SMC-Based Iterative Joint MUD Data Detection, Code Delay and Multipath Estimation in DS-CDMA Systems

Erdoğan Aydın[1] and Hakan A. Çırpan[2]

[1] Electronics Engineering Department, Maltepe University, 34857 Maltepe, Istanbul, Turkey
[2] Electronics & Communications Department, Istanbul Technical University, 34469 Maslak, Istanbul, Turkey
eaydin@maltepe.edu.tr, cirpanh@itu.edu.tr

Abstract

In this work, based on a sequential Monte Carlo Method (particle filtering) a new Bayesian based iterative receiver is presented for jointly estimation of channel parameters and multiuser detection (MUD) of transmitted data sequence over the multipath fading channel in Direct Sequence Code Division Multiple Access (DS CDMA) systems. The parameter estimation of the transmission channel together with transmitted data sequence is difficult in the multipath environment since time delay of the transmission channel represents nonlinear characteristic. Therefore, we used completely Bayesian approach to overcome nonlinear characteristic of the channel with respect to time delay.

1. Introduction

Joint estimation of linear and nonlinear channel parameters and transmitted data sequence can not be obtained with classical estimation techniques. In many applications of wireless communication, Bayesian based methods offer a numeric solution to the nonlinear models. To estimate nonlinear/non-Gaussian models parameters, Bayesian based methods can be adopted to wireless communication. For a such model, numeric methods are required for the estimation of nonlinear parameters because conventional techniques can not estimate these parameters. We therefore turn to Sequential Monte Carlo (SMC) Methodologies, also known as particle filtering (PF) [1], to provide an efficient numerical approximation to the dynamic models. From this perspective, SMC methodologies are used to offer an analytic solution, and online estimation to joint estimation of code delays, channel coefficients, and transmitted symbol sequence of the multiuser.

Joint estimation techniques for single user and the works on the multi user case have been also addressed in the literature [2]. Furthermore, joint delay and multipath estimation has been proposed in [3], and channel estimation is suggested in [4, 5]. For the applications to DS-CDMA systems, the conventional Kalman Filter and Particle Filter technique were combined for the problem of single user detection [6, 7]. The solutions to the blind detection problem for multiuser systems have been presented in [8, 9], where decision-directed method was used to obtain channel information necessary for Kalman filtering. The combined (mixture) Kalman Filtering and Particle Filtering have been applied to blind MUD over fading channels [10].

Previous nonlinear estimators for DS-CDMA benefit from EKF to estimate nonlinear parameters. In contrast to previous approaches, in [11], the proposed approach for the single user is considered. In this paper, [11] is generalized for the MUD and adapted to SMC-based method that is completely PF based receiver for DS-CDMA system in order to estimate joint symbols, code delays, and multipath channel coefficients for MUD. We show how PF can be appied to proposed iterative MUD in multipath environments.

2. Signal Model

In a DS CDMA system, the transmitted complex baseband signal assigned to the of kth user can be expressed as follows:

$$y_k(t) = \sum_{n=0}^{M-1} s_{k,n} \sum_{m=0}^{N_c-1} c_{k,m} g_k(t - mT_c - nT), \quad (1)$$

transmitted, $\{c_m\}_{m=0}^{N_c-1}$ is a spreading sequence code where each chip, c_m, takes values in set $\{\pm 1\}$, and N_c is the spreading factor. T_c and T are chip and symbol period respectively and $g(t)$ is the time-limited causal raised-cosine pulse. In Figure 1, the equivalent lowpass transmission system model considered in this paper is shown. The received signal can be expressed as follows:

$$\begin{aligned} Y(t) &= \sum_{k=1}^{K} \sum_{n=0}^{M-1} s_{k,n} \sum_{m=0}^{N_c-1} c_{k,m} \sum_{l=0}^{L_h-1} h_{k,l}(t) \\ &\times g_k(t - mT_c - nT - \tau_{k,l}(t)) + n(t), \quad (2) \end{aligned}$$

where $h_k(t,\tau) = \sum_{l=0}^{L_h-1} h_{k,l}(t)\delta(\tau - \tau_{k,l}(t))$ denotes impulse response of the time varying multipath propagation channel of the kth user. L_h is the number of independents paths. $h_{k,l}(t)$ and $\tau_{k,l}(t)$ are respectively time varying complex fading channel coefficient and spreading code delay of lth channel path at time t of the kth user. After matched filter operation, the signal at the output of the matched filter is given

$$\begin{aligned} r(t) &= \langle Y(t), g^*(-t)\rangle = \int_{-\infty}^{+\infty} Y(t) g^*(-t) dt \\ &= \sum_{k=1}^{K} \sum_{n=0}^{M-1} s_{k,n} \sum_{m=0}^{N_c-1} c_{k,m} \sum_{l=0}^{L_h-1} h_{k,l}(t) \\ &\times R_g(t - mT_c - nT - \tau_{k,l}(t)) + \eta(t), \quad (3) \end{aligned}$$

where $\eta(t)$ represents the expression of transmission channel's additive white gaussian noise (AWGN) $n(t)$ at the output of the matched filter and $R_g(t) = \int_{-\infty}^{+\infty} g^*(\tau) g(t+\tau)$ is the total

