AS A GENERATION FUNCTION: BOUNDARY VALUE OF A *pn*-JUNCTION

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ABSTRACT

A *pn*-junction boundary value has been evaluated as a generation function by Green's function approach. It has been shown that the contribution of a boundary value can be calculated by the solution of the homogeneous boundary value problem for a derived delta generation function. The method promises reduction in overall calculation payload and focuses on the physical insight of the boundary value.

I. INTRODUCTION

In conventional solution of differential equations, the boundary values are mainly utilised in determining unknown coefficients of the solution. In Green's function approach, a physical insight has been imposed on the boundary values. It has been shown that the solution of continuity problem by Green's function approach facilitates envisaging of contributions of initial value, each boundary value and generation function separately[1]. The presentation of solution is phenomenological and easier to interpret. In this paper, as a further step in device analysis, it has been shown that each boundary value can be treated as a generation function. Contribution of each boundary value requires only adapting the solution of homogeneous boundary value problem for a derived delta generation function[2]. This opportunity reduces overall calculation payload and signifies on the physical insight of the boundary values.

For focusing on mathematical method in depth, many secondary effects (interactions between particles, etc.) have not been considered. It has been assumed that the semiconductor is *uniformly doped* and all doping atoms have been *ionised*. All possible recombination processes have been presented with an *effective recombination time constant*. *Constant* and *isotropic* physical parameters have been assumed. *Abrupt junction* shape has been considered. For sake of analytical solution, *drift component* of minority carrier current has been neglected. All calculations are carried out under *low-level injection* and *quasineutrality* assumptions.

In this paper, solution of one-dimensional (1D) time dependent problem has been summarised in Section II. Then, this solution has been adapted for non-homogeneous boundary values in Section III. Finally, for a set of device parameters, outcome of the method has been presented.

II. GREEN'S FUNCTION APPROACH

Previously developed *drift-diffusion* based analytical device analysis has been summarised for ready use[1]. The outcome of the method has been further implemented for evaluation of the boundary values.

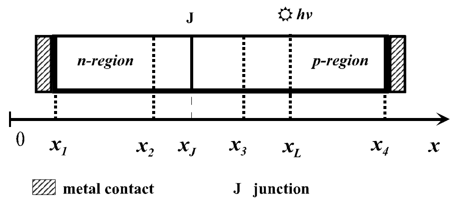

Fig. 1 Schematic presentation of a *pn*-junction.

One-dimensional *p*-region of a *pn*-junction has been considered as analysis domain (Fig. 1). x_3 and x_2 denote the edge of depletion regions; x_4 and x_1 denote boundary of *p* and *n*-regions (metal contacts), respectively; x_J denotes metallurgical junction. A delta type light source has been situated at the position, $x=x_L$.

One can construct the following well-known continuity equation for excess minority carriers (electrons, n) in time (t) and space (x) [3,4].

$$\frac{\partial^2 n}{\partial x^2} - \frac{n}{L_n^2} + \frac{g_L}{D_n} = \frac{1}{D_n}\frac{\partial n}{\partial t} \quad (1)$$

$$\begin{aligned} x &= x_3 \quad , \quad n \cong 0 \equiv f_{n1}(t) \quad , \quad t > 0 \\ x &= x_4 \quad , \quad n = 0 \equiv f_{n2}(t) \quad , \quad t > 0 \\ t &= 0 \quad , \quad n = 0 \equiv F_n(x) \end{aligned}$$

where, n is excess electron population (1/cm^3); D_n is diffusion constant; and L_n is diffusion length for electrons.

The procedure for the solution of the problem is as follows: The continuity equation is transformed to the heat conduction problem, first. Then, the transformed problem has been solved by Green's function. Finally, the solution of the original problem has been found by inverse transformation[1]. The following are summary of this calculation process:

The *Green's function of the transformed problem* has been found as

$$\begin{aligned} &\text{for} \quad t > \tau, \\ &\quad G_n(u,t|v,\tau) = \\ &\quad \left[\left(\frac{2}{L}\right) \sum_{j=1}^{\infty} e^{-D_n \beta_j^2 (t-\tau)} \sin(\beta_j u) \sin(\beta_j v) \right] \quad (2) \\ &\text{for} \quad t < \tau, \quad G_n(u,t|v,\tau) \equiv 0 \end{aligned}$$

where u is new space variable and defined as $u \equiv x - x_3$; τ is new time variable and defined as ($\tau<t$) ; and L is space (1D) length and defined as $L \equiv x_4 - x_3$. The series' sum parameter is given as

