Robust H_∞ Control of Switched Uncertain Time-varying Delay Discrete-time Systems via Dynamic Output Feedback

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Abstract

Robust H_∞ output feedback control problem for a class of switched linear discrete-time systems possessing norm-bounded uncertainty and time-varying delay is investigated. A switched dynamic output feedback controller is sought that can robust exponentially stabilize and reduce to a prescribed level the effect of disturbance input on the controlled output in the closed loop. A delay depen dent linear matrix inequality based sufficient condition for the problem to be solvable is derived via the switched Lyapunov functions. Controller design is performed by solving a set of linear matrix inequalities. An illustrative example along with simulation results is given to demonstrate the validity of these novel derivations.

1. Introduction

REAL world technological systems are known to exhibit time delays, due to long transmission lines or distributed physical parameters in the plant, as well as uncertainties, due measurement errors or noise and exogenous disturbance. Considerable efforts were devoted to the problem of robust H_∞ stability and stabilization of uncertain time-delay systems [1-6] due to the rapid development of and world-wide applications of communication networks.

For the considered class of systems the existing stability and stabilization criteria are often classified as delay independent [1-3] and delay dependent [4-6] criteria. In general, the delay-dependent stability and stabilization criteria are less conservative than delay-independent ones when the size of the time-delay is small. Recent research effort is focused more on delay-dependent stabilization. The main objective of the delay-dependent H_∞ control is to obtain a controller that allows a maximum delay size for a fixed H_∞ performance bound or achieves a minimum H_∞ performance bound for a fixed delay size. However, it is difficult to realize state feedback when perfect information of the state is not available. As also pointed out in [7], a static (or dynamical) output feedback controller is more practical to deal with uncertain systems. Therefore, the problem of H_∞ dynamical output feedback for uncertain systems with time-varying delays has been studied, e.g. see works [3 -7]. Switched systems, as an important class of hybrid systems, have drawn considerable attention in control, communication and computer communities for both theoretical development and applications. A number of stability analysis and controller design results for switched systems have appeared recently [8-19]. Among the three basic issues of switched systems [11, 12], the problem of stabilization under arbitrary switching plays an important role because it provides a high probability of pursuing other control goals in addition to stability.

It has been shown [10] a switched system is asymptotically stable under arbitrary switching if and only if all subsystems share a common Lyapunov function [10]. Multiple Lyapunov function based approach was used in [8]. Switching Lyapunov functions were employed to study stability and control synthesis for linear discrete-time switched systems in [9], [13-15], and were extended further to systems with uncertainties in [14]. In there, two equivalent LMI-based sufficient conditions for the problem to be solvable are presented using switched static state feedback and switched static output feedback controllers. In particular, works [16-17] investigated the robust H_∞ dynamic output feedback control problem for linear discrete-time switched systems that do not possess uncertainties and time-delays. Generally, there considerably fewer references on robust control of switched systems with time-delay in comparison with the literature for switched non-delay systems. Especially this is the case about delay-dependent stability and stabilization criteria.

Following the idea and the proving argument in [9], the robust H_∞ output feedback control problem for a class of linear discrete-time switched systems with norm-bounded uncertainty and time-varying delay is further investigated and improved results with ameliorated restrictions derived in this paper. A dynamic output feedback control that stabilizes the closed-loop switched system in robust exponential mode and reduces the effect of the disturbance on the controlled output to a prescribed level for all admissible uncertainties is derived. Switched Lyapunov function technique is used to find a delay–dependent linear matrix inequality (LMI) based sufficient condition for the problem to be solvable. The design of switched output feedback controllers is performed by solving a set of LMI and in conjunction with the rule of arbitrary switching [11, 12] albeit simple regular switching is preferable. The paper is further organized in the traditional standard.

2. Notation and Preliminaries

In the sequel, the Euclidean norm is used for vectors. W^T, W^{-1}, $\mu_{\max}(W)$, $\lambda_{\min}(W)$ and $\lambda_{\max}(W)$ denote, respectively, the transpose, the inverse, the maximum singular value, the minimum and the maximum eigenvalue of any square matrix W. The notation $W > (\geq, <, \leq)\ 0$ is used to denote a symmetric positive-definite (positive-semidefinite, negative-definite, negative-semidefinite, respectively) matrix, I is the identity matrix of appropriate dimension. Letter N denotes the set of nonnegative integers. The space $l_2[0,\infty)$ consists of square-summable infinite vector sequences over $[0,\infty)$. Symbol $*$ is used in some matrix expressions to indicate a symmetric structure.

