# On-line Rotor Time Constant Estimation for Induction Motor using Two New Methods, RLS and SDBP Algorithms

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## Abstract

This paper presents two new methods of online estimation for the rotor time constant of the induction motor for indirect vector control drives. These methods are presented using artificial neural networks with steepest descent back propagation training algorithm and recursive least square algorithm. These methods use measurements of the stator voltages, stator currents and the rotor speed. The problem is formulated as finding the rotor flux error, which occurs as a result of rotor time constant variation, and then the actual value of the rotor resistance is estimated. The effectiveness of these algorithms is demonstrated with simulation.

Keywords-artificial neural networks, recursive least square algorithm, rotor time constant, parameter estimation, induction motor drive

#### Nomenclature

 $v_{as}^{s}, v_{ds}^{s}$  Stator voltages in stationary reference frame

 $\psi_{as}^{s}, \psi_{ds}^{s}$  Stator flux linkages in stationary reference frame

 $i_{qs}^{s}, i_{ds}^{s}$  Stator currents in stationary reference frame

 $v_{ar}^{s}, v_{dr}^{s}$  Rotor voltages in stationary reference frame

 $\psi_{ar}^{s}, \psi_{dr}^{s}$  Rotor flux linkages in stationary reference frame

 $i_{ar}^{s}, i_{dr}^{s}$  Rotor currents in stationary reference frame

 $\psi_{ma}^{s}, \psi_{md}^{s}$  Mutual flux linkages in stationary reference frame

 $r_s$ ,  $r_r$  Stator and Rotor resistance respectively

- $x_s, x_r$  Stator and Rotor inductance respectively
- $\omega_r$  Electrical rotor speed
- $\omega_{h}$  Base speed
- "'" Rotor parameters or quantities referred to the stator

#### 1. Introduction

Indirect field oriented vector controlled induction motor drives are widely used in industrial applications for highperformance drive systems. The main problem in the indirect control, is the rotor open circuit time constant  $\tau_r$ , which is sensitive to both temperature and flux level. When the value of this parameter is incorrect in the controller, the calculated slip frequency will be uncorrected and the flux angle will be no longer appropriate for field orientation [1]. Therefore a mismatch between the actual rotor flux angle and estimated rotor flux angle, as a result of rotor time constant variations, leads to error between the actual motor torque and the estimated torque and concludes disturbed dynamic performance. Therefore it is important the value of rotor time constant is continuously estimated.

The parameter sensitivity effects have been quantified in the steady state and the transient state, considering both open and closed outer speed loop in the indirect vector controlled induction motor drive in [2]. The fast variations of rotor resistance due to rotor temperature have been shown in [3]. The slip dependency of rotor time constant which is due to motor losses is usually ignored. A slip frequency calculation procedure has been proposed by [3]. A model reference adaptive control scheme has been used to track the variations of the rotor time constant in [4]. No ideal characteristics of the power drives and stator resistance variations have been included. A model reference adaptive control scheme is based on Luenberger observer has been presented by [5]. This model has been compared with the classical adaptive algorithm. Also a current model rotor flux observer based on model reference adaptive system scheme, has been proposed in [6]. This method uses the desired value of the rotor flux along the Q axis, which should ideally be zero. An extended Kalman filter has been used for rotor time constant identification in [7] and [8]. Fuzzy logic principles have been used for rotor resistance estimation in [9] and [10]. Also a simple PI controller has been proposed in [10] and a variable gain PI controller, a generalization of a classical PI controller, has been proposed in [11]. Recently sliding mode control is introduced for induction motor control drives. A fourth-order sliding mode flux observer has been developed in [12]. A Lyapunov based estimator has been designed in [13]. Recursive least square error was used for rotor time constant estimation in [14].

