MODEL FOR CALCULATING THE CONDUCTIVITY TO GROUND OF POWER CABLES WITH METALLIC SHIELD PLACED ON A TWO LAYER SOIL

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Abstract-The cables with conductive to ground metallic shield are excellent as grounding grids. These types of cables are presented by a chain of π four-extremity where the per unit length impedance $\underline{z}$ and the admittance to ground $\underline{y}$ of the cables shields figure. For this type of cable "classical IPO" placed on a homogenous soil with a constant specific electrical resistance of the ground ρ, the well known relations are valid, mentioned in text below. However, if the cable is placed on a multy layer environment, the discontinuity of the environment should be taken into consideration, when calculating the admittance to ground. In this case, $\underline{y}$ is calculated by the method of medium potentials, used on a model of two-layer environment, where an "infinite" number of cables images is calculated because of the inhomogeneousness of the soil. Depending on the ground upper layer height for a two-layer environment with different specific electrical resistances and for different lenghts of the cable, the obtained results are illustrated and analysed.

Key words- metallic shield, conductivity to ground, resistance to ground, two-layer soil, medium potentials, grounding grids.

Introduction

After few weeks or months from the placing of the cables with metallic shield, moisture penetrates gradually through the layers of the impregnated jute and paper, indirectly restoring a conductive connection between the lead shield and the soil and the cable becomes an excellent grounding grid. These grounding grids (cables with conductive shield) have a small reduction factor and they loosen the grounding grids of the substations where the cables are connected and ease the realisation of the conditions of security, and at the same time they reduce the exit potential. We obtain the calculation of the effects of the shield of the cable as a grounding grid, from the equivalent scheme of a cable with a length l placed between two substations 1 and 2 presented by distributed parameters as a chain od π four-extremities. Each of the four-extremities consists of a per unit length impedance $\underline{z}$ calculated according to the Carson's relations [1], and two admittance's to ground per unit length $\underline{y}$. These admittances are nearly equal to the conductivity of a horizontal ground grid in shape of a tape with an equivalent diameter d (a diameter of the cable's shield) placed on a depth H with length l and they are calculated according to the relation:

$$\underline{y} \approx g = \frac{\pi}{\rho} \cdot \frac{10^3}{\ln \frac{l}{\sqrt{H \cdot d}}} \; (\mathrm{S/km}) \qquad (1)$$

By applying the common equation for lines with distributed parameters:

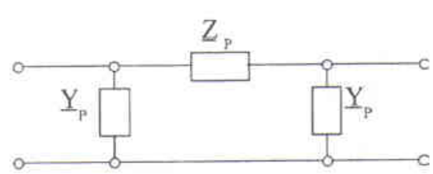
$$\underline{U}_1 = \underline{U}_2 \cdot \mathrm{ch}\underline{\gamma} \cdot l + \underline{I}_2 \cdot \underline{Z} \cdot \mathrm{sh}\underline{\gamma} \cdot l$$
$$\underline{I}_1 = \frac{\underline{U}_2}{\underline{Z}} \cdot \mathrm{sh}\underline{\gamma} \cdot l + \underline{I}_2 \cdot \mathrm{ch}\underline{\gamma} \cdot l \qquad (2)$$

where the distribution constant $\underline{\gamma}$ and the characteristic impendance of the shield of cable $\underline{Z}$ are calculated from the relations:

$$\underline{\gamma} = \sqrt{\underline{z}\underline{y}} \qquad \underline{Z} = \sqrt{\frac{\underline{z}}{\underline{y}}} \qquad (3)$$

the cable can be closely examined as a π four-extremity with a serial-connected impendance and two placed admittances to ground with parameters:

$$\underline{Z}_p = \underline{Z}\mathrm{sh}\underline{\gamma} \cdot l \qquad \underline{Y}_p = \frac{\mathrm{ch}\underline{\gamma} \cdot l - 1}{\underline{Z} \cdot \mathrm{sh}\underline{\gamma} \cdot l} \qquad (4)$$



Picture 1. A cabel with metallic conductive shield presented as a π four-extremity

By applying the upper relations [1], all the further parameters that are of interest to this problem, can be calculated. However, these calculations are valid only for a homogenous soil. In case of the soil not being homogenous, during the calculation of g, the discontinuity of the soil has to be taken into consideration. For this aim, the method of medium potentials applied on a two-layer soil can be used to calculate conductivity to ground, because a two-layer model with no major faults can generally present the multy-layer environments.