The class of switched systems

$$\begin{aligned} x(k+1) &= (A_\sigma + \Delta A_\sigma)x(k) + (A_{d\sigma} + \Delta A_{d\sigma})x(k-d(k)) \\ &\quad + (B_{1\sigma} + \Delta B_{1\sigma})u(k) + H_{1\sigma}w(k), \\ y(k) &= (C_\sigma + \Delta C_\sigma)x(k) + (C_{d\sigma} + \Delta C_{d\sigma})x(k-d(k)) \\ &\quad + (B_{2\sigma} + \Delta B_{2\sigma})u(k) + H_{2\sigma}w(k), \\ z(k) &= D_\sigma x(k) + B_{3\sigma}u(k), \\ x(k) &= \phi(k), \quad k \in [-d_2, 0], \end{aligned} \tag{1}$$

is considered. In there: $k \in N$; $x(k) \in R^n$ is the plant state, $u(k) \in R^l$ is control input, $y(k) \in R^r$ is the measurement output, $z(k) \in R^p$ is the controlled output, and $w(k) \in R^q$ is the disturbance input assumed to belong to $l_2[0,\infty)$; the sequence $\sigma(k): N \to \overline{M} = \{1,2,\cdots,m\}$ represents a piecewise constant switching signal; $d(k)$ is a positive integer representing the time-varying delay in the system and satisfying

$$0 < d_1 \leq d(k) \leq d_2, \tag{2}$$

where d_1 and d_2 are known positive integers. Function $\phi(k)$ is a real-valued initial condition on $[-d_2, 0]$. For a $\sigma(k)$, $A_\sigma, A_{d\sigma}, B_{1\sigma}, B_{2\sigma}, B_{3\sigma}, C_\sigma, C_{d\sigma}, D_\sigma, H_{1\sigma}$ and $H_{2\sigma}$ are known real constant matrices $\Delta A_\sigma, \Delta A_{d\sigma}$, $\Delta B_{1\sigma}, \Delta B_{2\sigma}, \Delta C_\sigma$ and $\Delta C_{d\sigma}$ are unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form

$$\begin{bmatrix} \Delta A_\sigma & \Delta A_{d\sigma} & \Delta B_{1\sigma} \\ \Delta C_\sigma & \Delta C_{d\sigma} & \Delta B_{2\sigma} \end{bmatrix} = \begin{bmatrix} E_{x\sigma} \\ E_{y\sigma} \end{bmatrix} \Gamma_\sigma \begin{bmatrix} F_{x\sigma} & F_{dx\sigma} & F_{u\sigma} \end{bmatrix}, \tag{3}$$

where Γ_σ is an unknown time-varying matrix function satisfying $\mu_{\max}(\Gamma_\sigma) \leq 1$, $E_{x\sigma}, E_{y\sigma}$, $F_{x\sigma}, F_{dx\sigma}$ and $F_{u\sigma}$ are known real constant matrices.

2.1 Exponential Stability Analysis

In this sub-section, we consider the unforced disturbance-free system (1), i.e. when $u(k) \equiv 0$ and $w(k) \equiv 0$ hence

$$\begin{aligned} x(k+1) &= (A_\sigma + \Delta A_\sigma)x(k) + (A_{d\sigma} + \Delta A_{d\sigma})x(k-d(k)), \\ x(k) &= \phi(k), \quad k \in [-d_2, 0], \end{aligned} \tag{4}$$

Definition 1. The system (4) is said to be robust exponentially stable if there exist constant scalars $0 < a < 1$ and $b > 0$ such that

$$|x(k)|^2 \leq ba^k \sup_{-d_2 \leq i \leq 0} |\phi(i)|^2,$$

for all admissible uncertainties and arbitrary switching signal. The following proposition is used in proving Theorem 1.