In recent years, the use of artificial neural networks for identification and control of nonlinear dynamic systems in power electronics and ac drives have been proposed [15], [16], as they are capable of approximating wide range of nonlinear functions to a high degree of accuracy [17]. An adaptation algorithm which uses the artificial neural networks has been proposed in [18]. There are two problems in this method. First is training data acquisition and second is complicated and time consumption training. A simple two layer feed forward neural network, trained by the momentum modification to back propagation algorithm (MOBP), has been used to estimate rotor time constant in [17].

In this paper a single neuron feed forward neural network, trained by steepest descent back propagation algorithm (SDBP), has been proposed. MOBP algorithm is one of the variations of back propagation that provides speedup and makes the algorithm more practical [19]. In this case, a neural network model based method is used, in which the neural network model has trained in each sample time and training set, has one member. Therefore MOBP does not improve the network performance certainly. One of the advantages of the proposed method is simplicity in calculations. All weights of the neural network are described as the functions of induction motor parameters and one weight. This leads to accurate and online rotor time constant estimation. Another advantage is stability of this method.

A simple recursive least square method has been presented in [20]. A nonlinear recursive least square error algorithm was used in [21]. In this paper a new recursive least squares algorithm (RLS) has been proposed. One of the advantages of the proposed method is simplicity in calculations. All of the coefficients of the model are described as the functions of induction motor parameters and one of these coefficients.

The rest of the paper is outlined as follows. Section (2) describes the induction motor model, which is used. In sections (3) and (4), SDBP and RLS algorithm are discussed respectively. Rotor time constant estimation, using SDBP and RLS algorithms is considered in section (5). Simulation results are presented in section (6). Finally, section (7) concludes the paper.

# 2. Induction Motor Model

Standard models of induction motors are available in the literature. Stator flux linkage equations in three-phase induction motor modeled by d-q stationary reference frame can be derived as [22]:

$$\psi_{qs}^{s} = \omega_{b} \int \{v_{qs}^{s} + \frac{r_{s}}{x_{ls}} (\psi_{mq}^{s} - \psi_{qs}^{s})\} dt$$
(1)

$$\psi_{ds}^{s} = \omega_{b} \int \{ v_{ds}^{s} + \frac{r_{s}}{x_{ls}} (\psi_{md}^{s} - \psi_{ds}^{s}) \} dt$$
(2)

$$\psi_{qs}^s = x_{ls} i_{qs}^s + \psi_{mq}^s \tag{3}$$

$$\psi_{ds}^s = x_{ls} i_{ds}^s + \psi_{md}^s \tag{4}$$

By substituting (3) and (4) in (1) and (2) respectively, after some calculations, equations (5) and (6) can be expressed as following.

$$\frac{1}{\omega_b} \frac{d\psi_{qs}^s}{dt} = v_{qs}^s - r_s i_{qs}^s \tag{5}$$

$$\frac{1}{\omega_b} \frac{d\psi^s_{ds}}{dt} = v^s_{ds} - r_s i^s_{ds} \tag{6}$$

By using stator flux linkage equations and (7) to (10), equations (11) and (12) can be written as following.

$$\psi_{mq}^{s} = x_{m} (i_{qs}^{s} + i_{qr}^{'s}) \tag{7}$$

$$\psi_{md}^s = x_m (i_{ds}^s + i_{dr}^{'s}) \tag{8}$$

$$\psi_{ar}^{'s} = x_{br}^{'} i_{ar}^{'s} + \psi_{ma}^{s} \tag{9}$$

$$\psi_{dr}^{'s} = x_{lr}^{'}i_{dr}^{'s} + \psi_{md}^{s} \tag{10}$$

$$\frac{1}{\omega_b} \frac{d\psi_{qr}^s}{dt} = \frac{x_r}{x_m} (v_{qs}^s - r_s i_{qs}^s) + \frac{1}{\omega_b} \frac{x_m^2 - x_s x_r}{x_m} \frac{di_{qs}^s}{dt}$$
(11)

$$\frac{1}{\omega_b} \frac{d\psi_{dr}^s}{dt} = \frac{x_r}{x_m} (v_{ds}^s - r_s i_{ds}^s) + \frac{1}{\omega_b} \frac{x_m^2 - x_s x_r}{x_m} \frac{di_{ds}^s}{dt}$$
(12)

Equations (11) and (12) are based on stator voltages and currents, which calculate the actual rotor flux linkage.

