E01.46/A4-46

Generalized Approach on Modeling, Analysis and Simulation of Salient Pole-Like Permanent Magnet Synchronous Motors

Tarık DURU
University of Kocaeli Dept. of Electrical Eng. Turkey duru@cs.kou.edu.tr

Abstract

Presented paper is concerned with modeling, analysis and simulation of generalized magnetically asymmetrical type permanent magnet synchronous motor (ARPMSM). The asymmetry of the machine originates on the rotor-embedded permanent magnets and creates a salient pole-like magnetic characteristic. Although in the high performance applications the PMSM's -called in general "Brushless DC Servo Motor"- are designed with magnetically symmetrical rotors, magnetically asymmetric rotor shapes are also used because of some advantages: such as being more rugged against centrifugal forces and allowing the use of flux concentration principle in the rotor design /1/. The salient pole-like magnetic characteristic not only results in causing the reluctance torque to be induced, but also creates some difficulties during the simulation and controlling of this type motors. A generalized approach for the analysis of this phenomena is given in the following. Two basic operation, voltage and current regulated cases are examined via simulation. Results are discussed briefly.

1- Introduction

The developments on the magnet materials, control techniques and power electronics have caused to increase the applications of the permanent magnet synchronous motors in variable speed and high performance industrial drives. When compared to the dc servo motor, the PMSM is more efficient, light, compact, safer, has no maintenance requirements and no commutation limits on the operating area. Efficiency, near unity power factor and simple controlling are the key features of PMSM against induction motors. PMSM's are designed in general as the stator having either lumped, or semi distributed and short-pitched three phase windings and rotor mounted permanent magnets. The geometry and the place of the permanent magnets in the rotor is basically determines the magnetic symmetry of the machine. From this point of view, a coarse distinction can be made. If the magnetic reluctance of magnetic circuit for any of the stator phases varies as a periodic function of the rotation angle as in the case of salient pole machines, this machine can be so called magnetically asymmetric. In that case reluctance torque is presented, otherwise machine can be called as magnetically symmetric and has no reluctance torque. In Fig.1. some rotor configurations are given. In fact, more or less in any PM machine magnetic reluctance seen from the stator or rotor is a periodic function of rotation angle due to slotting, anisotropy of magnetic materials and saturation effects.

Fig. 1. Examples of salient pole-like rotor designs

Presented method of modeling is based on a series of Finite Elements (FE) field solutions which are repeated for certain angular steps. In these solutions, two basic parameters are involved: Variation of the self and mutual inductance of phase windings and the flux linkage of the phase windings due to PM excitation. Mathematical model is based on the inductance matrix and the flux linkage vector obtained from the harmonic analysis of periodic variations of above mentioned parameters. Due to the fact that such machine is poor to retain the conditions of symmetrical machine model of generalized machine theory, the model is constructed in natural a.b.c phase variables rather than in terms of transformed d.q.o frame variables. Analysis of saliency effects in the solution of the mathematical model are made for voltage and current regulated operations.

2- Mathematical Model

In general, PMSM is modeled as three-terminal magnetically coupled electrical network and one mechanical terminal. Basic representation is given in Fig.2. According to the generalized machine theory electrical and mechanical terminal equations can be written in compact form as given below /2/.

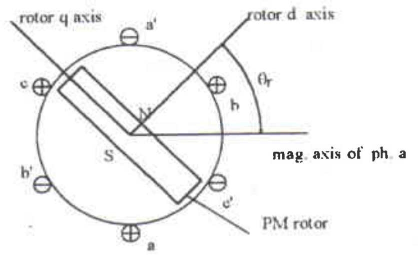

Fig 2. General PM machine

$$[L]\frac{d[I]}{dt} = \{ [V]-[R][I]-\omega_r[\lambda_m]^* - \omega_r[L]^*[I]\} \quad (1)$$

$$Te = p\left\{\frac{1}{2}[I]^T[L]^*[I]+[I]^T[\lambda_m]^*\right\} \quad (2)$$

$$\frac{d\omega_r}{dt} = J(T_e - T_l) \quad (3)$$

Where,
$[I]$: Three phase currents
$[V]$: Phase to neutral voltages.
$[\lambda_m]$:Flux linkages due to PM
$[L]$: Inductance matrix
w_r : Angular speed of rotor (electrical rad/s)
θ_r : Rotor angle (electrical rad)
T_e : Electromagnetic torque
Tl : Load Torque
J: Inertia reflected to shaft
p : Pole pair.
* : Partial derivative respect to rotor angular position (θ_r)

In (2), the first term is the reluctance torque, the second term is the torque due to the flux-current interaction. If the terms of inductance matrix are independent of rotor angle, then the reluctance torque term becomes zero. As introduced above, mathematical model parameters $[\lambda_m]$ and $[L]$ are based on a series of (FE) field analysis. Beside of FE method, mathematical model parameters can be obtained analytically or experimentally. As an example, two of these solutions for a 6 pole sample machine is given in Figure 3.