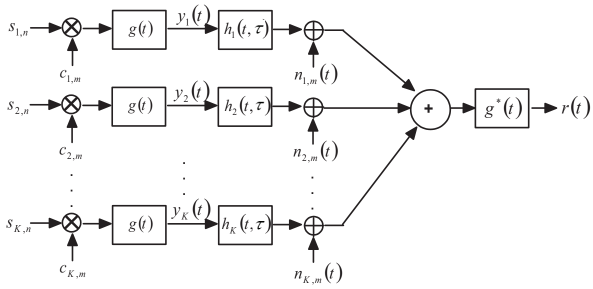

Figure 1: Structure of the transmission system model

impulse response of the transmission and receiver filters. It is noted that, the code delay must be provided $0 < \tau_{k,l}(t) < T$. The discrete time signal of (3) can be expressed as fallows

$$\begin{aligned} r_m &= \sum_{k=1}^{K}\sum_{n=0}^{M-1} s_{k,n} \sum_{i=m-L_s}^{m} c_{k,i} \sum_{l=0}^{L_h-1} h_{l,n}(k) \\ &\times \; R_g(mT_c - iT_c - nT - \tau_{l,n}(k)) + \eta_m, \quad (4) \end{aligned}$$

where we have assumed that $L_s + 1$ expresses the amount of the interference on the mth spreading sequence of the kth user, $c_{k,m}$, with assumption $(L_s + 1) < m$, $h_{l,n}(k)$ and $\tau_{l,n}(k)$ for $l = 0, \ldots, L_{h-1}$ denote respectively complex coefficient and delay of the lth channel path at time n. If the sum on the symbol in equation (4) is separated, the information of mth chip sequence in the nth symbol can be expressed as follows

$$\begin{aligned} r_{m,n} &= \sum_{k=1}^{K}\sum_{l=0}^{L_h-1} h_{l,n}(k) s_{k,n} \sum_{i=m-L_s}^{m} c_{k,i} \\ &\times \; R_g(mT_c - iT_c - nT - \tau_{l,n}(k)) + \eta_{m,n}. \quad (5) \end{aligned}$$

Then, equation (5) can be written in vector-matrix notation as

$$r_{m,n} = \underbrace{[\ \mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_K\ ]}_{\mathbf{U}} \underbrace{[\mathbf{h}_n(1), \mathbf{h}_n(2), \ldots, \mathbf{h}_n(K)]^T}_{\mathbf{h}_n} + \eta_{m,n}, \quad (6)$$

where $\mathbf{U}_k$ is $s_k \mathbf{c}_{k,m}^T$, where $(.)^T$ stands for matrix transpose, $\mathbf{h}_n, \tau_n(k)$ and $\mathbf{c}_m^T$ are $\mathbf{h}_n^T = [h_{0,n}(1), h_{1,n}(1), \ldots, h_{(L_h-1),n}(1), h_{0,n}(2), h_{1,n}(2), \ldots, h_{(L_h-1),n}(2), \ldots, h_{0,n}(K), h_{1,n}(K), \ldots, h_{(L_h-1),n}(K)]_{(1\times KL_h)}$, $\tau_n^T(k) = [\tau_{0,n}(k), \tau_{1,n}(k), \ldots, \tau_{(L_h-1),n}(k)]_{(1\times L_h)}$, $\mathbf{c}_{k,m}^T = [c_{1,m-L_s}, c_{1,m-L_s+1}, \ldots, c_{1,m}$, respectively, and $\mathbf{R}_{g_k}(\tau_n(k))$ is expressed as follows

$$\begin{bmatrix} g_1(mT_c-(m-L_s)T_c-nT-\tau_{0,n}(k)) & ,\ldots, g_K\left(mT_c-(m-L_s)T_c-nT-\tau_{(L_h-1),n}(k)\right) \\ g_1(mT_c-(m-L_s+1)T_c-nT-\tau_{0,n}(k)) & ,\ldots, g_K\left(mT_c-(m-L_s+1)T_c-nT-\tau_{(L_h-1),n}(k)\right) \\ \vdots & \vdots \quad\quad \vdots \\ g_1(mT_c-mT_c-nT-\tau_{0,n}(k)) & ,\ldots, g_K\left(mT_c-mT_c-nT-\tau_{(L_h-1),n}(k)\right) \end{bmatrix}$$

