparent power and fundamental harmonic unbalanced apparent power:

$$S_{e1} = \sqrt{\left(S_1^+\right)^2 + S_{U1}^2} \quad (22)$$

Fundamental harmonic positive-sequence apparent power can be calculated as $S_1^+ = \sqrt{\left(P_1^+\right)^2 + \left(Q_1^+\right)^2}$ by means of fundamental harmonic positive-sequence active power ($P_1^+=3V_1^+I_1^+\cos\theta_1^+$) and fundamental harmonic positive-sequence reactive power ($Q_1^+=3V_1^+I_1^+\sin\theta_1^+$).

In addition, non-fundamental apparent power is decomposed into the power components as follows;

$$S_{eN}^2 = D_{eI}^2 + D_{eV}^2 + S_{eH}^2 \quad (23)$$

where

$$D_{eI} = 3V_{e1}I_{eH} \quad (24)$$

is the current distortion power,

$$D_{eV} = 3V_{eH}I_{e1} \quad (25)$$

is the voltage distortion power,

$$S_{eH} = 3V_{eH}I_{eH} = \sqrt{P_{eH}^2 + D_{eH}^2} \quad (26)$$

is the harmonic apparent power, which is the vector sum of the total harmonic active power (P_{eH}) and the harmonic distortion power (D_{eH}).

According to the concept of the IEEE definition, both voltages and currents should be sinusoidal & balanced after compensation process.

3.4. Apparent Power Defined By Mayordomo & Usaola

In [7], Mayormo and Usaola modified Buchollz's apparent power as;

$$S_{MU} = V_{MU}I_{MU} \quad (27)$$

by redefining collective rms voltage,

$$V_{MU} = \sqrt{\sum_m 1/\alpha_m V_{m0}^2} \quad (28)$$

and collective rms current,

$$I_{MU} = \sqrt{\sum_m \alpha_m I_m^2} \quad (29)$$

where the phase-to-virtual neutral point voltage is calculated as;

$$v_{m0} = \left[v_{mn} - \frac{\sum_m \frac{v_{mn}}{\alpha_m}}{\sum_m \frac{1}{\alpha_m}} \right] \quad (30)$$

In the equations from (28) to (30), α_m of m^th line is the ratio among the resistance of respective line (r_m) and a reference resistance value (r_L), which can be one of the resistances of m-line.

4. Application

In order to quantitatively analyse their ability on the measurement of the system efficiency, the power factor and apparent power definitions related with total line loss [7] is referenced in the analysis. The reference definition of power factor used in the analysis has the meaning of the power factor well expressed for sinusoidal & balanced three-phase systems with identical supply line resistances. The power factor defined for such systems can be expressed in terms of the total line loss as below:

Firstly, power factor is written as;

$$pf_T = \frac{P}{S} = \frac{I_{\min}}{I} \quad \text{for } S = \sqrt{3}VI \text{ and } P = \sqrt{3}VI_{\min} \quad (31)$$

where I_{min} and I are the minimum and actual rms line currents transports the same active power (P) with keeping the same rms value of line-to-line voltage (V).

Second, the ratio among the minimum (ΔP_{min}) and actual (ΔP) total line losses is expressed as;

$$\frac{\Delta P_{\min}}{\Delta P} = \left(\frac{I_{\min}}{I}\right)^2 \; for\; \Delta P_{\min} = 3rI_{\min}^2 \text{ and } \Delta P = 3rI^2 \quad (32)$$

where r denotes supply line resistances. It should be underlined that active power has the same value for the minimum and actual total line loss cases.

And then; the equivalent of I_{min}/I found in (31) is substituted in (32):

$$pf_T^2 = \frac{\Delta P_{\min}}{\Delta P} \quad (33)$$

Finally, (33) can be arranged as;

$$pf_T = \sqrt{\frac{\Delta P_{\min}}{\Delta P}} \quad (34)$$

Therefore, a fictitious apparent power is calculated with respect to this power factor definition as;

$$S_T = P/pf_T \quad (35)$$

For all possible conditions, the apparent power definition proposed by Mayordomo & Usaola gives S_T expressed in (35) when the minimum rms current compensation, which is utilized to transfer the same active power with the minimum total line loss, is applied to the system. However, the rest of the reviewed apparent power definitions may not give S_T in nonsisusoidal and unbalanced systems.