A model for calculation of conductivity to ground

Because the cables with metallic shield are infact grounding grids, with a calculation of the resistance to ground R of these grounding grids, the conductivity G can be calculated, i.e conductivity to ground per unit length g through:

$$G = \frac{1}{R} \qquad g = \frac{G}{l} \tag{5}$$

Conductivity to ground for a homogenous environment can be calculated this way, as well, and that's been done below for comparison, and it can be noticed that the results according to the relation (1) are not much different from the results obtained by the method of medium potentials. To obtain a bigger punctuality of the results, although according to punctuality of 10^{-4} (a larger number of images are taken), the cable is divided to as larger number of elements (electrodes 10, 20, 30, 40) in all the calculations, with 20 meters length. If it is a two-layer soil, each element (electrode) of the grounding grids that are closely examined is generally divided to elements, so that each element with its own length lies in one of the layers. It is not necessary to do this with the cables because the cable is placed either in the upper layer with depth h and specific resistance of the soil ρ_1 or in the bottom one with ρ_2 and an infinite depth.

The method of medium potentials which is especially convenient for calculation of grounding grids consisted of straight electrodes with infinitely small section is based on the following two hypotheses **[5]**:

1. It is supposed that the surface density of the current length wise the electrodes of the grounding grids is a constant.
2. The potential φ_{kj} of the electrode k, created by the current field of the element j with current I_j, is equal to the medium value of the potentials in the separate points length wise the element k.

For an isolated grounding grid placed on a two-layer soil, the mutual resistances of the electrodes are calculated approximately with the hypothesis that the medium potential of the second electrode is equal to the arithmetic environment of potentials in several representative points on it, calculated according to the well known Cejtlin's formula with current of 1 A in the first electrode. To achieve a satisfactory exactness, the electrodes should be divided to the largest possible number of segments. That way, the number of the elements of the grounding grids multiplies. However, this number of segments is not fixed and it depends on the mutual distance of the electrodes of the grounding grid. Because of the effect of equalising of the potential in distance, the potential in a more distant electrode can be calculated as a medium value of the potential in a very small number of points of the same. With help of the algorhytm, the optimum number of points on each electrode of the grounding grid can be chosen, so that the procedure of calculating reduces, it doesn't vanish in the exactness of the obtained results **[3]**. This way, the mutual resistance between the point M and the straight electrode k in an infinitely homogenous soil is determined from the phrase:

$$r_{kM} = \frac{\rho}{4\pi l_k} \ln\left(\frac{G_{kM} + l_k}{G_{kM} - l_k}\right) \tag{6}$$

$$G_{kM} = R_k(P_k, M) + R(Q_k, M) \tag{7}$$

where P_k, Q_k are starting and ending points of the element k, and the distance is indicated by R_k. If the point M lies on the element k the self resistance is:

$$r_{kM} = \frac{\rho}{4\pi l_k} \ln\left(\frac{2l_k}{d_k}\right)^2 \tag{8}$$

where l_k, d_k are the length i.e. the diameter of the electrode k.

The inhomogenousness of the environment is taken into account with help of the inhomogenousness factor:

$$k_g = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \tag{9}$$

If the element k is in a two layer-soil, then, according the Nehman **[2]**, the potential in the point M can be calculated by the superposition of the potentials of appropriate infinite image lines of elements, and the following relations are valid:

If the element k and the point M lie in the upper layer, for the mutual resistance is valid:

$$r_{kM} = \frac{\rho_1}{4\pi l_k} \cdot \left\{A_{kM}(0) + B_{kM}(0) + \sum_{s=1}^{\infty} k_g^s \left[A_{kM}(s) + B_{kM}(s) + C_{kM}(s) + D_{kM}(s)\right]\right\} \tag{10}$$

If the element is in the bottom layer, and the point is in the upper one, is valid:

$$r_{kM} = \frac{\rho_1}{4\pi l_k}(1 + k_g)\sum_{s=0}^{\infty} k_g^s \left[A_{kM}(s) + B_{kM}(s)\right] \tag{11}$$

and in the opposite case

$$r_{kM} = \frac{\rho_1}{4\pi l_k}(1 + k_g)\sum_{s=0}^{\infty} k_g^s \left[B_{kM}(s) + D_{kM}(s)\right] \tag{12}$$

