EXCITATION OF RECTANGULAR CAVITY BY WALSH FUNCTIONS

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ABSTRACT

In this study, Analytical electromagnetic field solution is obtained in time domain for rectangular cavity excited by time dependent external force for different kind of Walsh functions. Resonance phenomena in rectangular cavity is obtained related to period of Walsh functions. As a method of solution of time-domain problem, Evolutionary Approach to Electromagnetic (EAE) is used.

I. INTRODUCTION

Rayleigh first investigated the transmission and oscillation of electromagnetic waves in waveguides and cavities about 100 years ago [1]. It seems that all investigations are restricted to sinusoidal excitation of waveguides and cavities. Because the widely held belief that a cavity can oscillate and resonate without distortion only by sinusoidal excitation is based on solutions of Maxwell's equation. After Rayleigh research, a few scientists were investigated about non-sinusoidal wave oscillation and propagation in cavities and waveguides [2-3]. Mathematical solution of sinusoidal based electromagnetic problems is rigorously considered and analysed because Fourier transformation method is very good agreement with sinusoidal excitation. During twentieth century, the most important functions for communications, military purposes, radar and computer technologies were pulses and Walsh functions. Nowadays, pulses and Walsh functions have been widely in use for ground penetrating radar (GPR) applications and cryptology of communication signals [4,5]. The solution of electromagnetic problems by using Fourier transformation for different kind of excitation such as pulse and Walsh function in different kind of media may not be exact or sometimes impossible. Fourier transformation is useful tool for solution of vast class of electromagnetic problems in frequency domain. But some main important disadvantages of Fourier transformation in literature are given at below [6-7],

- The mathematical and physical difficulties for Fourier transformation problem of ultra short signals,
- The Fourier transformation problem of non-linear terms which is appeared because of exact constitutive relations for different kind of media,
- The difficulties of Fourier transformation in electromagnetic problems with new materials which have time and space dependencies of electromagnetic parameters,
- Fourier expansion and convergence problem of singular functions to Fourier domain,
- Computer time and error problems of numerical methods such as using FDTD instead Fourier transformation method,

If Walsh function type excitation is considered in rectangular cavity, the solution of problem by using Fourier expansion, convergence problem of singular function like Walsh function has been faced for exact solution. To overcome of this problem for Walsh function, EAE is used and resonance condition in rectangular cavity is obtained in time domain analytically.

II. WALSH FUNCTIONS

Walsh functions are a kind of orthogonal functions and their amplitude jumps between +1 and –1. In every jump points, they have singularities. They may be periodic or non-periodic signals with non-sinusoidal form. The derivation of Walsh functions do not give another type of orthogonal Walsh functions which is possible for sin-cos functions. Walsh functions can be reconstructed from Hadamard matrix by using unit pulse function and shown as digit forms. The complete system of Walsh functions seems to have been found around 1900 by J.A.Barrett [8]. They have been used for digital signals modelling for coding and transmitting of signal in digital system. A general set of Walsh functions arranged in sequence order is given in Figure 1.

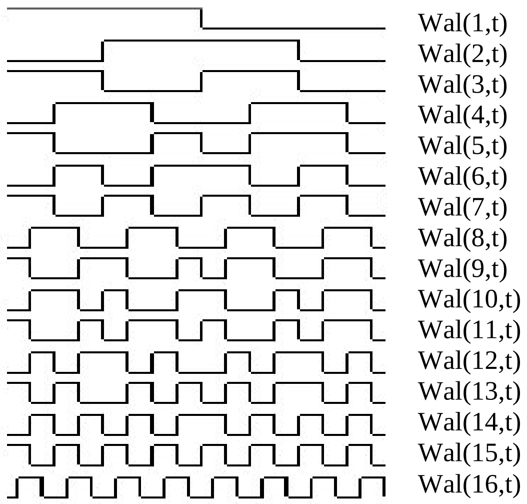

Figure 1. A set of Walsh functions arranged in sequency order

Modelling of Walsh functions can be constructed by Hadamard matrixes and unit pulse $p(t)$ function