Proposition 1 [6]**.** Let Φ, Ω, Ψ, Ξ and Γ be real matrices of appropriate dimensions satisfying $\Xi > 0$ and $\mu_{\max}(\Gamma) \leq 1$. Then, for any scalar $\varepsilon > 0$ satisfying $\Xi - \varepsilon\Phi\Phi^T > 0$, it holds that

$$\begin{aligned} &(\Omega + \Phi\Gamma\Psi)^T \Xi^{-1} (\Omega + \Phi\Gamma\Psi) \\ &\qquad \leq \Omega^T (\Xi - \varepsilon\Phi\Phi^T)^{-1} \Omega + \varepsilon^{-1}\Psi^T\Psi. \end{aligned}$$

Theorem 1. The system (4) is robust exponentially stable if there exist a set of scalars $\varepsilon_i > 0$, matrices $P_i > 0$ and matrix $Q > 0$ such that the following matrix inequalities hold

$$\begin{bmatrix} -P_i & * & * & * & * \\ 0 & -Q & * & * & * \\ A_i & A_{di} & \varepsilon_i^{-1}E_{xi}E_{xi}^T - P_j^{-1} & * & * \\ F_{xi} & F_{dxi} & 0 & -\varepsilon_i^{-1}I & * \\ \tilde{d}I & 0 & 0 & 0 & -\tilde{d}Q^{-1} \end{bmatrix} < 0, \tag{5}$$

for $\forall i, j \in \overline{M}$, where $\tilde{d} = d_2 - d_1 + 1$.

2.2 The H∞ Performance

In this sub-section, we investigate the H_∞ performance of the unforced system (1), i.e.

$$\begin{aligned} x(k+1) &= (A_\sigma + \Delta A_\sigma)x(k) + (A_{d\sigma} + \Delta A_{d\sigma})x(k-d(k)) \\ &\quad + H_{1\sigma}w(k), \\ z(k) &= D_\sigma x(k), \\ x(k) &= \phi(k), \quad k \in [-d_2, 0]. \end{aligned} \tag{6}$$

Definition 2. Given $\gamma > 0$. The system (6) is said to be robust exponentially stable with an H_∞-norm bound γ , if the following conditions are satisfied:

(i) The disturbance-free system (24) is robustly exponential stable;

(ii) Under the zero initial condition, the controlled output z_k satisfies

$$\sum_{k=0}^{\infty} \|z(k)\|^2 < \gamma^2 \sum_{k=0}^{\infty} \|w(k)\|^2,$$

for any nonzero $w(k) \in l_2$ for all admissible uncertainties and arbitrary switching signal.

Theorem 2. Given $\gamma > 0$. The system (6) is robust exponentially stable with an H_∞-norm bound γ if there exists a set of scalars $\varepsilon_i > 0$, matrices $P_i > 0$ and matrix $Q > 0$ such that the following matrix inequalities

$$\begin{bmatrix} -P_i & * & * & * & * & * & * \\ 0 & -Q & * & * & * & * & * \\ 0 & 0 & -\gamma I & * & * & * & * \\ A_i & A_{di} & H_{1i} & \varepsilon_i^{-1}E_{xi}E_{xi}^T - P_j^{-1} & * & * & * \\ F_{xi} & F_{dxi} & 0 & 0 & -\varepsilon_i^{-1}I & * & * \\ D_i & 0 & 0 & 0 & 0 & -\gamma I & * \\ \tilde{d}I & 0 & 0 & 0 & 0 & 0 & -\tilde{d}Q^{-1} \end{bmatrix} < 0, \tag{7}$$

hold for $\forall i, j \in \overline{M}$. It presents the sufficient conditions for robust exponentially stability with a H_∞-norm bound γ of (7).

Remark 2. Inequality (7) is not a LMI. Thus via pre- and post-multiplying (7) by $diag\{P_i^{-1}, Q^{-1}, I, I, I, I, I\}$, it can be converted into the following LMI

$$\begin{bmatrix} -P_i^{-1} & * & * & * & * & * & * \\ 0 & -Q^{-1} & * & * & * & * & * \\ 0 & 0 & -\gamma I & * & * & * & * \\ A_iP_i^{-1} & A_{di}Q^{-1} & H_{1i} & \varepsilon_i^{-1}E_{xi}E_{xi}^T - P_j^{-1} & * & * & * \\ F_{xi}P_i^{-1} & F_{dxi}Q^{-1} & 0 & 0 & -\varepsilon_i^{-1}I & * & * \\ D_iP_i^{-1} & 0 & 0 & 0 & 0 & -\gamma I & * \\ \tilde{d}P_i^{-1} & 0 & 0 & 0 & 0 & 0 & -\tilde{d}Q^{-1} \end{bmatrix} < 0,$$

for $\forall i, j \in \overline{M}$, which can be easily solved by the respective toolbox of the MATLAB.