Then, (5) can be written final vector-matrix form as

$$r_{m,n} = \mathbf{U}\mathbf{h}_n + \eta_{m,n}, \quad (7)$$

The proposed iterative receiver is shown in Figure 2. As expressed in it, during the nth symbol interval, the received chip samples, $r_{m,n}$ for $m = 0, 1, \ldots, N_c - 1$, can be expressed as vector form as follows

$$\mathbf{r}_n = \mathbf{\Phi}_n + \eta_{\mathbf{n}} \quad (8)$$

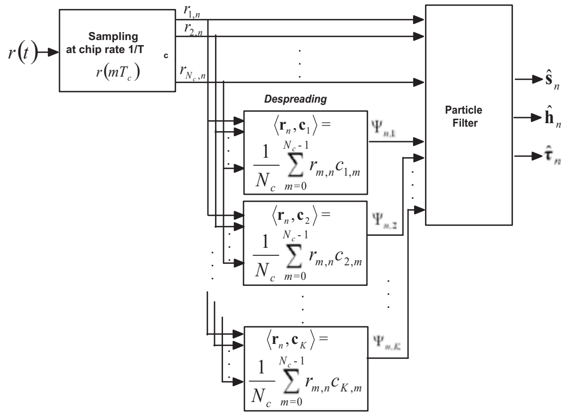

Figure 2: Structure of the proposed iterative receiver

where $\mathbf{r}_n = [r_{1,n}, r_{2,n}, , r_{N_c,n}]^T$, $\Phi_n = [\mathbf{U}\mathbf{h}_n, \ldots, \mathbf{U}\mathbf{h}_n]^T$ and $\eta_{\mathbf{n}} = [\eta_{1,n}, \ldots, \eta_{N_c,n}]^T$ After despreading operation, the obtained signals in symbol rate is given by

$$\Psi_n = \underbrace{\begin{bmatrix} Q_{1,1} & Q_{1,2} & \cdots & Q_{1,K} \\ Q_{2,1} & Q_{2,2} & \cdots & Q_{2,K} \\ \vdots & \cdots & \ddots & \vdots \\ Q_{K,1} & Q_{K,2} & \cdots & Q_{K,K} \end{bmatrix}}_{\mathbf{Q}_n} \underbrace{\begin{bmatrix} s_{1,n} \\ s_{2,n} \\ \vdots \\ s_{K,n} \end{bmatrix}}_{\mathbf{s_n}} + \underbrace{\begin{bmatrix} \overline{\eta}_{1,n} \\ \overline{\eta}_{2,n} \\ \vdots \\ \overline{\eta}_{K,n} \end{bmatrix}}_{\overline{\eta}_{\mathbf{n}}} \quad (9)$$

where $\overline{\eta}_{k,n}$ represents despreading of $\eta_{\mathbf{n}} \in \mathbb{C}^{N_c\times 1}$ in the kth spreading code, and Q_{k_1,k_2} can be expressed as $Q_{k_1,k_2} = \rho_{k_1,k_2}\widetilde{\mathbf{R}}_{g_{k_2}}(\tau_n(k_2))\mathbf{h}_n(k_2)$, where $\widetilde{\mathbf{R}}_{g_{k_2}}(\tau_{k_2,n})$ asigned to the k_2 user is $\mathbf{R}_{g_{k_2}}(end, :)$, $\mathbf{h}_{k_2,n}$ denotes the time varying complex fading channel coefficient of the k_2th user, and ρ_{k_1,k_2} represents the cross-correlation between the spreading code of the k_1th and the k_2th user and is defined as $\rho_{k_1,k_2} = \langle \mathbf{c}_{k_1}, \mathbf{c}_{k_2} \rangle = \frac{1}{N_c}\sum_{m=1}^{N_c} c_{k_1,m}c_{k_2,m}$ The set of desipreading filter outputs $\mathbf{\Psi}_n = [\Psi_{n,1}, \Psi_{n,2}, \ldots, \Psi_{n,K}]^T$ can be represented in vector-matrix form as

$$\mathbf{\Psi}_n = \mathbf{Q}_n\mathbf{s}_n + \overline{\eta}_{\mathbf{n}}. \quad (10)$$

3. Particle Filter Based Parameters Estimation

Following [6], the time varying complex fading coefficients $h_{l,n}(k)$ and spreading code delays $\tau_{l,n}(k)$ for $l = 1, \ldots, L_h, k = 1, \ldots, K$ can be described as a first-order Auto Regressive (AR) processs:

$$\begin{aligned} \tau_{1,n}(1) &= \alpha_{1,1}\tau_{1,n-1}(1) + \nu_{1,n}(1), \\ &\vdots \\ \tau_{L_h,n}(1) &= \alpha_{1,L_h}\tau_{L_h,n-1}(1) + \nu_{L_h,n}(1), \\ \tau_{1,n}(2) &= \alpha_{2,1}\tau_{1,n-1}(2) + \nu_{1,n}(2), \\ &\vdots \\ \tau_{L_h,n}(2) &= \alpha_{2,L_h}\tau_{L_h,n-1}(2) + \nu_{L_h,n}(2), \\ &\vdots \\ \tau_{L_h,n}(K) &= \alpha_{K,L_h}\tau_{L_h,n-1}(K) + \nu_{L_h,n}(K), \end{aligned} \quad (11)$$