Also rotor flux linkage equations can be derived as [22]:

$$\psi_{qr}^{'s} = \omega_b \int \{ v_{qr}^{'s} + \frac{\omega_r}{\omega_b} \psi_{dr}^{'s} + \frac{r_r^{'}}{x_{hr}^{'}} (\psi_{mq}^s - \psi_{qr}^{'s}) \} dt \quad (13)$$

$$\psi_{dr}^{'s} = \omega_b \int \{ v_{dr}^{'s} - \frac{\omega_r}{\omega_b} \psi_{qr}^{'s} + \frac{r_r}{x_{lr}} (\psi_{md}^s - \psi_{dr}^{'s}) \} dt \quad (14)$$

By using equations (7) to (10), equations (13) and (14) can be written as:

$$\frac{1}{\omega_b}\frac{d\psi_{qr}^{'s}}{dt} = +\frac{\omega_r}{\omega_b}\psi_{dr}^{'s} - \frac{r_r^{'}}{x_r^{'}}\psi_{qr}^{'s} + x_m\frac{r_r^{'}}{x_r^{'}}i_{qs}^{s}$$
(15)

$$\frac{1}{\omega_b} \frac{d\psi_{dr}^{'s}}{dt} = -\frac{\omega_r}{\omega_b} \psi_{qr}^{'s} - \frac{r_r^{'}}{x_r^{'}} \psi_{dr}^{'s} + x_m \frac{r_r^{'}}{x_r^{'}} i_{ds}^{s}$$
(16)

Equations (15) and (16) are based on stator currents and rotor speed. In these equations, rotor flux linkage calculated using the inverse value of rotor time constant. Therefore the error between the actual value of rotor time constant and the assuming value, leads to error in the rotor flux linkage calculation. The error between rotor flux linkages based on (11) and (12) and based on (15) and (16) can be used to calculate the actual value of the rotor time constant.

## 3. SDBP Algorithm

The back propagation algorithm uses the mean square error as it is performance index. The algorithm is provided with a set of examples of proper network behavior [19]:

$$\{p_1, t_1\}, \{p_2, t_2\}, \dots, \{p_3, t_3\}$$
(17)

Where p is an input to the network, and t is the corresponding target output.

The algorithm should adjust the network parameters in order to minimize the mean square error [19].

$$F(x) = E[e^{T}e] = E[(t-a)^{T}(t-a)]$$
(18)

Where "t" is the target vector and "a" is the output vector. The steepest descent back propagation algorithm (SDBP) for the approximate mean square error is [19]:

$$w_{i,j}^m(k+1) = w_{i,j}^m(k) - \alpha \frac{\partial F}{\partial w_{i,j}^m}$$
(19)

$$b_i^m(k+1) = b_i^m(k) - \alpha \frac{\partial F}{\partial b_i^m}$$
(20)

Where  $\alpha$  is the learning rate, w is weight, b is bias, i is number of the neuron, j is number of the input and m is number of the layer and  $b_i^m$  is the bias.

#### 4. RLS Algorithm

The model output can be written as:

$$Y = X_1 \theta_1 + X_2 \theta_2 + \ldots + X_n \theta_n \tag{21}$$

Gradient of loss function with respect to the parameter vector  $\hat{\theta}$  has to be equal to zero. This leads to the least square estimate [23]:

When the LS method is required to run online in real time, a recursive least square (RLS), calculates a new update for parameter vector, each time new data comes in. The RLS update is [23]:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + P(t+1)x(t+1) \{y(t+1) - x^{T}(t+1)\hat{\theta}(t)\}$$
(23)  
$$P^{-1}(t+1) = P^{-1}(t) + x(t+1)x^{T}(t+1)$$
(24)