3. A Solution to Dynamic Output Feedback

This section presents the main results on robust H_∞ control for the system (1) via switched dynamic output feedback. The switched dynamic output feedback (DOF) controller is constituted of a family of dynamic output feedbacks such that each of which is designed for a particular subsystem.

Considering the following switched full-order dynamic output feedback controller

$$\begin{aligned} x_c(k+1) &= A_{K\sigma}x_c(k) + B_{K\sigma}y(k), \\ u(k) &= C_{K\sigma}x_c(k), \end{aligned} \tag{8}$$

where $x_c(k) \in R^n$ is the controller state vector and $A_{K\sigma}, B_{K\sigma}, C_{K\sigma}$ are system matrices to be determined later. Application of the control law (8) to the system (1) yields

$$\begin{aligned} \xi(k+1) &= (A_{c\sigma} + \Delta A_{c\sigma})\xi(k) \\ &\quad + (A_{cd\sigma} + \Delta A_{cd\sigma})H\xi(k-d(k)) + H_{c\sigma}w(k), \\ z(k) &= C_{c\sigma}\xi(k), \end{aligned} \tag{9}$$

where

$$\xi(k) = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}, \; A_{cd\sigma} = \begin{bmatrix} A_{d\sigma} \\ B_{K\sigma}C_{d\sigma} \end{bmatrix},$$

$$A_{c\sigma} = \begin{bmatrix} A_\sigma & B_{1\sigma}C_{K\sigma} \\ B_{K\sigma}C_\sigma & A_{K\sigma} + B_{K\sigma}B_{2\sigma}C_{K\sigma} \end{bmatrix},$$

$$\Delta A_{c\sigma} = \begin{bmatrix} \Delta A_\sigma & \Delta B_{1\sigma}C_{K\sigma} \\ B_{K\sigma}\Delta C_\sigma & B_{K\sigma}\Delta B_{2\sigma}C_{K\sigma} \end{bmatrix},$$

$$\Delta A_{cd\sigma} = \begin{bmatrix} \Delta A_{d\sigma} \\ B_{K\sigma}\Delta C_{d\sigma} \end{bmatrix}, \; H_{c\sigma} = \begin{bmatrix} H_{1\sigma} \\ B_{K\sigma}H_{2\sigma} \end{bmatrix},$$

$$C_{c\sigma} = [D_\sigma \quad B_{3\sigma}C_{K\sigma}], \quad H = [I \quad 0].$$

Definition 3. Given $\gamma > 0$. The system (1) is said to be robust exponentially stabilizable with an H_∞-norm bound γ, if there exists a switching dynamical output feedback controller (8) such that the closed-loop system (9) of the plant (1) is robust exponentially stable with an H_∞-norm bound γ.

3.1. Exponentially Stabilizing Switched DOF Control

First, we investigate the robust exponential stability for the disturbance-free system (9) and give a novel result.

Proposition 2. The disturbance-free system (9) is robust exponentially stable if there exist a set of scalars $\varepsilon_i > 0$, matrices $P_i > 0$ and $Q > 0$ such that the following matrix inequalities

$$\begin{bmatrix} -P_i & * & * & * & * \\ 0 & -Q & * & * & * \\ A_{ci} & A_{cdi} & \varepsilon_i^{-1}\tilde{E}_i\tilde{E}_i^T - P_j^{-1} & * & * \\ \tilde{F}_i & \tilde{F}_{di} & 0 & -\varepsilon_i^{-1}I & * \\ \tilde{d}H & 0 & 0 & 0 & -\tilde{d}Q^{-1} \end{bmatrix} < 0, \tag{10}$$

where

$$\tilde{F}_i = [F_{xi} \quad F_{ui}C_{Ki}], \quad \tilde{F}_{di} = F_{dxi},$$

$$\tilde{E}_i^T = [E_{xi}^T \quad E_{yi}^T B_{Ki}^T], \quad \tilde{d} = d_2 - d_1 + 1.$$