If the element and the point are in the bottom layer:

$$r_{kM} = \frac{\rho_2}{4\pi d_k}\left[A_{kM}(0) - k_g \cdot C_{kM}(1) + (1 - k_g^2)\sum_{s=0}^{\infty} k_g^s \cdot B_{kM}(s)\right] \tag{13}$$

Because the cable is a horizontal grounding grid, the relations (10) and (13) are valid, while the rest of the are mentioned in case of calculating an arbitrary grounding grid with elements in both the layers. Then the parameters $A_{kM}, B_{kM}, C_{kM}, D_{kM}$ are LN-functions in the phrase (6) for the image lines A, B, C, D of the element k. The co-ordinates x,y of the ending points of the images, are hatched with the same ones from the element k, while the z-coordinates are determined from (14), where the z -coordinates of the appropriate ending (starting and ending) points of the element k are maked by z_k. The first associate of the line A which corresponds to s=0 is the element itself and B(0) is its image:

$$z_{Ak} = z_k + 2sh \qquad z_{Ck} = -z_k + 2sh \qquad (14)$$
$$z_{Bk} = -z_k - 2sh \qquad z_{Dk} = z_k - 2sh$$

The self resistance of the elements of grounding grid can be determined as a mutual resistance of the element alone with itself, by applying the relations (10), i.e (13). The model for calculating the ground grid in a homogenous soil can be obtained by the general two-layer model, if we suppose that the first layer height is infinitely big. Then all the electrodes of the grounding grid will be in the upper layer and the mutual resistances will be calculated according to (10). The influence of the images for $s \geq 1$ will be omitted, because these images according to (14) are infinitely distant, so we have:

$$r_{kM} = \frac{\rho}{4\pi l_k}\{A_{kM}(0) + B_{kM}(0)\} \qquad (15)$$

The grounding grid voltage and the currents taken away from a single electrode are connected by the relations:

$$[1] \cdot U = [r] \cdot [I] \qquad (16)$$

$[r]$ - a square matrix of the self and mutual resistances of the grounding grid electrodes, taking into account the infinite image number of the grounding grid

$[1], [I]$ - a single column vector and column vector of taking away to ground currents from the electrodes appropriately.

The taking away current to ground is

$$I_z = [1]^T \cdot [I] \qquad (17)$$

The resistance to ground of the grounding grid is obtained from:

$$R = \frac{U}{I_z} = \left\{[1]^T \cdot [r]^{-1} \cdot [1]\right\}^{-1} \qquad (18)$$

and further on, the conductivity to ground per unit length g is determined from the relation (5).

On the basis of the exposed method, a computer program has been made for calculating the arbitrary grounding grids consisted of straight elements used when calculating conductivity to ground of this type of cables. Because of the limited frames of this article, a part of the analysis is presented on a cable IPO13 $3 \times 150(mm^2)$ with $d = 46(mm)$ placed in height $H = 0.7(m)$ for one and two layer soil with different specific soil resistances, for different lengths of the cable and different upper layer heights of the soil. For the layer height $h = 0.5(m)$ the cable is in the bottom layer, so the relation (13) is valid, and in the rest of the cases it is in the upper layer and relation (10) is valid.

Analysis results

Table 1. $g = f(l)$ for different $\rho=100, \rho=30, \rho=300(\Omega m)$ of the homogenous soil

l(m)	200	400	600	800
100 A	4.4776	4.0750	3.8714	3.7389
100 B	4.5754	4.1569	3.9454	3.8080
100 C	4.5929	4.1726	3.9988	3.8213
30 A	14.925	13.584	12.905	12.463
30 B	15.251	13.856	13.152	12.693
30 C	15.301	13.909	13.199	12.738
300 A	1.4925	1.3584	1.2905	1.2463
300 B	1.5251	1.3856	1.3152	1.2693
300 C	1.5301	1.3909	1.3199	1.2738

A- g calculated according to the formula

B- g calculated by the method (the cable is one element)

C- g calculated by the method (divided to elements with 20 meters length)

By increasing the cable length, and the specific soil resistance, as well, generally, the conductivity reduces. It can be noticed that the method (which is more exact when the number of segments with smaller cable lengths is bigger) gives higher results from the relation for 2,1%-2,5% and this percentage declines with the length increasing. Theoretically, the cable can be divided to infinite number of parts and the comparison between the results of second and third case won't (increase the exactness too much). According to the exposed method the relation for a homogenous soil should be multiplied with the factor c, which is equal to any, ρ it can be seen from Table 2.