# 5. Rotor Time Constant Estimation using SDBP and RLS Algorithms

With respect to equations (15) and (16), neural network model of the induction motor is confirmed. The sample-data model of equations (15) and (16), can be written as following.

$$\psi_{qr}^{'s}(k) = \omega_{b} \left(\frac{1}{\omega_{b}} - \frac{T_{s}r_{r}}{x_{r}}\right) \psi_{qr}^{'s}(k-1) + T_{s}\omega_{b} \frac{r_{r}x_{m}}{x_{r}} i_{qs}^{s}$$

$$(25)$$

$$\psi_{dr}^{'s}(k) = \omega_{b} \left(\frac{1}{\omega_{b}} - \frac{T_{s}r_{r}}{x_{r}}\right) \psi_{dr}^{'s}(k-1) - T_{s}\omega_{r} \psi_{qr}^{'s}(k-1) + T_{s}\omega_{b} \frac{r_{r}x_{m}}{x_{r}} i_{ds}^{s}$$

$$(26)$$

Where  $T_s$  is the sampling period.

#### 5.1. SDBP Estimation

Equations (25) and (26) represents matrix form of (15) and (16) [17].

$$\overline{\psi}_{r}^{'s}(k) = W_1 X_1 + W_2 X_2 + W_3 X_3 \tag{25}$$

Where:

$$\begin{aligned} \overline{\psi}_{\mathbf{r}}^{'s}(\mathbf{k}) &= \begin{bmatrix} \psi_{q\mathbf{r}}^{'s}(\mathbf{k}) \\ \psi_{d\mathbf{r}}^{'s}(\mathbf{k}) \end{bmatrix}, X_1 = \begin{bmatrix} \psi_{qr}^{'s}(k-1) \\ \psi_{dr}^{'s}(k-1) \end{bmatrix}, X_3 = \begin{bmatrix} i_{qs}^s(k-1) \\ i_{ds}^s(k-1) \end{bmatrix}, \\ W_1 &= \omega_b (\frac{1}{\omega_b} - \frac{T_s r_r^{'}}{x_r^{'}}), W_2 = T_s \omega_r, W_3 = T_s \omega_b \frac{r_r^{'} x_m}{x_r^{'}}. \end{aligned}$$

Equation (25) represents a single neuron neural network model, where  $W_1$ ,  $W_2$  and  $W_3$  are the weights of the inputs and  $X_1$ ,  $X_2$ ,  $X_3$  are the inputs of the neuron and a linear function is the transfer function of the neuron. The single neuron neural network is shown in Fig. 1.

The error between the actual rotor flux linkage and output of single neuron neural network, is given by:

$$\overline{\varepsilon}(k) = T(k) - \overline{\psi}_r^{'s}(k)$$
(26)

Where T is the vector of rotor flux linkage. The performance index is described as following.

$$E = \frac{1}{2}\bar{\varepsilon}^{2}(k) = \frac{1}{2}(T(k) - \bar{\psi}_{r}^{'s}(k))^{2}$$
(27)

 $W_2$  is known and  $W_1$  can be described by using induction motor parameters and also  $W_3$ . So  $W_3$  needs to be updated and then  $W_1$  can be computed.

The weight variation based on SDBP are presented by equation (28).

$$\Delta W_{3}(k) = \alpha(\frac{\partial E}{\partial W_{3}}) = \alpha(\frac{\partial E}{\partial \overline{\psi}_{r}^{s}(k)} \frac{\partial \overline{\psi}_{r}^{s}(k)}{\partial W_{3}})$$

$$= -\alpha(\overline{\psi}_{r}^{s}(k) - \overline{T}(k))X_{3}$$
(28)

Where  $\alpha$  is the learning rate and  $\gamma$  is the momentum coefficient

Actual value of the rotor time constant can be found out form equation (30).

