A Fast and Straightforward Solver for Generation Allocation Problem Including Losses using A Hopfield Network

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Abstract

In this paper, a fast solver for generation allocation problem including transmission losses using a Hopfield Neural Network (HNN) approach is presented. The proposed HNN is distinguished by a direct computation method mapped to the generation allocation problem of thermal generators commonly known as economic dispatch (ED). The developed HNN employs a linear input-output model for the transfer function of neurons. Formulations for solving the ED problem are explored, through the application of these formulations; direct computation instead of iterations for solving the problem without losses becomes possible. Not like the usual Hopfield methods, which select the weighting factors of the energy function by trials, the proposed method determines the corresponding factors only by calculations. To include the transmission losses, a dichotomy method is combined to the Hopfield Neural Network iteratively. The effectiveness of the developed method is identified through its application to the 15-unit system. Computational results manifest that the method has a lot of excellent performances.

1. Introduction

The objective of generation allocation problem is to minimize production cost while satisfying demand and working area constraints for a given combination of active units. Aside from using the solutions of the ED (combination of units with the least production cost) for its own merits in system operation, they are used to guide the solution method that solves the combinatorial part of the unit commitment problem. When the combinatorial part of the unit commitment problem is solved, solutions from the ED are used to estimate the quality of different unit combinations.

In this paper the construction and implementation of an exact method using the Hopfield Neural Network that solves the economic dispatch problem is presented. The performance of such method with respect to time and solution quality is a crucial part in the solution process of solving the unit commitment problem. The use of the Hopfield neural network methods to solve ED is therefore justifiable if the method produces optimal solutions and outperforms near-optimal solver with respect to computation time.

2. Problem Formulation

Generation allocation is defined as the process of allocating generation levels to the thermal generating units in service within the power system, so that the system load is supplied entirely and most economically [1] and [2]. The objective of the generation allocation or economic dispatch ED problem is to calculate, for a single period of time, the output power of every generating unit so that all demands are satisfied at minimum cost, while satisfying different technical constraints of the network and the generators. The system consists of N generating units connected to a single bus-bar serving an electrical load D. The input to each unit shown as Fi, represents the generation cost of the unit. The output of each unit Pi is the electrical power generated by that particular unit. The total cost of the system is the sum of the costs of each of the individual units. The essential constraint on the operation is that the sum of the output powers must equal the load demand.

The standard ED problem can be described mathematically as an objective with two constraints as:

$$\min F_T = \sum_{i=1}^N F_i(P_i) \tag{1}$$

Subject to the following constraints:

$$\sum_{i=1}^{N} P_i = D + L$$

$$P_i^{\min} \le P_i \le P_i^{\max}$$
(2)

where, N is the total number of units in service; P_i is the real power output of i-th generator (MW); F_T is the total operating cost (\$ /h); F_i (P_i) is the operating cost of unit i (\$ /h); D is the total demand (MW); L is the transmission losses (MW); P_i^{min} , P_i^{max} are the operating power limits of unit *i* (MW).

The fuel cost function or input-output characteristic of the generator may be obtained from design calculations or from heat rate tests. Many different formats are used to represent this characteristic. The data obtained from heat rate tests or from the plant design engineers may be fitted by a polynomial curve. It is usual that, quadratic characteristic is fit to these data. A series of straight-line segments may also be used to represent the input-output characteristic [1]. The fuel cost function of a generator that usually used in power system operation and control problem is represented with a second-order polynomial.

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^{\ 2}$$
(3)

where, a_i , b_i and c_i are the cost coefficients (non-negative constants) of the *i* th generating unit.

For some generators such as large steam turbine generators, however, the input-output characteristic is not always as smooth as Eq. (3). Large steam turbine generators will have a number of steam admission valves that are opened in sequence to obtain ever-increasing output of the unit [3], [4].

3. Solving Economic Dispatch with Hopfield Neural Network

The EDP has been widely studied and reported by several authors in books and journals on power system analysis. Many techniques have been developed to solve this problem, e.g. the lambda-iterative method, gradient technique, Interior Point, Lagrange technique, linear programming, Quadratic Programming, Dynamic Programming, Simulated Annealing, Genetic algorithm (GA), Evolutionary Programming (EP), Neural Network and methods combining two or more of the above methods [6] and [7]. Most of these methods often suffer from the large amount of computational requirement or give just a good estimate (near optimal) of the solution to the ED problem.