Table 2. Correction factor according to the exposed method

l	200	400	600	800
c	1.0251	1.0234	1.0223	1.0216

The case of a two-layer soil is closely examined for most frequent upper layer heights, as well as different combinations of specific soil resistances. The tabled results can be used as a base during working with this kind of cables. By linear approximation, we can obtain the value of conductivity to ground and of other cable lengths and upper layer heights for the presented specific soil resistances. The difference in the results in respect of one layer soil shows the necessity of regarding the inhomogenousness of the soil. From the tables 3-8 it can be noticed that by reducing the upper layer height h, the value of the conductivity to ground g for any combination of specific soil resistances, approaches the value it would have when the soil would be homogenous with a specific resistance ρ_2. This is already noticeable for $h = 0.5(m)$, although theoretically it should be $h = 0(m)$ for gaining equal values. Vice versa, by increasing the upper layer height h, the value of g for any combination of specific soil resistances, approaches the value it would have when the soil would be homogenous with a specific resistance ρ_1. Theoretically, it should be $h = \infty$, so that equal values would be obtained.

Table 3. $g = f(l,h)$ for $\rho_1 = 300(\Omega m), \rho_2 = 100(\Omega m)$

l(m)	200	400	600	800
h=0.5	4.7959	4.3384	4.1131	3.9684
h=0.7	3.9193	3.6016	3.4443	3.3476
h=1.0	2.8647	2.6908	2.6017	2.5451
h=2.0	2.4747	2.3427	2.2744	2.2316
h=3.0	2.3172	2.1997	2.1389	2.1008

Table 4. $g = f(l,h)$ for $\rho_1 = 100(\Omega m), \rho_2 = 300(\Omega m)$

l(m)	200	400	600	800
h=0.5	1.4552	1.3357	1.2771	1.2394
h=0.7	1.9616	1.7431	1.6374	1.5705
h=1.0	2.1642	1.9020	1.7771	1.6986
h=2.0	2.4143	2.0956	1.9460	1.8526
h=3.0	2.5701	2.2156	2.0410	1.9471

Table 5. $g = f(l,h)$ for $\rho_1 = 30(\Omega m), \rho_2 = 100(\Omega m)$

l(m)	200	400	600	800
h=0.5	4.3686	4.0133	3.8404	3.7296
h=0.7	6.0461	5.3574	5.0254	4.8170
h=1.0	6.6978	5.8657	5.4707	5.2249
h=2.0	7.5347	6.5095	6.0301	5.7342
h=3.0	8.0670	6.9155	6.3807	6.0521

Table 6. $g = f(l,h)$ for $\rho_1 = 100(\Omega m), \rho_2 = 30(\Omega m)$

l(m)	200	400	600	800
h=0.5	16.012	14.481	13.727	13.244
h=0.7	12.927	11.915	11.369	11.046
h=1.0	9.0060	8.5000	8.2178	8.0471
h=2.0	7.6619	7.2896	7.0801	6.9524
h=3.0	7.1585	6 8293	6.6440	6.5310

Table 7. $g = f(l,h)$ for $\rho_1 = 30(\Omega m), \rho_2 = 300(\Omega m)$

l(m)	200	400	600	800
h=0.5	1.6123	1.5098	1.4637	1.4349
h=0.7	2.7528	2.3535	2.1710	2.0599
h=1.0	3.1175	2.6205	2.3976	2.2635
h=2.0	3.7990	3.1088	2.8067	2.6277
h=3.0	4.2967	3.4601	3.0982	2.8852

Table 8. $g = f(l,h)$ for $\rho_1 = 300(\Omega m), \rho_2 = 30(\Omega m)$

l(m)	200	400	600	800
h=0.5	16.004	14.491	13.744	13.262
h=0.7	12.198	11.314	10.851	10.520
h=1.0	4.3763	4.2581	4.1912	4.1417
h=2.0	3.2654	3.1927	3.1528	3.1237
h=3.0	2.9129	2.8501	2.8157	2.7913

Conclusion

1. The model enables calculation of electrical values of an arbitrary grounding grid type consisted of straight elements in one layer and two layer environment.
2. Such calculated conductivity to ground of cables with metallic shield is much more punctual, and with combination with up-to-date developed computer technology, the calculation time is minimal (less than a second).
3. The results show that it is necessary to respect the two layerness of the soil.
4. In combination with the Maxwel's equations, more cables placed in the same ditch can be closely examined, and by taking into consideration their mutual influence.

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