$$\beta_j = j\frac{\pi}{L} \quad , \quad \mathrm{j} = 1,2,3,... \quad (3)$$

General solution of the transformed problem has been calculated[1] as

$$\begin{aligned} \psi_n(u,t) &= D_n \int_{v=0}^{L} G_n(u,t|v,\tau)\Big|_{\tau=0} \frac{e^{\tau/\tau_n}}{D_n} F(v)dv \\ &+ D_n \int_{\tau=0}^{t} d\tau \int_{v=0}^{L} G_n(u,t|v,\tau) \frac{e^{\tau/\tau_n}}{D_n} g_L(v,\tau)dv \\ &+ D_n \int_{\tau=0}^{t} G_n(u,t|v,\tau)\Big|_{v=x_3} \frac{e^{\tau/\tau_n}}{D_n} f_1(\tau)d\tau \\ &+ D_n \int_{\tau=0}^{t} G_n(u,t|v,\tau)\Big|_{v=x_4} \frac{e^{\tau/\tau_n}}{D_n} f_2(\tau)d\tau \end{aligned} \quad (4)$$

where, $\psi_n(u,t)$ is the solution of the transformed problem. f_1 and f_2 are boundary values, and *F(v)* is initial value of the original problem. v is introduced for integration in space. τ_n is recombination time constant for electrons and has the following relation:

$$L_n^2 = D_n \ \tau_n \quad (5)$$

For homogeneous boundary values (f_1 and f_2 are zero) and zero initial value (*F(v)* is zero), the only generation function related term should be considered.

Let's consider a delta generation function, which has been defined as

$$g_L(u) \equiv M\delta(u - u_L) \quad (6)$$

where, $u_L \equiv x_L - x_3$. *The solution of original problem* for the given delta generation function is obtained[1] as

$$\begin{aligned} n(x,t) = M\left(\frac{2}{L}\right)\sum_{j=1}^{\infty} \frac{\tau_n}{(L_n\beta_j)^2 + 1}(1 - e^{-\frac{(L_n\beta_j)^2+1}{\tau_n}t}) \\ \times \sin[\beta_j(x - x_3)] \sin[\beta_j(x_L - x_3)] \end{aligned} \quad (7)$$

The last expression describes *the excess electron population* in the *p-region* of a planar *p-n* junction, in *time* and *space* under the injection of *M electron-hole pairs* per *cm^2* per *second at the position, x=x$_L$*. This solution forms the base for the evaluation of the non-homogeneous boundary value in the following section. It will be referred as *the solution of the homogeneous boundary value problem for a delta generation function.*

III. EVALUATION OF A BOUNDARY VALUE

A *pn*-junction device under an external electrical field can be analysed by Green's function approach[2]. To do so, one needs to set up the excess minority continuity equation in the particular region and under the particular boundary values. The *p*-region of a *pn*-junction under an external electrical field has the following boundary values:

On *the metal contact*,

$$x = x_4 \quad , \quad n = 0 \tag{8}$$

On *the depletion edge*, as a result of *Shockley* diode relation[4],

$$x = x_3 \quad , \quad n = n_{p0}\left(e^{\frac{V}{V_T}} - 1\right) \tag{9}$$

where n_{p0}, electron concentration of *p*-region in thermal equilibrium; V is external applied voltage; and V_T is thermal voltage.

The problem can be solved by Green's function general procedure-contribution of each boundary value can be calculated by integration in time (Eq.5). Instead, one can solve the homogeneous boundary value problem for a derived delta generation function, first. Then, this solution can be adapted for the non-homogeneous boundary value. The later one promises less calculation payload.

The new problem has been posed as follows:

(i) The non-homogeneous boundary value has been presented with a delta generation function (derived delta generation function) as

$$g_{n0}(x,t) = S_n n_{p0}\left(e^{\frac{V}{V_T}} - 1\right)\delta\left(x - x_3\right) \tag{10}$$

where, S_n [cm/s] is a calibration parameter which transforms electron population to the current density. As the general case in application, it is assumed that the time dependency can only come via applied external voltage (V). S_n is time independent.

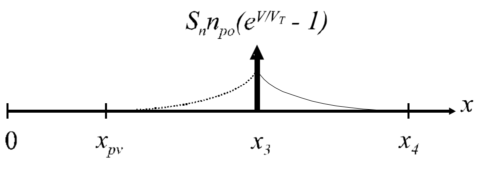

Fig. 2 Extended *p*-region.