Accordingly, the relative difference values between S_T (or S_{MU}) and the rest of the reviewed apparent powers (S_{Ar}, S_V, S_e and S_Σ), which are calculated as in (36), are analyzed to understand the capabilities and limitations of S_{Ar}, S_V, S_e and S_Σ:

$$RD_X(\%) = \frac{S_X - S_{MU}}{S_{MU}} \cdot 100 \quad X\text{: } Ar,\ V,\ e \text{ and } \Sigma \quad (36)$$

In addition, the effect of α, which is the ratio between neutral line and phase line resistances ($\alpha=R_n/R_p$), on the relative difference values is taking into account. The test system used for the analysis is given in Fig. 1.

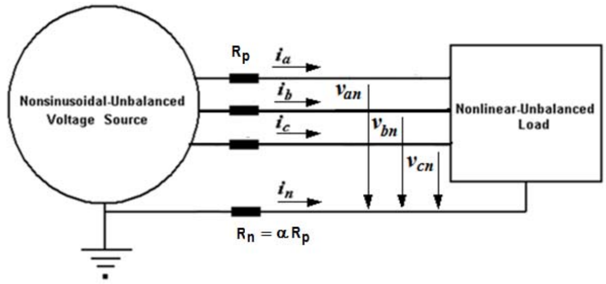

Fig. 1: Test system.

In the test system, the wave shapes of the phase-to-neutral voltages and the phase currents are given in Fig. 2 and Fig. 3, respectively.

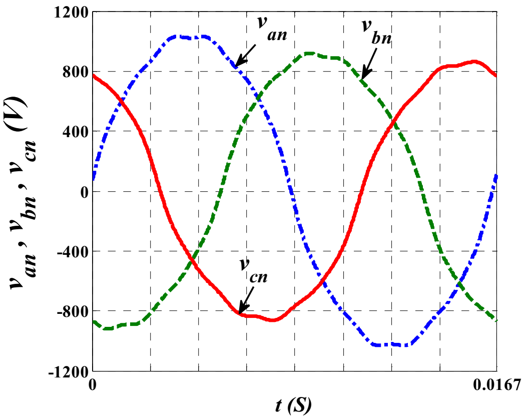

Fig. 2: The wave shapes of phase-to-neutral voltages.

It is shown from Fig. 2 that phase-to-neutral voltages have unbalanced and nonsinusoidal wave shapes, which have $THDV_{a,b,c}$, V_1^-/V_1^+ and V_1^0/V_1^+ measured as 8%, 2.05% and 10.25%, respectively.

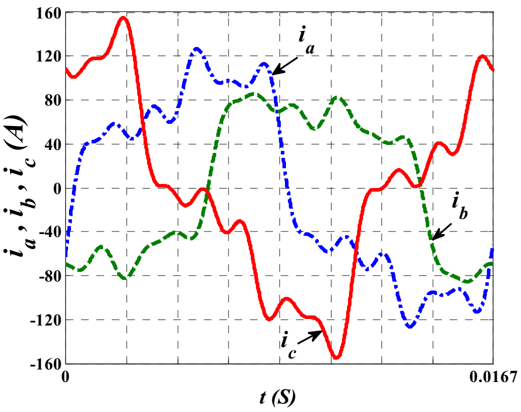

Fig. 3: The wave shapes of phase currents.

It is shown from Fig. 3 that the phase currents have unbalanced and nonsinusoidal wave shapes, which have $THDI_{a,b,c}$, I_1^-/I_1^+ and I_1^0/I_1^+ measured as 35.00 %, 22.30 % and 36.30 %, respectively.

For the interval of α from 0.1 to 3, the variation of active power (P) and the apparent power (S_{MU}) and power factor ($pf_{MU}=P/S_{MU}$) of Mayordomo & Usaola's definition are plotted in Fig. 4.