$$
\begin{aligned}
h_{1,n}(1) &= \beta_{1,1}h_{1,n-1}(1) + \upsilon_{1,n}(1),\\
&\vdots\\
h_{L_h,n}(1) &= \beta_{1,L_h}h_{L_h,n-1}(1) + \upsilon_{L_h,n}(1),\\
h_{1,n}(2) &= \beta_{2,1}h_{1,n-1}(2) + \upsilon_{1,n}(2),\\
&\vdots\\
h_{L_h,n}(2) &= \beta_{2,L_h}h_{L_h,n-1}(2) + \upsilon_{L_h,n}(2),\\
&\vdots\\
h_{L_h,n}(K) &= \beta_{K,L_h}h_{L_h,n-1}(K) + \upsilon_{L_h,n}(K),
\end{aligned}
\tag{12}
$$

where parameters of $\alpha_{1,1},\ldots,\alpha_{1,L_h},\alpha_{2,1},\ldots,\alpha_{K,L_h}$ and $\beta_{1,1},\ldots,\beta_{1,L_h},\beta_{2,1},\ldots,\beta_{K,L_h}$ indicate the changing of multipath channel with time.$\nu_{1,1},\ldots,\nu_{1,L_h},\nu_{2,1},\ldots,\nu_{K,L_h}$ and $\upsilon_{1,1},\ldots,\upsilon_{1,L_h},\upsilon_{2,1},\ldots,\upsilon_{K,L_h}$ are AWGN with zero mean and respectively variance $\sigma_\nu^2,\sigma_\upsilon^2$, i.e., $\mathcal{N}(0,\sigma_\nu^2),\mathcal{N}(0,\sigma_\upsilon^2)$.

The equivalent vector-matrix representation of the equations (11) and (12) is given by

$$
\underbrace{\begin{bmatrix}\tau_n(1)\\ \tau_n(2)\\ \vdots\\ \tau_n(K)\end{bmatrix}}_{\tau_n}=\underbrace{\begin{bmatrix}\mathbf{A}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}_2 & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_K\end{bmatrix}}_{\mathbf{A}}\underbrace{\begin{bmatrix}\tau_{n-1}(1)\\ \tau_{n-1}(2)\\ \vdots\\ \tau_{n-1}(K)\end{bmatrix}}_{\tau_{n-1}}+\underbrace{\begin{bmatrix}\nu_n(1)\\ \nu_n(2)\\ \vdots\\ \nu_n(K)\end{bmatrix}}_{\nu_n}
\tag{13}
$$

$$
\underbrace{\begin{bmatrix}\mathbf{h}_n(1)\\ \mathbf{h}_n(2)\\ \vdots\\ \mathbf{h}_n(K)\end{bmatrix}}_{\mathbf{h}_n}=\underbrace{\begin{bmatrix}\mathbf{B}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{B}_2 & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & 0 & \mathbf{B}_K\end{bmatrix}}_{\mathbf{B}}\underbrace{\begin{bmatrix}\mathbf{h}_{n-1}(1)\\ \mathbf{h}_{n-1}(2)\\ \vdots\\ \mathbf{h}_{n-1}(K)\end{bmatrix}}_{\mathbf{h}_{n-1}}+\underbrace{\begin{bmatrix}\upsilon_n(1)\\ \upsilon_n(2)\\ \vdots\\ \upsilon_n(K)\end{bmatrix}}_{\upsilon_n}
\tag{14}
$$

where $\tau_n(k) = [\tau_{1,n}(k),\ldots,\tau_{L_h,n}(k)]^T$ and $\mathbf{h}_n(k) = [h_{1,n}(k),\ \ldots,h_{L_h,n}(k)]^T$ are $(L_h\times 1)$ vectors , $\mathbf{0}$ is an $(L_h\times L_h)$ all zeros matrix , $\mathbf{A}_k = diag\{\alpha_{k,1},\ldots,\alpha_{k,L_h}\}$ and $\mathbf{B}_k = diag\{\beta_{k,1},\ldots,\beta_{k,L_h}\}$ are diagonal matrixes of size $(L_h\times L_h)$, $\nu_{\mathbf{n}}(\mathbf{k}) = [\nu_{1,n}(k),\ldots,\nu_{L_h,n}(k)]^T$ and $\upsilon_{\mathbf{n}}(\mathbf{k}) = [\upsilon_{1,n}(k),\ldots,\upsilon_{L_h,n}(k)]^T$ are $(L_h\times 1)$ AWGN vectors. Then, (13) and (14) can be rewritten in final vector-matrix form as

$$
\tau_n = \mathbf{A}\tau_{n-1}+\nu_{\mathbf{n}},\qquad \mathbf{h}_n = \mathbf{B}\mathbf{h}_{n-1}+\upsilon_{\mathbf{n}},
\tag{15}
$$

where $\mathbf{A}$ and $\mathbf{B}$ are block diagonal matrixes, i.e., $\mathbf{A} = diag\{\mathbf{A}_1,\ldots,\mathbf{A}_K\}$ and $\mathbf{B} = diag\{\mathbf{B}_1,\ldots,\mathbf{B}_K\}$.

