# Phasor Measurement Unit (PMU) Allocation in power system with different Algorithms 

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#### Abstract

One of the most important measurement devices in Power systems is Phasor Measurement Unit (PMU). The PMU is a power system device, capable of measuring the synchronized voltage and current phasor in a power system. In this paper we introduce the PMU and then we acclaim according to the PSAT, MATLAB toolbox, and different algorithms we can allocate the PMU in IEEE 14 -bus and 57 -bus system simulated, optimally. By comparing them, we found that, we can't say an algorithm is optimal for all systems. good allocation can reserves the finance and obtain better power network monitoring.


## 1. INTRODUCTION

The Phasor Measurement Unit (PMU) is a power system device capable of measuring the synchronized voltage and current phasor in a power systems.
PMU is considered to be one of the most important measuring devices in the future of power systems. The distinction comes from its unique ability to provide synchronized phasor measurements of voltages and currents from widely dispersed locations in an electric power grid. The commercialization of the global positioning satellite (GPS) with accuracy of timing pulses in the order of 1 microsecond made possible the commercial production of phasor measurement units.[1]
Simulations and field experiences suggest that PMUs can revolutionize the ways that power systems are monitored and controlled. However, it is perceived that costs and communication links will affect the number of PMUs to be installed in any power system. Furthermore, defining the appropriate PMU system application is a utility problem that must be resolved.

## 2. PMU Placement

The static state estimation problem is generally formulated as a non-linear set of equations, as follows:
$z=h(x)+\varepsilon$
Where:
$z\left(z \in R^{m}\right)$ : Measurement vector;
$x\left(x \in R^{n}\right)$ : State vector;
$\varepsilon\left(\varepsilon \in \mathrm{R}^{\mathrm{m}}\right)$ : Measurement errors vector;
$h\left(h: R^{n} \rightarrow R^{m}\right)$ : Vector of the relationships between states and measurements;
Equation (1) is typically solved by means of a Newton-Raphson technique [Schweppe et al. 1970, Allemong et al. 1982, Monticelli and Garcia 1990]. Using devices able to provide voltage and current phasors, such as PMUs, yields a linear
relationship between state variables and measurements variables, as follows:
$z=H x+\varepsilon$
Where $H\left(H \in R^{n \times m}\right)$ is the "state" matrix of the system. Typically $m>n$, and the solution of (2) is obtained by a least mean square technique [Thorp et al. 1985].
By splitting the vector $z$ into the $m_{V} X l$ voltage and $m_{I} X_{1}$ current subvectors, $Z_{V}$ and $Z_{b}$, and the vector $x$ into the $n_{M} \times l$ and $n_{C} X 1$ non-measured subvectors, $V_{M}$ and $V_{C}$, relationship (2) becomes
$\left[\begin{array}{c}z_{V} \\ z_{I}\end{array}\right]=\left[\begin{array}{cc}I & 0 \\ Y_{I M} & Y_{I C}\end{array}\right]\left[\begin{array}{l}V_{M} \\ V_{C}\end{array}\right]+\left[\begin{array}{c}\varepsilon_{V} \\ \varepsilon_{C}\end{array}\right]$
Where $I$ is the identity matrix, and $Y_{I M}, Y_{I C}$ are submatrices whose elements are series and shunt admittances of the network branches.
Neglecting shunts, the matrix H is as follows:

$$
H=\left[\begin{array}{cc}
I & 0  \tag{4}\\
M_{I B} Y_{B B} A_{M B}{ }^{T} & M_{I B} Y_{B B} A^{T}{ }_{C B}
\end{array}\right]
$$

Where $M_{I B}$ is the $m_{I} \times b$ measurement-to-branch incidence matrix associated with the current phasor measurements, $Y_{B B}$ is the $b \times b$ diagonal matrix of the branch admittances, and $A_{M B}$ and $A_{C B}$ are the $n_{M} \times b$ and $n_{C} \times b$ calculated node-to branch incidence submatrices, respectively [3].

## 3. PMU Placement Rules

The following PMU placement rules were proposed in [3]:
Rule 1: Assign one voltage measurement to a bus where a PMU has been placed, including one current measurement to each branch connected to the bus itself (Fig. 1.a).
Rule 2: Assign one voltage pseudo-measurement to each node reached by another equipped with a PMU.
Rule 3: Assign one current pseudo-measurement to each branch connecting two buses where voltages are known (Fig.1.b). This allows interconnecting observed zones.
Rule 4: Assign one current pseudo-measurement to each branch where current can be calculated by the Kirchhoff current law (Fig.1.c). This rule applies when the current balance at one node is known, i.e. if the node has no power injections (if $\mathrm{N}-1$ currents incident to the node are known, the last current can be computed by difference).


## 4. Algorithms

## A. Depth First

This method uses only Rules from 1 to 3 (it does not consider pure transit nodes).
The first PMU is placed at the bus with the largest number of connected branches if there is more than one bus with this characteristic, one is randomly chosen. Following PMUs are placed with the same criterion, until the complete network visibility is obtained, as depicted in (Fig.2)
B. Graph Theoretic Procedure

This method was originally proposed in [Baldwin et al. 1993] and is similar to the Depth first].
Algorithm, except for taking into account pure transit nodes (Rule 4).