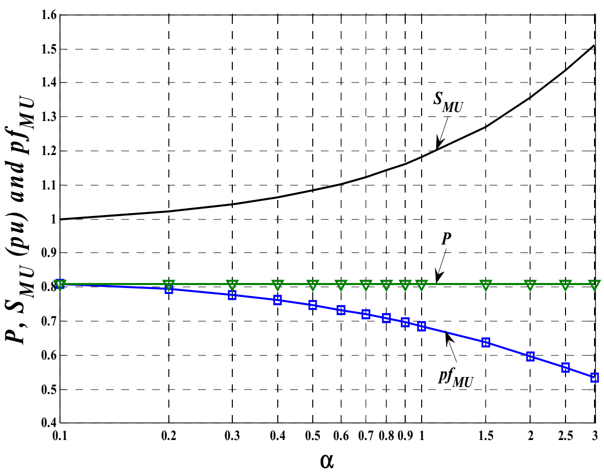

Fig. 4: *The variations of P, S_{MU} and pf_{MU} for α values from 0.1 to 3.*

Fig. 4 shows that P has the same value measured as 0.8104 pu for the α values from 0.1 to 3. However, S_{MU} increases from 1.0000 pu to 1.5122 pu and pf_{MU} decreases from 0.8104 to 0.5359 with the increment of α. This case points out that the ratio between neutral line and phase line resistances highly affects the values of S_{MU} and pf_{MU}.

The relative difference values of Buchollz's (DIN standard) apparent power (RD_Σ), IEEE standard (RD_e), Arithmetic (RD_{Ar}) and Vector (RD_V) apparent powers are plotted in Fig. 5.

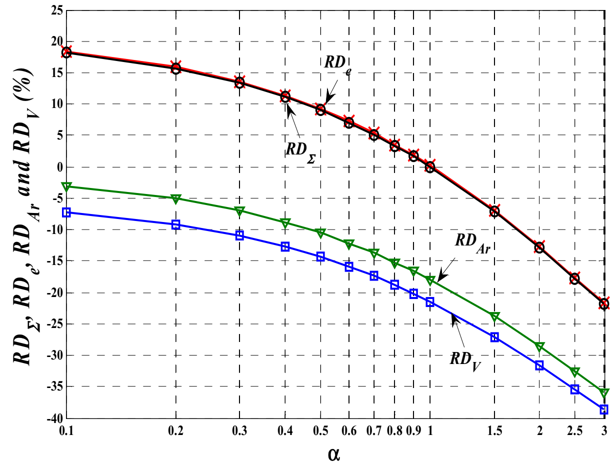

Fig. 5: The variations of RD_{Ar}, RD_V, RD_e and RD_Σ for α values from 0.1 to 3.

Fig. 5 shows that RD_{Ar} varies from -3.0538 to -35.8956, RD_V varies from -7.2031 to -38.6391, RD_e varies from 18.3815 to -21.7347, and RD_Σ varies from 18.1541 to -21.8841 with the increment of α. In addition, for α=1, RD_Σ is equal to zero. It is also seen from this figure that RD_{Ar} and RD_V are smaller than the relative difference values of two major definitions (RD_e and RD_Σ) for α<0.5 and α≤0.3, respectively. This case points out that Arithmetic and Vector apparent powers have closer values to Mayordomo & Usaola's apparent power, which is the apparent power definition keeping (34) and (35), than IEEE and DIN apparent powers for these α intervals. Finally, from the figure, it is provided that RD_e and RD_Σ are very close to each other for all α values.

5. Conclusion

In this paper, the ability of the widely recognized apparent power definitions on the measurement of the system efficiency is analysed by considering the apparent power, which is calculated in terms of the minimum and actual total line loss of the system. In this comparative analysis, the minimum total line loss is considered as the total line loss when the system is compensated using minimum rms current compensation.

The qualitative investigation shows that the apparent power definition proposed by Mayordomo & Usaola gives the apparent power definition based on the total line loss. On the other hand, the ratios of the supply line resistances should be known for the calculation of the apparent power of Mayordomo & Usaola. However, this is not possible in the practical systems.

Therefore, the comparison of Mayordomo & Usaola's apparent power and the rest reviewed apparent powers is done by taking into account the effect on the system efficiency of unbalance between neutral line and phase line resistances.

From the comparison, it can be concluded that;

- The relative difference values between Mayordomo & Usaola's apparent power and the rest reviewed apparent power definitions (Bucholz's apparent power, Arithmetic, Vector, IEEE standard apparent powers) can be considerable values.

- Arithmetic apparent power can effectively be used to measure the system efficiency for the system with the α value varying from 0.1 to 0.5.
- DIN standard and IEEE standard apparent powers, which are very close each other, can effectively be used for the systems with α values between 0.5 and 3.
- DIN standard apparent power gives the value of the apparent power proposed by Mayordomo & Usaola for α=1.
- In addition to these results, Vector apparent power has the poorest ability for the measurement of the system efficiency.

7. References

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