Theorem 3. The disturbance-free system (9) is robust exponentially stabilizable if there exist matrices $X > 0$, $Y_i > 0$ and Π_i, Φ_i, Ψ_i such that the following matrix inequalities

$$\begin{bmatrix} -J_{1i} & * & * & * \\ L_{1i} & -J_{2j} & * & * \\ 0 & L_{3i}^T & -J_{3i} & * \\ L_{2i} & 0 & 0 & -J_{4i} \end{bmatrix} < 0, \quad \forall i, j \in \overline{M}, \tag{11}$$

hold for some given matrix $Q > 0$ and a set of scalars $\varepsilon_i > 0$, where

$$J_{1i} = \begin{bmatrix} Y_i & * & * \\ I & X & * \\ 0 & 0 & Q \end{bmatrix}, \; J_{2j} = \begin{bmatrix} Y_j & * \\ I & X \end{bmatrix},$$

$$J_{3i} = \varepsilon_i I, \; J_{4i} = \begin{bmatrix} \varepsilon_i^{-1}I & 0 \\ 0 & \tilde{d}Q^{-1} \end{bmatrix},$$

$$L_{1i} = \begin{bmatrix} A_iY_i + B_{1i}\Psi_i & A_i & A_{di} \\ \Pi_i & XA_i + \Phi_iC_i & XA_{di} + \Phi_iC_{di} \end{bmatrix},$$

$$L_{2i} = \begin{bmatrix} F_{xi}Y_i + F_{ui}\Psi_i & F_{xi} & F_{dxi} \\ \tilde{d}Y_i & \tilde{d}I & 0 \end{bmatrix},$$

$$L_{3i} = \begin{bmatrix} E_{xi} \\ XE_{xi} + \Phi_iE_{yi} \end{bmatrix}.$$

Then a desired switched dynamic output feedback controller (8) is parameterized as follows:

$$A_{Ki} = S^{-1}(\Pi_i - XA_iY_i - \Phi_iC_iY_i - XB_{1i}\Psi_i - \Phi_iB_{2i}\Psi_i)W_i^{-T}, \tag{12}$$

$$B_{Ki} = S^{-1}\Phi_i, \; C_{Ki} = \Psi_iW_i^{-T}, \tag{13}$$

where S and W_i are any non-singular matrices satisfying

$$SW_i^T = I - XY_i. \tag{14}$$

3.2. H_∞ Performance Switched DOF Control

In this sub-section, the focus is on the robust exponential stabilization with an H_∞-norm bound γ for the system (1). The next Proposition 3 plays a key role in proving the main result.

Proposition 3. Given $\gamma > 0$. The system (9) is robustly exponentially stable with an H_∞-norm bound γ if there exist a set of scalars $\varepsilon_i > 0$, matrices $P_i > 0$ and $Q > 0$ such that the following matrix inequalities

$$\begin{bmatrix} -P_i & * & * & * & * & * & * \\ 0 & -Q & * & * & * & * & * \\ 0 & 0 & -\gamma I & * & * & * & * \\ A_{ci} & A_{cdi} & H_{ci} & \varepsilon_i^{-1}\tilde{E}_i\tilde{E}_i^T - P_j^{-1} & * & * & * \\ \tilde{F}_i & \tilde{F}_{di} & 0 & 0 & -\varepsilon_i^{-1}I & * & * \\ C_{ci} & 0 & 0 & 0 & 0 & -\gamma I & * \\ \tilde{d}H & 0 & 0 & 0 & 0 & 0 & -\tilde{d}Q^{-1} \end{bmatrix} < 0, \tag{15}$$

hold for $\forall i, j \in \overline{M}$, where $A_{ci}, A_{cdi}, H_{ci}, C_{ci}, H$ and $\tilde{E}_i, \tilde{F}_i, \tilde{F}_{di}, \tilde{d}$ are given as in Proposition 2.

Via considering the H_∞ performance, Theorem 4 below summarizes the main result.