Fig. 1. Single neuron neural network model

## 5.2. RLS Estimation

Equations (23) and (24), can be written as following.

$$Y = X\theta \tag{9}$$

Where  $T_s$  is the sampling period and:

$$Y(K) = \begin{bmatrix} \psi_{qr}^{s}(k) \\ \psi_{dr}^{s}(k) \end{bmatrix}, \quad X = \begin{bmatrix} X_1, X_2, X_3 \end{bmatrix}, \quad X_1 = \begin{bmatrix} \psi_{qr}^{s}(k-1) \\ \psi_{dr}^{s}(k-1) \end{bmatrix},$$
$$X_3 = \begin{bmatrix} i_{qs}^{s}(k-1) \\ i_{ds}^{s}(k-1) \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}, \quad \theta_1 = \omega_b \left(\frac{1}{\omega_b} - \frac{T_s r_r}{x_r}\right), \quad \theta_2 = T_s \omega_r$$

and  $\theta_3 = T_s \omega_b \frac{r_r x_m}{x_r}$ .

Coefficients matrix updated based on RLS by equation (10).  $\hat{\theta}(k+1) = \hat{\theta}(k) + P(k+1)X(k+1)\{Y(k+1) - X(k+1)\theta(k)\} (10)$   $P^{-1}(k+1) = P^{-1}(k) + X(k+1)X^{T}$ (11)

Where  $\hat{\theta}$  is the estimated matrix.  $\theta_2$  is known and  $\theta_1$  can be described by using induction motor parameters and also  $\theta_3$ . Actual value of the rotor time constant can be found out from (12).

$$T_r = \frac{T_s x_m}{\theta_3} \tag{12}$$

#### 6. Simulation Results

A three-phase induction motor with a rotor flux oriented vector controller was used for simulations. The parameters of the motor are presented in table (1).

# 6.1. SDBP Algorithm

The actual value and the estimated value of the rotor time constant using SDBP are shown in Fig. 2. Also the estimation error is shown in Fig. 3. The maximum value of the error is 1.7% and the neural network is stable. In The next step, learning rate is increased. Results are shown Fig. 4 and Fig. 5. The maximum error is 0.8% and the neural network is stable.



Fig. 2. Actual value of the rotor time constant and estimated value using SDBP



Fig. 3. Rotor time constant estimation error using SDBP



Fig. 4. Actual value of the rotor time constant and estimated value using SDBP



Fig. 5. Rotor time constant estimation error using SDBP

In The third step, learning rate is increased again. Results are shown Fig. 6 and Fig. 7. In these figures, the maximum error is 1%. This means that the network is over trained and the learning rate should not to be increased.



Fig. 6. Actual value of the rotor time constant and estimated value using SDBP



Fig. 7. Rotor time constant estimation error using SDBP

Table 1. Induction Motor Parameters

14.92 KW, 220 V, 3 phase, 4 pole,	
60Hz	
R <sub>s</sub>	0.1062 Ω
R <sub>r</sub>	0.0764 Ω
L <sub>lr</sub>	0.5689 mH
L <sub>1s</sub>	0.5689 mH
L <sub>m</sub>	15.47 mH
J	2.8 Kgm2

# 6.2. RLS Algorithm

Results of the rotor time constant estimation using RLS are shown in Fig. 8 and Fig. 9. The maximum value of the error is 1.5% in steady state. As shown in these figures, the estimation error in the beginning of the estimation is high, because of matrix P(0). The initial value of the matrix P must be in rang of  $10^2$  up to  $10^3$  and this leads to consuming time.

## 7. Conclusion

Increasing rotor time constant estimation accuracy is an important subject to induction motor indirect vector controlled drive. In this paper, two new approaches for estimation of induction motor rotor time constant, were proposed and compared. In these methods SDBP and RLS algorithms are used. Using SDBP and RLS caused to reduce estimation error and SDBP improve transient state of the estimation, because of the initial value of the matrix P in RLS.

Important advantage of proposed SDBP and RLS estimation methods, that they are online accurate methods for different conditions.



Fig. 8. Actual value of the rotor time constant and estimated value using RLS



Fig. 9. Rotor time constant estimation error using RLS

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