(ii) To satisfy homogeneous boundary values *a virtual metal contact* (x_{pv}) has been situated at the image of the real metal contact (x_4):

$$x_{pv} \equiv 2x_3 - x_4 \tag{11}$$

(iii) The analysis domain has been extended as

$$\overline{\Omega}_a = \{x | x \in [x_{pv}, x_4]\} \tag{12}$$

The new posed problem has the same properties as that of one which its solution has been summarised in preceding section. In case of time independent boundary value, the solution of the new problem can be obtained by just replacing the following parameters in Eq.7:

(i) The amplitude of the generation function should be replaced as

$$M \rightarrow S_n n_{p0}\left(e^{\frac{V}{V_T}} - 1\right) \tag{13}$$

(ii) Extended space (1D) length should be replaced as

$$L \rightarrow 2L \tag{14}$$

(iii) Displacement parameters should be replaced as

$$x_L \rightarrow x_3 \quad , \quad x_3 \rightarrow x_{pv} \tag{15}$$

The solution of the new problem–excess electron population in *p*-region– is as

$$n_e(x,t) = S_n n_{p0}\left(e^{\frac{V}{V_T}} - 1\right)\left(\frac{2}{2L}\right) \times \sum_{i=1}^{\infty} \Gamma_n(\beta_i, t) X_n(\beta_i, x_3) X_n(\beta_i, x) \tag{16}$$

$$\beta_i = i\frac{\pi}{2L} \tag{17}$$

$$\Gamma_n(\beta_{i,} t) \equiv \frac{\tau_n}{(L_n\beta_i)^2 + 1}\left(1 - e^{-\frac{(L_n\beta_i)^2 + 1}{\tau_n}t}\right) \tag{18}$$

$$X_n(\beta_i, x) \equiv \sin\left[\beta_i(x - x_{pv})\right] \tag{19}$$

Calibration parameter can be determined from non-homogeneous boundary value of original problem.

$$n_e(x,t)\Big|_{x=x_3} = n_{p0}\left(e^{\frac{V}{V_T}} - 1\right) \tag{20}$$

$$\frac{1}{S_n} = \left(\frac{1}{L}\sum_{i=1}^{\infty} \Gamma_n(\beta_i, t \to \infty) X_n^2(\beta_i, x_3)\right) \tag{21}$$

Solution domain is restricted to

$$\overline{\Omega}_s = \{x | x \in [x_3, x_4]\} \tag{22}$$

As it has been claimed at the beginning of the paper, no calculation has been carried out for a time independent boundary value. The same analysis procedure can be applied to the other non-homogeneous boundary values.

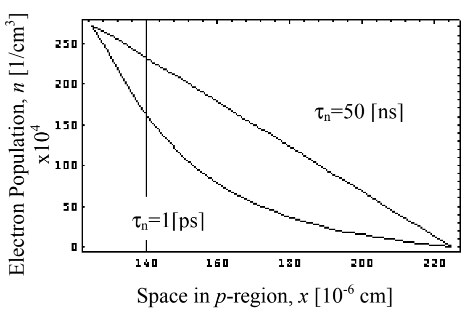

Fig. 3 Excess electron population. Utilised parameters [2] are $N_A = 5.0\ 10^{17}$ [1/cm^3]; Dn = 745.7 [cm^2/s]; L = 1 [μm]; V = 0.7 [V].

Because of the linear property of the system under consideration, contribution of each boundary, initial value, and generation function should be summed up for the general solution of the problem.

For a general continuity problem, calculation payload involves integration in space (v) for initial value, integration in time and space for a specific generation function, and integration in time (τ) for the derived delta generation function. However, for a particular continuity problem, calculation payload is application dependent. In transient problems, both initial value and boundary values are involved. In steady state problems, boundary values and specific generation function–if exists– are involved. In applications where the boundary values are time independent, the replacement of some parameters is only required.

In problems where prescribed assumptions are not satisfied properly, the method may not give desired results. In these circumstances, the available numerical methods should be sought.

IV. CONCLUSION

In analytical device analysis, it has been shown that a boundary value can be treated as a generation function by utilising Green's function approach. Only the solution of homogeneous boundary value problem for a derived delta generation function is required. Contribution of each non-homogeneous boundary value can be calculated by just adapting solution of the homogeneous boundary value problem for the delta generation function. The method can be generalised to any finite number of boundary values. Multi-dimensional problems can be treated similarly. The presented method provides a clear and consistent interpretation of the physical problems, and promises less overall calculation payload.

V. REFERENCES

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