Fig. 2. Flowchart of the Graph Theoretic Procedure
C. Bisecting Search Method

Figures. 3 and4 depict the Pseudo-code of the simulated annealing procedure and the flowchart of the bisecting search method. Refer to [3] for the complete description of this method.

```
begin
\(\frac{\text { evaluate coverage of } P M U \text { placement set } S}{\text { eval }}\)
\(E:=N-\) number of buses in the observed region
\(T:=15\)
\(M:=\min \left\{0.002\left({ }^{\text {Vown }}{ }^{N}\right), M_{\text {max }}\right\}\)
for \(i:=1\) to 40 do
    \(\underline{\text { for }} j:=\bar{I}\) to \(\bar{M}\) do
            randomly setect a PMU
            save the bus location of the selected PMU
            randomly select a non-PMU bus
            evaluate coverage of modified placement set
            \(E_{\text {new }}:=N\) - number of buses in the observed region
            if \(E_{\text {new }}:=0 \underline{\text { then }}\)
                return with 'system observable'
                and the modified placement set
            \(\frac{\mathbf{f i}}{\Delta} E\) :
            \(\Delta E:=E_{\text {new }}-E\)
            if \(\Delta E>0\) then
                    \(\overline{\text { generate a random accept/reject value }}\)
                    with a probability \(\exp (-\triangle E T)\)
                        if reject then
                                return selected PMU to
                                    pervious bus location
                            fi
            \(\xrightarrow[\mathbf{o d}_{T} \quad \underline{\mathbf{f i}}]{0.879 T}\)
\(\frac{\mathrm{od}}{\text { ret }}\)
\(\frac{\mathbf{o d}}{\text { return with 'system not observable' }}\)
end
```

Fig. 3. Pseudo-code of the simulated Annealing Algorithm


Fig. 4. Flowchart of the Bisecting Search
D. Recursive Security N Algorithm

This method is a modified depth first approach. The procedure can be subdivided into three main steps:

1) Generation of N minimum spanning trees: Fig. 5 depicts the flowchart of the minimum spanning tree generation algorithm. The algorithm is performed N times ( N being the number of buses ), using as starting bus each bus of the network.


Fig.5. Recursive N Security Method
2) Search of alternative patterns: The PMU sets obtained with the step (a) are reprocessed as follows: one at a time, each PMU of each set is replaced at the buses connected with the node where a PMU was originally set, as depicted in Fig. 6. PMU placements which lead to a complete visibility are retained.

(a)

(b)

(c)

Fig. 6. Search of alternative placement sets
3) Reducing PMU number in case of pure transit nodes: In this step it is verified if the network remains observable taking out one PMU at a time from each set, as depicted in Fig. 7. If the network does not present pure transit nodes, the procedure ends at step (2).The placement sets which present the minimum numbers of PMUs are finally selected.


Fig. 7. Pure transit node filtering

## E. Single Shot Security N Algorithm

This method was proposed in [Denegri et al. 2002]. The algorithm is based only on topological rules, and determines a single spanning tree, as illustrated in Fig.8.


Fig. 8. Single-Shot N Security Method
F. Recursive and Single-Shot Security N-1 Algorithms

The rules for minimal PMU placement assume a fixed network topology and a complete reliability of measurement devices. Simple criteria which yield a complete visibility in case of line outages (N-1 security) are proposed in [Denegri et al. 2002] and are based on the following definition:
A bus is said to be observable if at least one of the two following conditions applies:

Rule 1: a PMU is placed at the node;
Rule 2: the node is connected at least to two nodes equipped with a PMU.
Rule 2 is ignored if the bus is connected to single-end line. Figures 10 and 11 depict the algorithms for obtaining the $\mathrm{N}-1$ security placement proposed in [Denegri et al. 2002]. The first method is a slightly different version of the recursive technique described in Section D, whereas the second method is a variant of the algorithm described in Section E.[4]

## 5. SIMULATED SYSTEM

In this section the IEEE 14-bus test system simulated and results for each Algorithms shown as reported. Finally IEEE 57-bus system simulated and efficiency of each algorithms and comparing of number of suggested bus for obviousability, between two mentioned systems, have showed at table1.


Fig. 9. IEEE 14-bus test system
PMU PLACEMENT REPORT
PS AT_2.1.3

1. Placement Method: Depth First

| PMUs | $\mathbf{6}$ |
| :--- | :---: |
| PMU Sets | 1 |
| Meas. Currents | 16 |
| Pseudo-Meas. Currents | 0 |
| PMU PLACEMENT |  |
| Bus Name | Set 1 |
| Bus 01 | 1 |
| Bus 02 | 0 |
| Bus 03 | 0 |
| Bus 04 | 1 |
| Bus 05 | 0 |
| Bus 06 | 1 |
| Bus 07 | 0 |
| Bus 08 | 1 |
| Bus 09 | 0 |
| Bus 10 | 1 |
| Bus 11 | 0 |
| Bus 12 | 0 |
| Bus 13 | 0 |
| Bus 14 | 1 |




Fig. 10. Single Shot N-1 Security Method


Fig. 11. Recursive N-1 Security Method.
Table 1. comparing of number of suggested bus for obviousability between IEEE 14 and 57 buses systems

| METHODE | 14-BUS | 57-BUS |
| :---: | :---: | :---: |
| Depth First | 6 | 21 |
| Graph Theoretic Procedure | 5 | 13 |
| Annealing Method | 4 |  |
| Direct Spanning Tree | 4 |  |
| Direct (N-1) Spanning Tree | 4 |  |
| Minimum Spanning Tree | 3 | 30 |



Fig. 12. IEEE 57-bus test system

## 6. CONCLUSIONS

We used PSAT for test different algorithm and understood that we can't say an algorithm is optimal for all systems. So it's necessary to test different algorithms, before any allocation of PMU, to obtain optimal placement. Optimal PMU placement decreases number of PMUs that redounds costs declining. Using PMU in power system increases reliability of power system stability. It is therefore possible to fully monitor the system by using relatively less number of PMUs than the number of system buses.

## 7. References

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