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \; H_{2n} = \begin{bmatrix} H_n & H_n \\ H_n & \bar{H}_n \end{bmatrix}$$

$$W_n(t) = \sum_{i=1}^{N} h_n(i)\, p(t-(i-1)) \qquad (1)$$

$$h_n(i) = line(H_{2n})_i \;,\; h_n(i)h_n(j) = 0 \;,\; i \neq j$$

where H_2 is lowest order Hadamard matrix, H_{2n} is higher order Hadamard matrix, $\bar{H}_{2n}$ is inverse replacing of 1's and –1's of the given H_2 matrix, $h_n(i)$ is ith line of nth order Hadamard matrix. Every ith line of Hadamard matrix is known as 2n-bit Walsh series and ordering of Walsh series is referred to in the literature more recently Lexicographic ordering.

III. THE METHOD OF EVOLUTIONARY APPROACH TO ELECTROMAGNETIC FOR RECTANGULAR CAVITY

EAE is based on separation of a self-adjoint operator in Maxwell operator that is obtained from Maxwell equation in cavity [9]. Eigenvector set of self-adjoint operator is complete and originates a basis. The electromagnetic field term in cavity can be presented in terms of eigenvector of self-adjoint operator. Physically, it corresponds to modal expansion in cavity and modal expansion of amplitude which is function of time must be calculated. Problem of modal expansion can be obtained via projecting of Maxwell equation onto the same basis. According to EAE, the modal expansion of cavity modes,

$$\vec{E}_n(\vec{r},t) = \sum_n e_n(t)\vec{E}_n(\vec{r})$$
$$\vec{H}_n(\vec{r},t) = \sum_n h_n(t)\vec{H}_n(\vec{r}) \qquad (1)$$

where space dependent terms $\vec{E}_n(\vec{r})$ and $\vec{H}_n(\vec{r})$ can be found by solving scalar Dirichlet and Neuman boundry value problems,

$$(\Delta + k_n^2\varepsilon_o\mu_o)\vec{E}_n = 0 \,,\, \vec{H}_n = -irot\vec{E}_n / k_n\mu_o$$
$$div\vec{E}_n = 0 \,,\; k_n = k_{+n} > 0 \,,\, \left[\vec{n}\times\vec{E}_n\right]_S = 0 \qquad (2.a)$$

$$(\Delta + k_n^2\varepsilon_o\mu_o)\vec{H}_n = 0 \,,\, \vec{E}_n = irot\vec{H}_n / k_n\varepsilon_o$$
$$div\vec{H}_n = 0 \,,\; k_n = k_{+n} > 0 \,,\left(\vec{n}.\vec{H}_n\right)_S = 0 \qquad (2.b)$$

Normalization condition for (2.a) and (2.b) is given at below,

$$\frac{\varepsilon_o}{v}\int_v \left|\vec{E}_n\right|^2 dv = 1 \;,\; \frac{\mu_o}{v}\int_v \left|\vec{E}_n\right|^2 dv = 1$$

Time dependent terms can be found by solving second order diferential equation,

$$\frac{d}{dt}e_n(t) + ik_n h_n(t) = -j_n^e(t) \,,\; e_n(t)\big|_{t=0} = e_n^o \qquad (3.a)$$
$$\frac{d}{dt}h_n(t) + ik_n e_n(t) = 0 \,,\; h_n(t)\big|_{t=0} = h_n^o \qquad (3.b)$$

where n shows mode number, $j_n^e(t)$ is external source.. The solution of equation (3a-3b) is time dependent terms of electromagnetic field in equation (1).

V. APPLICATION

Lossy-free rectangular cavity is considered with unit dimension (10 cm x 10 cm x 10 cm) and some class of Walsh functions are applied to rectangular cavity. Resonance condition of Walsh functions for TE_{111} modes is considered and analysed. It is shown that if the period of Walsh function is equal to fundemantal period which can be calculated for $k_n = k_{111}$ (TE_{111} mode) and chosen for different applications, resonance condition can be obtained in rectangular cavity. Otherwise chosen mode will not get resonance. For TE_{111} mode with periodic 4-bit 1000 and non-periodic 0100 Walsh functions which are external time dependencies of source and resonance condition in lossy-free rectangular cavity is shown in Figure 3.a-b) and Figure 4.a-b).