We can then combine (8) and (15) to express the following state-space representation

State Equations:

$$
\tau_n = \mathbf{A}\tau_{n-1}+\nu_{\mathbf{n}},\quad \mathbf{h}_n = \mathbf{B}\mathbf{h}_{n-1}+\upsilon_{\mathbf{n}},\quad \mathbf{s}_n = \mathbf{S}\mathbf{s}_{n-1}+\mathbf{d}_n
$$

Observation Equation:

$$
\mathbf{r}_n = \mathbf{\Psi}_n+\eta_{\mathbf{n}},
\tag{16}
$$

where $\mathbf{S}$ is an $(K\times K)$ shifting matrix whose all elements are zero and $\mathbf{d}_k$ is the $(K\times 1)$ vector that includes the new sysmbols,s_k, $\mathbf{d}_k = [s_{1,n},s_{2,n},\ldots,s_{K,n}]^T$,$k = 1,2,\ldots,K$.

By means of Bayesian perspective, the estimation of the state sequence $\{\mathbf{s}_{1:n},\tau_{1:n},\mathbf{h}_{1:n}\}$ can be obtained with joint posterior probability distribution of the system state $p(\mathbf{s}_{1:n},\tau_{1:n},\mathbf{h}_{1:n}|\mathbf{r}_{1:n})$. Since the estimation of the later density of the system state can not be analytically obtained over the time, the closed form solution of the system sequence is not possible. Therefore, we appeal to PF technique. According to this technique, the system state is approximated using N discrete random measures that are defined with particles,$\{(\mathbf{s}_{1:n},\tau_{1:n},\mathbf{h}_{1:n})^{(i)}\}_{i=1}^N$, and associated importance weights, $w_n^{(i)}$.

The next important factor in the PF have been defined importance function. Since particles can be generated according to importance function, $\pi(\mathbf{s}_{1:n},\tau_{1:n},\mathbf{h}_{1:n})$, instead of $p(\mathbf{s}_{1:n},\tau_{1:n},\mathbf{h}_{1:n}|\mathbf{r}_{1:n})$, this function must have same support with probability distribution function that is obtained approximately. Then, normalized weights are computed as $w_n^{(i)}(\mathbf{s}_{1:n},\tau_{1:n},\mathbf{h}_{1:n}|\mathbf{r}_{1:n})/\pi(\mathbf{s}_{1:n},\tau_{1:n},\mathbf{h}_{1:n})$ where, $w_n^{(i)} = w_n^{*(i)}/\sum_{i=1}^N w_n^{*(i)}$ where $w_n^{*(i)}$ is nonnormalized weight. If the importance density is factorized as $\pi(\mathbf{s}_{1:n},\tau_{1:n},\mathbf{h}_{1:n}) = \prod_{i=1}^n \pi_i(\mathbf{s}_i,\tau_i,\mathbf{h}_i)$, then the particles and associated importance weights are updated according to equations (17) and (18)

$$
(\mathbf{s}_{n+1},\tau_{n+1},\mathbf{h}_{n+1})^{(i)}\sim\pi_{n+1}(\mathbf{s}_{n+1},\tau_{n+1},\mathbf{h}_{n+1})
\tag{17}
$$

$$
w_{n+1}^{(i)}\propto w_n^{(i)}\frac{p(\mathbf{r}_{n+1}|\mathbf{s}_{n+1}^{(i)},\tau_{n+1}^{(i)},\mathbf{h}_{n+1}^{(i)})p(\mathbf{s}_{n+1}^{(i)},\tau_{n+1}^{(i)},\mathbf{h}_{n+1}^{(i)}|\mathbf{s}_n^{(i)},\tau_n^{(i)},\mathbf{h}_n^{(i)})}{\pi_{n+1}(\mathbf{s}_{n+1}^{(i)},\tau_{n+1}^{(i)},\mathbf{h}_{n+1}^{(i)})}
\tag{18}
$$

In literature, there are two frequently used importance functions which are priori and optimal importance function. Optimal importance function minimizes the variance of the importance weights. This function is given by

$$
\begin{aligned}
&\pi_{n+1}(\mathbf{s}_{n+1},\tau_{n+1},\mathbf{h}_{n+1})\\
&\quad= p(\mathbf{s}_{n+1},\tau_{n+1},\mathbf{h}_{n+1}|\mathbf{s}_n^{(i)},\tau_n^{(i)},\mathbf{h}_n^{(i)},\mathbf{r}_{n+1})\\
&\quad\propto p(\mathbf{s}_{n+1}|\mathbf{s}_n^{(i)},\tau_{n+1},\mathbf{h}_{n+1},\mathbf{r}_{n+1})\\
&\quad\times p(\tau_{n+1},\mathbf{h}_{n+1}|\tau_n^{(i)},\mathbf{h}_n^{(i)},\mathbf{r}_{n+1}).
\end{aligned}
\tag{19}
$$