Theorem 4. Given $\gamma > 0$. The system (9) is robust exponentially stabilizable with an H_∞-norm bound γ if there exist matrices $X > 0$, $Y_i > 0$ and $\Pi_i, \Phi_i, \Psi_i, i \in \overline{M}$ such that the following matrix inequalities

$$\begin{bmatrix} -\Theta_{1i} & * & * & * & * \\ 0 & -\Theta_{2i} & * & * & * \\ \Omega_{1i} & \Omega_{3i} & -\Theta_{3j} & * & * \\ 0 & 0 & \Omega_{4i}^T & -\Theta_{4i} & * \\ \Omega_{2i} & 0 & 0 & 0 & -\Theta_{5i} \end{bmatrix} < 0, \tag{16}$$

where

$$\Theta_{1i} = \begin{bmatrix} Y_i & * & * \\ I & X & * \\ 0 & 0 & Q \end{bmatrix}, \Theta_{2i} = \gamma I, \ \Theta_{3j} = \begin{bmatrix} Y_j & * \\ I & X \end{bmatrix},$$

$$\Theta_{4i} = \varepsilon_i I, \ \Theta_{5i} = diag(\varepsilon_i^{-1}I, \gamma I, \tilde{d}Q^{-1}),$$

$$\Omega_{1i} = \begin{bmatrix} A_iY_i + B_{1i}\Psi_i & A_i & A_{di} \\ \Pi_i & XA_i + \Phi_iC_i & XA_{di} + \Phi_iC_{di} \end{bmatrix},$$

$$\Omega_{2i} = \begin{bmatrix} F_{xi}Y_i + F_{ui}\Psi_i & F_{xi} & F_{dxi} \\ D_iY_i + B_{3i}\Psi_i & D_i & 0 \\ \tilde{d}Y_i & \tilde{d}I & 0 \end{bmatrix},$$

$$\Omega_{3i} = \begin{bmatrix} H_{1i} \\ XH_{1i} + \Phi_iH_{2i} \end{bmatrix}, \ \Omega_{4i} = \begin{bmatrix} E_{xi} \\ XE_{xi} + \Phi_iE_{yi} \end{bmatrix}.$$

hold for $\forall i, j \in \overline{M}$, some given matrix $Q > 0$ and a set of scalars $\varepsilon_i > 0$. Furthermore, then a desired switched dynamic output feedback controller is given in the terms of (8) along with its parameters defined by (12-14).

Remark. It is easy to see that (16) is not a LMI with respect to the parameters $Q > 0$ and $\varepsilon_i > 0, i \in \overline{M}$, since ε_i and Q appear non-linearly in (16). Thus parameters ε_i and Q must be fixed such that (16) is an LMI in X, Y_i, Π_i, Φ_i and Ψ_i.

4. An Illustrative Example

In this section, an illustrative example along with numerical and simulation results are presented to demonstrate the effectiveness and applicability of the novel theory in the preceding section with the simple regular switching law. Consider the system (1) with $m = 2$, and with its parameters defined as follows:

$$A_1 = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 1.2 \end{bmatrix}, A_2 = \begin{bmatrix} 1.2 & 0.3 \\ 0.4 & 0.8 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, B_{11} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B_{12} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix},$$

$$H_{11} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, H_{12} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0.5 \\ -1 & 1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.5 & -1 \\ 1 & 1 \end{bmatrix}, C_{d1} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}, C_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 1 & 2 \\ 0 & 1.5 \end{bmatrix}, B_{22} = \begin{bmatrix} 0 & 1.5 \\ 1 & 2 \end{bmatrix}, H_{21} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, H_{22} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, E_2 = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, B_{31} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{32} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$E_{x1} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, E_{x2} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, E_{y1} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, E_{y2} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

$$F_{x1} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, F_{x2} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, F_{dx1} = \begin{bmatrix} 0 & 0.1 \end{bmatrix},$$

$$F_{dx2} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, F_{u1} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, F_{u2} = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}.$$

The time-varying delay is assumed to satisfy (2) with values $d_1 = 1$ and $d_2 = 3$. In this example, the admissible disturbance attenuation level is assumed as $\gamma = 1.0007$. Next, the parameters $\varepsilon_1 = 3$, $\varepsilon_2 = 3$, $Q = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ are chosen. The use of Matlab LMI Control Toolbox for (50) yields:

$$X = \begin{bmatrix} 208.5882 & 158.3043 \\ 158.3043 & 302.3206 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 0.1250 & 0.0108 \\ 0.0108 & 0.1549 \end{bmatrix}, Y_2 = \begin{bmatrix} 0.1140 & 0.0031 \\ 0.0031 & 0.1596 \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix} -226.1820 & -91.8668 \\ -309.1995 & -165.1335 \end{bmatrix}, \Phi_2 = \begin{bmatrix} -139.3667 & -304.2529 \\ -65.2926 & -355.1541 \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} -0.0771 & -0.0696 \\ 0.0982 & -0.0393 \end{bmatrix}, \Psi_2 = \begin{bmatrix} 0.1750 & -0.0234 \\ -0.1279 & -0.0523 \end{bmatrix},$$

$$\Pi_1 = \begin{bmatrix} 0.0488 & 0.0122 \\ -0.1708 & 0.1032 \end{bmatrix}, \Pi_2 = \begin{bmatrix} -0.2821 & 0.0249 \\ -0.4996 & 0.0256 \end{bmatrix}.$$