The continuous or deterministic model of the Hopfield Neural Network is based on continuous variables. The output variable of neuron *i* has the range $y_i^0 < y_i < y_i^{\ I}$ and the input-output function is a continuous and monotonically increasing function of the input x_i to neuron *i*. The model is a mutual coupling neural network and of non-hierarchical structure. Architecture of a HNN of three neurnes sample is shown in figure 1. The processing elements are modeled as a neurone in conjunction with feedback circuits to model the basic computational features of neurons and synapses connecting different neurons. Usually the neurones have sigmoidal monotonic input-output relations.



Fig. 1. Architecture of the Hopfield Neural Network

The dynamic characteristic of each neuron can be described by the following differential equation [9] and [10].

$$\frac{dx_i}{dt'} = \sum_{j=1}^{N} \omega_{ij} y_j + I_i \tag{4}$$

where x_i is the neuron input; y_i is the neuron output; ω_{ii} is the self-connection conductance of neuron i; ω_{ij} is the connection conductance between neuron i and neuron j; I_i is the external conductance of neuron i

The output of neuron *i* is given by:

 $y_i = f_i\left(x_i\right) \tag{5}$

where $f_i(x_i)$ is the input-output function of the neuron *i*. The energy function of the continuous Hopfield model can be defined as:

$$E = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_{ij} y_i y_j - \sum_{i=1}^{N} I_i y_i$$
(6)

The time derivative of the energy function can be proved to be negative [11]. Therefore, in the computation process the model state always moves in such a way that energy function gradually reduces and converges to a minimum.

The Hopfield model of neural networks [12] has been employed to solve the ED problem for units having continuous or piece wise quadratic fuel cost function [9] and [13], and even for units having prohibited zones constraint [14, 15]. The conventional Hopfield model belongs to the kind of continuous and deterministic model, and the input-output relationship for its neurons is described by a modified sigmoidal function. Due to the use of sigmoidal function in the conventional Hopfield model, in solving the ED problems, a method involving numerical iterations is inevitably applied; this numerical iteration method often suffers from large amount of computational requirements. Adopting a modified sigmoidal function causes two other problems. The first, it incurs unreasonable or incorrect generation dispatch, which is attributable to the serious saturation phenomena existing in the input-output relationship represented by the sigmoidal function. The second; it is troublesome to select shape constant of the sigmoidal function.

A fast Hopfield Neural Network method to solve the ED problem is presented. The method employs a linear input-output model for the neurons. Formulations for solving the ED problem are explored. Through the application of these formulations, direct computation instead of iterations for solving the problem becomes possible. Not like the usual Hopfield methods, which select the weighting factors of the energy function by trials, this method determines the corresponding factors by calculation.

The adoption of a linear model describing the input-output relationship of the neuron has resulted in the avoidance of the aforementioned problems.

To solve the ED problem using the Hopfield method, energy function including both power mismatch, P_m and total fuel cost F is defined as follows:

$$E = (A/2) \left((D+L) - \sum_{i=1}^{N} P_i \right)^2 + .$$

(B/2) $\sum_{i=1}^{N} \left(a_i + b_i P_i + c_i P_i^2 \right)$ (7)

A and B: introduce the relative importance of their respective associated terms.

Comparing Eq. (7) with Eq. (6), we get:

$$\omega_{ii} = -A - B \cdot c_i \tag{8}$$

$$\omega_{ij} = -A \tag{9}$$

$$I_i = A (D + L) - B (b_i/2)$$
(10)

At this stage the transmission losses L can be neglected and reconsidered later in section 4. Substituting Eq. (8), Eq. (9) and Eq. (10) into Eq. (4), the dynamic equation becomes,

$$dx_{i}/dt' = AP_{m} - (B/2)(dF_{i}/dP_{i})$$
(11)

Application of the conventional Hopfield method to the ED problem, the power output value can be represented by the output y_i of neuron *i* using a modified sigmoidal function, described as follows [13] and [14]:

$$P_{i} = y_{i} = f_{i}(x_{i})$$

. = $P_{i}^{\min} + (1/2) (P_{i}^{\max} - P_{i}^{\min}) (1 + \tanh(x_{i}/\alpha_{0}))$ (12)

where α_0 is the shape constant of the sigmoidal function.