Unfortunately, since the calculation of the last term in equation (19) is difficult, the following approach is provided as follows

$$
\begin{aligned}
&p(\tau_{n+1},\mathbf{h}_{n+1}|\tau_n^{(i)},\mathbf{h}_n^{(i)},\mathbf{r}_{n+1})\\
&\quad= p(\tau_{n+1}|\mathbf{h}_{n+1},\tau_n^{(i)},\mathbf{r}_{n+1})\\
&\quad\times p(\mathbf{h}_{n+1}|\mathbf{h}_n^{(i)},\mathbf{r}_{n+1})\\
&\quad\approx p(\tau_{n+1}|\tau_n^{(i)})p(\mathbf{h}_{n+1}|\mathbf{h}_n^{(i)}).
\end{aligned}
\tag{20}
$$

Note that to obtain final form of equation (20), we assume that the channel, $\mathbf{h}$, and code delay ,τ, are independent of each other. When equation (20) is put into (19), the importance function is obtained as follows:

$$
\begin{aligned}
\pi_{n+1}(\mathbf{s}_{n+1},\tau_{n+1},\mathbf{r}_{n+1}) &= p(\mathbf{s}_{n+1}|\mathbf{s}_n^{(i)},\tau_{n+1},\mathbf{h}_{n+1},\mathbf{r}_{n+1})\\
&\times p(\tau_{n+1}|\tau_n^{(i)})p(\mathbf{h}_{n+1}|\mathbf{h}_n^{(i)}).
\end{aligned}
\tag{21}
$$

The proposed iterative PF technique consists of *Initialization, Importance Sampling, Weight Update, Estimation* and *Resampling* which are described in detail in the following paragraphs.

3.1. Initialization

The *a priori* density of the code delay $p(\tau_{-1})$ is uniform in the interval $(0,T_c)$. We assume that spreading code sequences of all the users, i.e., $\mathbf{c}_k = [c_{k,1},c_{k,2},\ldots,c_{k,N_c}]^T$ for $k =$

$1, 2, \ldots, K$, are known by the receiver. In practice the fist symbol of all the users, $\mathbf{s}_{-1}$, is known by the receiver. Supposing that the transmission channel has a Gaussian distribution $\mathcal{N}(\hat{\mathbf{h}}_{-1}, \sigma_h^2)$, where $\hat{\mathbf{h}}_{-1}$ is the mean and σ_h^2 is the variance of the channel. The initial values of the channel paths, $\hat{\mathbf{h}}_{-1}$, have been estimated using pilot symbol during the first communication, and all the initial weights of the particles are equal.

3.2. Importance Sampling

Considering(21), it can be seen that the importance function consists of three distributions. Code delay and channel coefficient distributions are obtained by sampling according to

$$\tau_{n+1}^{(i)} \sim \mathcal{N}(\mathbf{A}\tau_n^{(i)}, \sigma_\nu^2), \quad \mathbf{h}_{n+1}^{(i)} \sim \mathcal{N}(\mathbf{B}\mathbf{h}_n^{(i)}, \sigma_v^2). \qquad (22)$$

Since the transmitted symbol sequence is not sampled directly using state equation, they are sampled from $p(\mathbf{s}_{n+1} |\mathbf{s}_n^{(i)}, \tau_{n+1}, \mathbf{h}_{n+1}, \mathbf{\Psi}_{n+1})$. Since these symbols are i.i.d. discrete uniform random variables, this distribution can be written as follows

$$\begin{aligned} &p(\mathbf{s}_{n+1}|\mathbf{s}_n^{(i)}, \tau_{n+1}^{(i)}, \mathbf{h}_{n+1}^{(i)}, \mathbf{\Psi}_{n+1}) \\ &\quad \propto \; p(\mathbf{\Psi}_{n+1}|\mathbf{s}_{n+1}, \mathbf{s}_n^{(i)}, \tau_{n+1}^{(i)}, \mathbf{h}_{n+1}^{(i)}) = \mathcal{N}(\mu_{\mathbf{n+1}}{}^{(i)}, \sigma_{\overline{\eta}}^{\mathbf{2}}) \\ &\quad = \; \mathcal{N}(\mu_{\mathbf{n+1}}{}^{(i)}, \sigma_{\overline{\eta}}^{\mathbf{2}}), \end{aligned} \qquad (23)$$

where,

$$\mu_{\mathbf{n+1}}^{(\mathbf{i})} = \underbrace{\begin{bmatrix} Q_{1,1}^{(i)} & Q_{1,2}^{(i)} & \cdots & Q_{1,K}^{(i)} \\ Q_{2,1}^{(i)} & Q_{2,2}^{(i)} & \cdots & Q_{2,K}^{(i)} \\ \vdots & \cdots & \ddots & \vdots \\ Q_{K,1}^{(i)} & Q_{K,2}^{(i)} & \cdots & Q_{K,K}^{(i)} \end{bmatrix}}_{\mathbf{Q_{n+1}^{(i)}}} \underbrace{\begin{bmatrix} s_{1,(n+1)} \\ s_{2,(n+1)} \\ \vdots \\ s_{K,(n+1)} \end{bmatrix}}_{\mathbf{s_{n+1}}} \qquad (24)$$