In order to make use of Theorem 4 for a switched dynamic output feedback controller, we choose $S = \begin{bmatrix} 15 & 8 \\ 6 & 24 \end{bmatrix}$, and

$$W_1 = \begin{bmatrix} -1.4691 & -0.5933 \\ -0.8406 & -1.7706 \end{bmatrix}, W_2 = \begin{bmatrix} -1.3032 & -0.4652 \\ -0.7691 & -1.7969 \end{bmatrix}.$$

such that the equalities (43) hold. Therefore, by Theorem 4, a desired switched dynamic output feedback H_∞ controller is found in the form of (29) with parameters as follows:

$$A_{K1} = \begin{bmatrix} -1.9361 & 1.0303 \\ -2.5153 & 1.2831 \end{bmatrix}, B_{K1} = \begin{bmatrix} -9.4704 & -2.8325 \\ -10.5157 & -6.1724 \end{bmatrix},$$

$$C_{K1} = \begin{bmatrix} 0.0453 & 0.0178 \\ -0.0938 & 0.0667 \end{bmatrix}, A_{K2} = \begin{bmatrix} 0.2598 & 0.8002 \\ -0.5391 & 0.0785 \end{bmatrix},$$

$$B_{K2} = \begin{bmatrix} -9.0463 & -14.2976 \\ -0.4589 & -11.2237 \end{bmatrix}, C_{K2} = \begin{bmatrix} -0.1640 & 0.0832 \\ 0.1036 & -0.0152 \end{bmatrix}.$$

For this control design, the simulation experiments were carried out for the initial conditions $\phi(k)=[-6,1]^T$ with $k\in[-4,0]$ and $x_c(0)=[-7,3]^T$. The main simulation results are shown in Figures 1, 2, and 3.

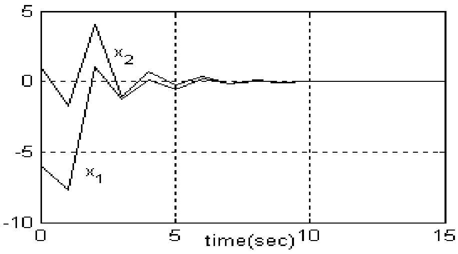

Fig. 1. The response of plant state vector x.

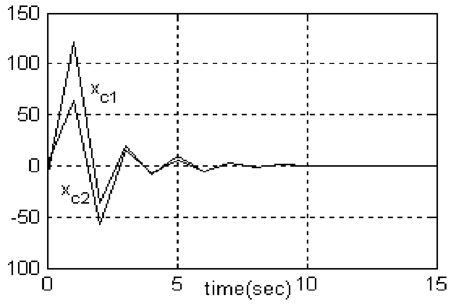

Fig. 2. The response of controller state vector x_c

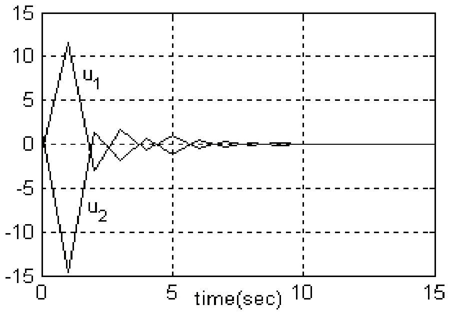

Fig. 3. The feedback control vector u generated.

5. Conclusion

A design for switched dynamical output feedback controller that guarantees the resulting closed-loop switched system is robust exponentially stable with a prescribed H_∞-norm bound is proposed. The sufficient condition for the problem to be solvable is delay-dependent, and it is given in terms of LMI. Moreover, the existence of a family of dynamical output feedback control laws is given in terms of the solvability of linear matrix inequalities. In order to account for all possible switching from each subsystem to another, the proposed conditions have to be satisfied for all pairs (i, j) under the rule of arbitrary switching.

7. References

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