To avoid the problems resulting from curve saturation, a linear model is used to describe the input-output relationship for the neuron instead of the sigmoidal function. Linear transfer function of the *i*-th neuron is defined as follows:

$$y_{i} = P_{i} = \begin{cases} \frac{x_{i} - x_{\min}}{x_{\max} - x_{\min}} (P_{i}^{\max} - P_{i}^{\min}) + P_{i}^{\min}, x_{\min} \le x_{i} \le x_{\max} \\ P_{i}^{\max}, x_{i} \ge x_{\max}, x_{i} \ge x_{\max} \\ P_{i}^{\min}, x_{i} \le x_{\min} \end{cases}$$
(13)

Substituting Eq. (13) in Eq. (11) the dynamic equation becomes:

$$dx_{i}/dt^{*} = AP_{m} - (B/2)(b_{i} + 2c_{i}(K_{1i}x_{i} + K_{2i}))$$
(14)

with $K_{1i} = \left(P_i^{\max} - P_i^{\min}\right) / \left(x_{\max} - x_{\min}\right)$

$$K_{2i} = P_i^{\min} - K_{1i} x_{\min}$$

Solving Eq. (14) the neuron's input function, $x_i(t')$ is obtained as:

$$x_i(t^{*}) = \left(x_i(0) + \left(K_{4i}/K_{3i}\right)\right)e^{K_{3i}t^{*}} - \left(K_{4i}/K_{3i}\right)$$
(15)

with:

$$K_{3i} = -Bc_i K_{1i}$$
(16)

$$K_{4i} = AP_m - (B/2)b_i - Bc_i K_{2i}$$
(17)

From Eq. (13), the neuron's output function, $y_i = P_i(t')$, is obtained as:

$$P_{i}(t^{*}) = \left(2K_{AB}P_{m} - b_{i}\right)/2c_{i} + \left(K_{1i}x_{i}(0) + K_{2i} - \left(2K_{AB}P_{m} - b_{i}\right)/2c_{i}\right)e^{K_{3i}t^{*}}$$
(18)

where $K_{AB} = A/B$

The second term in Eq. (18) decays exponentially, finally becomes vanishingly small and eventually setting $t' = \infty$, Eq. (18) gives,

$$P_i(\infty) = y_i(\infty) = \left(2K_{AB}P_m - b_i\right)/2c_i \tag{19}$$

Here $P_i(\infty)$ represents the optimal generation level of unit *i*, and the final neuron output y_i which is the required solution. Back substituting of Eq. (19) in Eq. (18), a more simple formula for the generation function is given as:

$$P_{i}(t^{*}) = P_{i}(\infty) + \left(P_{i}(0) - P_{i}(\infty)\right)e^{K_{3i}t^{*}}$$
(20)

where $P_i(0)$ is obtained from Eq. (18) by letting t'=0, to give:

$$P_i(0) = K_{2i} + K_{1i} x_i(0) \tag{21}$$

It should be noted here that t' is not representing real time, it is a dimensionless variable.

Using the power mismatch definition and Eq. (28) we obtain:

$$P_m = \left(D + \left(\frac{1}{2}\right)\sum_{i=1}^{N} \frac{b_i}{c_i}\right) / \left(1 + K_{AB}\sum_{i=1}^{N} \frac{1}{c_i}\right)$$
(22)

Eqs. (19) through Eq. (22) constitute the Hopfield model for the economic dispatch problem. A non iterative direct computation process is, therefore, possible.

4. Inclusion of Transmission Losses in a Hybrid Algorithm

The transmission losses L can be either given from a load flow study or approximated by traditional representation using B coefficients:

$$L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{cij} P_j + \sum_{i=1}^{N} B_{0i} P_i + B_{00}$$
(23)

A dichotomy solution method for solving the economic dispatch including transmission losses combined to the Hopfield Neural Network is presented in the following steps:

Step1: Initialization of the interval search $[Cr_3 \ Cr_1]$. Where Cr_3 and Cr_1 are the estimation of total production when neglecting losses and when including losses, respectively. ε : is a pre-specified tolerance; γ is maximum value losses to

load ratio.