i.e., $\mu_{\mathbf{n+1}}^{(\mathbf{i})} = \mathbf{Q_{n+1}^{(i)} s_{n+1}}$ where $Q_{j,k}^{(i)} = \rho_{j,k}^{(i)} \mathbf{R}_{gk}^{(i)}(\tau_n^{(i)}(k))\mathbf{h}_n^{(i)}(k)$ It is noted that, given $\mathbf{s}_n^{(i)}$, in order to generate the new symbols, only $\mathbf{s}_{n+1}^{(i)}$ have required. Therefore, the new symbols can be obtained by assigning posterior probabilities to alphabet $\mathcal{Y} = \{\mathcal{Y}_1, \mathcal{Y}_2, \ldots, \mathcal{Y}_j\}$ that is used in the modulation. Then probability mass function is calculated as

$$\begin{aligned} \xi^{(\mathbf{i})}(\mathbf{s}_{n+1}) \; &= \; p(\mathbf{s}_{n+1} = \mathcal{Y}_{\mathbf{j}}|\mathbf{s}_n^{(i)}, \tau_{n+1}^{(i)}, \mathbf{h}_{n+1}^{(i)}, \mathbf{\Psi}_{n+1}) \\ &= \; \frac{\mathcal{N}(\mu_{\mathbf{n+1}}^{(\mathbf{i})}(\mathcal{Y}_{\mathbf{j}}), \sigma_{\overline{\eta}}^2)}{\sum_{\mathcal{Y} \in \mathcal{Y}} \mathcal{N}(\mu_{\mathbf{n+1}}^{(\mathbf{i})}(\mathcal{Y}), \sigma_{\overline{\eta}}^2)} \end{aligned} \qquad (25)$$

for $i = 1, \ldots, N$. Finally, $p(\mathbf{s}_{n+1}|\mathbf{s}_n^{(i)}, \tau_{n+1}^{(i)}, \mathbf{h}_{n+1}^{(i)}, \mathbf{\Psi}_{n+1})$ is obtained by putting the $\xi^{(i)}(\mathbf{s}_{n+1})$ in place of $\mathbf{s}_{n+1}^{(i)}$.

3.3. Weight Update

The probability density $p(\mathbf{\Psi}_{n+1}|\mathbf{s}_{n+1}^{(i)}, \tau_{n+1}^{(i)}, \mathbf{h}_{n+1}^{(i)})$ can then be written at the symbol rate

$$p(\mathbf{\Psi}_{n+1}|\mathbf{s}_{n+1}^{(i)}, \tau_{n+1}^{(i)}, \mathbf{h}_{n+1}^{(i)}) = \mathcal{N}(\mu_{\mathbf{n+1}}{}^{(i)}, \sigma_{\overline{\eta}}^{\mathbf{2}}), \qquad (26)$$

Once the new particles have been sampled, the importance weights are updated. The updated weights are computed in equation (27) substituting (21), (23), (25) and (26) into (18).

$$\begin{aligned} w_{n+1}^{*(i)} \; &\propto \; w_n^{(i)} \frac{p(\mathbf{\Psi}_{n+1}|\mathbf{s}_{n+1}^{(i)}, \tau_{n+1}^{(i)}, \mathbf{h}_{n+1}^{(i)})}{\xi^{(i)}(s_{n+1}^{(i)})} p(s_{n+1}^{(i)}|s_n^{(i)}) \\ &\propto \; w_n^{(i)} \frac{p(\mathbf{\Psi}_{n+1}|\mathbf{s}_{n+1}^{(i)}, \tau_{n+1}^{(i)}, \mathbf{h}_{n+1}^{(i)})}{\xi^{(i)}(s_{n+1}^{(i)})} \\ &= \; w_n^{(i)} \sum_{\mathcal{Y} \epsilon \mathcal{Y}} \mathcal{N}(\mu_{n+1}^{(i)}(\mathcal{Y}), \sigma_\eta^2) \end{aligned} \qquad (27)$$

Then normalized weights are $w_{n+1}^{(i)} = w_{n+1}^{*(i)} / \left(\sum_{i=1}^N w_{n+1}^{*(i)} \right)$

3.4. Estimation

The minimum mean square estimate (MMSE) of the code delays and multipath channels are given by $\hat{\tau}_{0:n+1}^{MMSE} = \sum_{i=1}^N \tau_{0:n+1}^{(i)} w_{n+1}^{(i)}$, $\hat{\mathbf{h}}_{0:n+1}^{MMSE} = \sum_{i=1}^N \mathbf{h}_{0:n+1}^{(i)} w_{n+1}^{(i)}$ and maximum a posteriori (MAP) estimate of the symbols is presented as $\hat{\mathbf{s}}_{0:n+1}^{MAP} = \underbrace{\arg\max}_{\mathbf{s}_{0:n+1}} \left\{ \sum_{i=1}^N \delta(\mathbf{s}_{0:n+1}^{(i)} - \mathbf{s}_{0:n+1}) w_{n+1}^{(i)} \right\}$.

3.5. Resampling

Resampling [12] is a technique where particles with negligible weights are removed and particles with considerable weights are replicated. The resampling can be implemented whenever the effective sample size N_{eff} , estimated by $\hat{N}_{eff} = \frac{1}{\sum_{i=1}^N (w_{n+1}^{(i)})^2} \leq N/2$ is below a certain threshold N_{thres}.