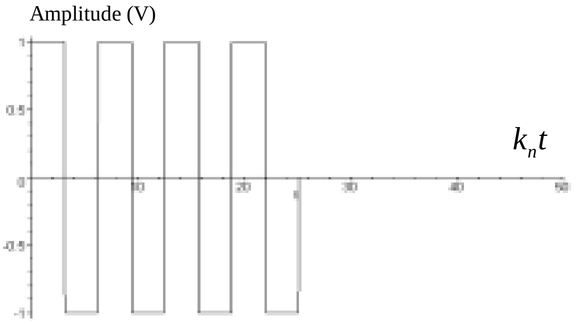

Figure 3-a) 4 bit-1000 Walsh function as a time dependency of external source

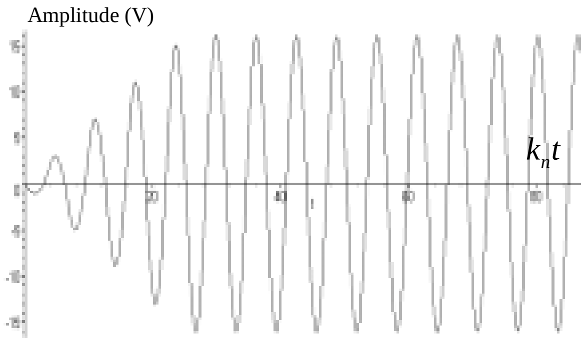

Figure 3-b) Resonance phenomena of TE_{111} cavity mode in lossy-free rectangular cavity

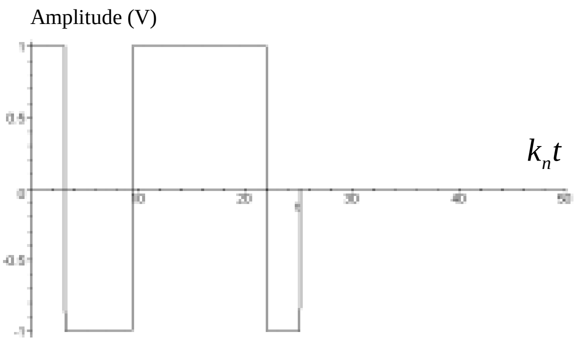

Figure 4-a) 4 bit-0100 Walsh function as a time dependency of external source

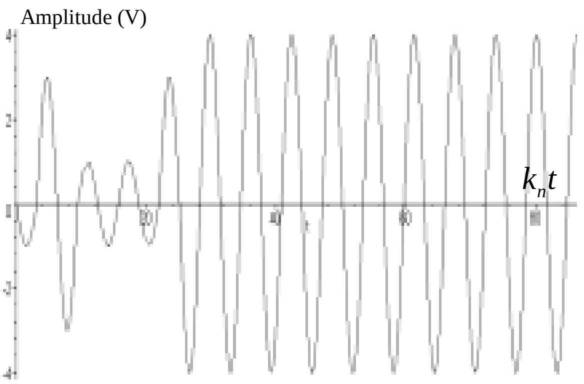

Figure 4-b) Resonance phenomena of TE_{111} cavity mode in lossy-free rectangular cavity

V. CONCLUSION

Walsh function that is periodic or non-periodic in lossy-free rectangular cavity for a chosen mode can resonate. The response of rectangular cavity for square wave that is a special form of Walsh function and non-periodic Walsh function is investigated and resonance condition is obtained by using EAM in time domain analytically. If note that Figure 3-b) and Figure 4-b), the response of cavity for different Walsh functions has two behavior as transient and stable forms. The stable forms are the same but transient forms are different from each other according to different Walsh functions. It means that rectangular cavity may be used to select excitation type by using response of cavity. For this aim, the class of all Walsh function should be analyzed which is aimed in future research plan.

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