Initialize the iteration counter k = 1.

$$Cr_{3}^{\ k} = D;$$

$$Cr_{1}^{\ k} = Cr_{3}^{\ k} + \gamma * Cr_{3}^{\ k};$$

$$Cr_{2}^{\ k} = Cr_{3}^{\ k} + (Cr_{1}^{\ k} - Cr_{3}^{\ k})/2;$$

Step2: Determine the optimal generators' power outputs P_i , i = 1,...,N using the Hopfield neural network algorithm, by neglecting losses and setting the power demand as $Cr^{\ k} = Cr_2^{\ k}$;

Step3: Calculate the transmission losses L^k for the current iteration k using Eq. (23);

Step 4: if $Cr_1^k - Cr_3^k < \varepsilon$, stop otherwise go to step 5;

Step5: if $Cr_2^k - L^k < D$, update Cr_3 and Cr_2 for the next iteration as follows:

$$Cr_{3}^{k+l} = Cr_{2}^{k} Cr_{2}^{k+l} = Cr_{2}^{k} (Cr_{1}^{k} - Cr_{2}^{k})/2;$$

Replace k by k+1 and go to step 2;

Step 6: if $Cr_2^k - L^k > D$, update Cr_1 and Cr_2 for the next iteration as follows:

$$Cr_{2}^{k+1} = Cr_{2}^{k} Cr_{2}^{k-1} = Cr_{2}^{k} - (Cr_{2}^{k} - Cr_{3}^{k})/2;$$

Replace k by k+1 and go to step 2.

5. Results and Discussion

To demonstrate the performance of the Hopfield based ED solver, A 15-unit test system [16] is used, where the convergence criteria considered here is the unit generation constraints must be not violated. The system consists of 15-units where data is given in table1. For comparison the case of a load demand of 2650 MW is considered as in [16].

The total operating cost of the system is represented by the following polynomial,

$$F_{T} = \sum_{i=1}^{N} F_{i}(P_{i}) = \sum_{i=1}^{N} \left(a_{i} + b_{i}P_{i} + c_{i}P_{i}^{2} \right).$$
(24)

The polynomials coefficients are listed in table 1, along with generator minimum and maximum operating limits.

The loss coefficients matrix B_c , vector B_0 and constant B_{00} are taken from [16].

The sixth column of table 1 shows the optimal generators' power outputs when the transmission losses is neglected. Total production cost is \$ 32542.30. The problem was carried out on Pentium M 1.73 MHz using the presented Hopfield method with $x_{min} = -0.1$, $x_{max} = 0.1$ and $P_m = 0.0001$. The computation time was about 0.14 s.

The same test system was solved in [16], the total production cost is obtained as \$ 32549.8. It can be seen that the presented Hopfield approach could provide a better solution within a much shorter time.

The last column of table 1 shows the optimal generators' power outputs when the transmission losses is taken into account. The pre-specified tolerance was taken as 0.001. Total production cost is \$ 32880.42, and the transmission losses equal to 32.1138 MW. The computation time was about 0.51 s for 21 iterations.

P_i^{\min}	P_i^{max}	а	b	С	P_i	P_i
(MW)	(MW)	\$/hr	\$/MWhr	\$/MW ² hr	(MW)	(MW)
150	455	671.03	10.07	0.000299	455	455
150	455	574.54	10.22	0.000183	455	455
20	130	374.59	8.8	0.001126	130	130
20	130	374.59	8.8	0.001126	130	130
150	470	461.37	10.4	0.000205	317.8331	348.77
135	460	630.14	10.1	0.000301	460.	460.
135	465	548.2	9.87	0.000364	465	465
60	300	227.09	11.5	0.000338	60	60
25	162	173.72	11.21	0.000607	25	25
20	160	175.95	10.72	0.001203	20	20
20	80	186.86	11.21	0.003586	20	20
20	80	230.27	9.9	0.005513	57.1659	58.32
25	85	225.28	13.12	0.000371	25	25
15	55	309.03	12.12	0.001929	15	15
15	55	323.79	12.41	0.004447	15	15
		Transmission losses L (MW)			0	32.11
		Total production power generation (MW)			2650	2682.09
		Total proc	luction cost	$F_T(\$)$	32542.30	32880.42

Table 1. Input data of unit System and the computational results

6. Conclusion

We have developed a fast-computation solver for ED problems solution with transmission losses, the sover is a hybrid Dichotomy - Hopfield neural network method. The developed method overcomes the drawbacks of the conventional segmoidal function based Hopfield neural network. This is done by adopting a linear input/output transfer function, which resulted

in a superior Hopfield neural network as one calculation process is required. This led to a very short computing time and suitability for on-line usage. The proposed method is relatively simple, straightforward, efficient, and easy to apply and requires no training. It is so far: (i) the determination of the energy function weighting factors is not necessary, (ii) it is mutual coupling network and nonhierarchical structure. Its connective conductances and external input can be determined directly by employing system data.

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