4. Simulation Results

In this section, we provide computer simulations to demonstrate the performance of the proposed iterative SMC receiver for Multiuser DS-CDMA system with BPSK modulation in multipath fading channel. Orthogonal Walsh-Hadamard codes are used for spreading. Spreading factor $N_c = 64$, transmitted symbols $M = 500$. L_s is limited to 7 samples. Roll-off factor, α_{rcp}, is 0.3. The number of multipaths $L_h = 3$ and $N = 200$. The code delay and multipath channel are modeled as first order AR process driven by a white Gaussian noise with equal variance, i.e., $\sigma_\nu^2 = \sigma_v^2 = 0.001$. $\alpha_{1,1} = \alpha_{1,2} =, \ldots, = \alpha_{1,L_h} =, \ldots, = \alpha_{K,L_h} 0.999$, $\beta_{1,1} = \beta_{1,2} =, \ldots, = \beta_{1,L_h}, \ldots, = \beta_{K,L_h} = 0.999$.

First, we consider the situation that the channel coefficients and symbols are unknown by the receiver to demonstrate the estimation of code delay tracking performance. Figure 3 shows the code delay tracking performance of the proposed Blind SMC receiver for the kth user with the three code delays. Assuming that in the Figure 3; E_s/N_0=25 dB where E_s is the energy per symbol and N_0 is unilateral spectral power density.

The channel paths tracking performance of the proposed Blind algorithm with unknown symbols and code delays is given in Figure 4 where E_s/N_0=25 dB.

In Figure 5, the bit-error-rate (BER) performance of the proposed iterative SMC receiver with the unknown channel coefficients and code delays as well as with the known channel coefficients and code delays in multipath fading channel is shown. The decorrelating decision feedback detector (DDF) [13] is used in the comparision of proposed method. We provide BER versus SNR performance for $K = 3$. For the scenario of $K = 3$, the number of multipaths L_h is 3.

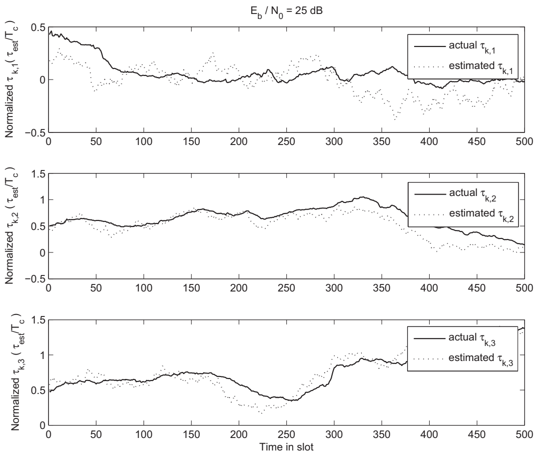

Figure 3: Code delays tracking performance of the proposed iterative SMC receiver for the kth user at E_s/N_0 = 25 dB.

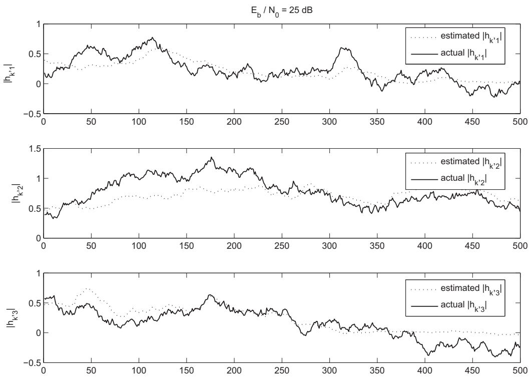

Figure 4: Channel paths tracking performance of the proposed iterative SMC receiver for the kth user at E_s/N_0 = 25 dB.

5. Conclusion

In this paper, we have developed a new recursive receiver joint estimation of code delays, channel coefficients, and transmitted symbol sequence based on Sequential Monte Carlo methodology for MUD in DS-CDMA systems with multipath fading channels. The proposed sequential approach presents the online joint estimation of channel parameters together with transmitted symbols without using training data for MUD in DS-CDMA systems.

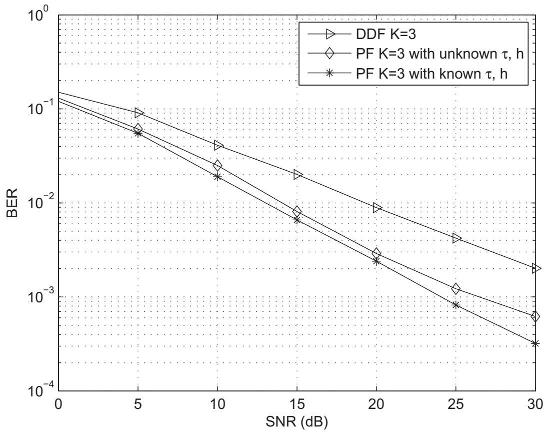

Figure 5: The total BER performance of the proposed iterative SMC receiver in the multipath environment with three paths

6. References

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