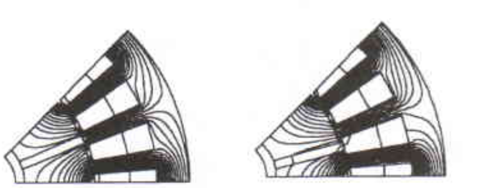

Fig.3. FE solutions for two sequential rotor angle

After a series of field solutions, the graphical view of the calculated self and mutual inductance and the flux linkage of the phase windings are obtained as seen in Fig. 4. Derivative of the variation of flux linkage respect to θ_r directly determines the shape of the speed voltage of the machine, while the self and mutual inductance's variations is a measure of the magnetic asymmetry of the rotor.

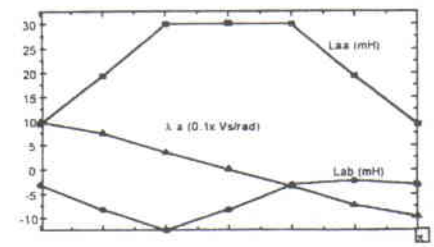

Fig 4. Variation of flux linkage, self and mutual inductance for phase "a"

A Numerical Fourier analysis of the obtained waveforms gives harmonic composition of inductance matrix and flux linkage elements. The elements of the inductance matrix are obtained as in following general form /2/.

$L_{aa} = L_{ao} - L_{b1}\cos(2\theta_r) - L_{b2}\cos(4\theta_r) - L_{b3}\cos(6\theta_r)$

$L_{ab} = -L_{bo} - L_{m1}\cos(2\theta_r - 2\pi/3) - L_{m2}\cos(4\theta_r - 4\pi/3) + L_{m3}\cos(6\theta_r - 12\pi/3)\ldots$

$\lambda_{ma} = \lambda_{m1}(\cos\theta_r) + \lambda_{m3}(\cos 3\theta_r) + \lambda_{m5}(\cos 5\theta_r) + \lambda_{m7}(\cos 7\theta_r)$

After having the terms of $[L]$ and $[\lambda_m]$ as finite numbered harmonic coefficients, the terms related with partial derivatives of $[L]$ and $[\lambda_m]$ in equation (1) and (2) now can be obtained using chain rule.

A closer look in to the elements of inductance matrix, indicates the existence of unusual terms. In short, this is because of the followings, which are almost invalid in general PMSM

- In ideal machine, the phase windings are perfectly distributed in order to have pure sinusoidal MMF.
- Self and mutual inductance variation of phase windings are harmonic-free.
- flux linkages due to excitation is harmonic-free.

Higher angular resolution in field solutions gives a better accuracy in fitting waveforms and ensures the model to have higher harmonic components. After these processes, together with [R] and J parameters, a general mathematical model in terms of a.b.c phase variables is obtained.

3- Analysis and Development of the Method for the General Solution of the Mathematical Model

In the derived mathematical model the input are the phase to neutral voltages and the load torque. the phase currents. angular velocity and the angular displacement of the rotor are the independent state variables. In the numerical solution of the model. at each step of integration. angular position dependent parameters of [L] are evaluated. Then both sides of equation (1) is multiplied by $[L]^{-1}$ to obtain the system of equations for currents in proper form. Evaluation of T_e from equation (2) and substituting in (3) completes the solution for this step.As mentioned above for the solution of this model, phase to neutral voltages should be known. This requires some additional computations. since considered three phase system has unbalanced nature by means of speed voltages and inductances. Equivalent circuit for the system seen in Fig. 5. Voltage equations for Fig.5. can be written as follows: where E denotes speed voltage ($w_r \lambda^*$).

$$V_{ao}= V_{an}+V_{no}$$
$$V_{bo}= V_{bn}+V_{no}$$
$$V_{co}= V_{cn}+V_{no} \quad (4)$$

$$V_{no}=(1/3)(V_{ao}+V_{bo}+V_{co})-(1/3)(E_a+E_b+E_c)$$

$$V_{an} = V_{ao}-V_{no} +k(I_a+I_b+I_c)$$
$$V_{bn} = V_{bo}-V_{no} +k(I_a+I_b+I_c) \quad (5)$$
$$V_{cn} = V_{co}-V_{no} +k(I_a+I_b+I_c)$$

the term " $(1/3)(E_a+E_b+E_c)$" is presented in order to block the currents in the neutral line caused by 3. harmonic voltages. The term "$k(I_a+I_b+I_c)$" (where k is a constant determined during simulation) is used for further correction the phase to neutral voltages in order to keep neutral current zero. This correction is required because of the unbalanced nature of the motor self and mutual inductances. By these definitions model can be solved numerically for any supply system without any constrain on balance and neutral connection. In Fig. 6., simulated results for balanced and unbalanced three phase supply operations with neutral disconnected are given. Correction terms are seemed to ensure to keep neutral current zero in both cases.

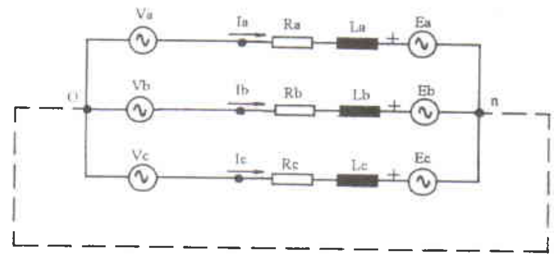

Fig. 5. Equivalent circuit for three phase operation

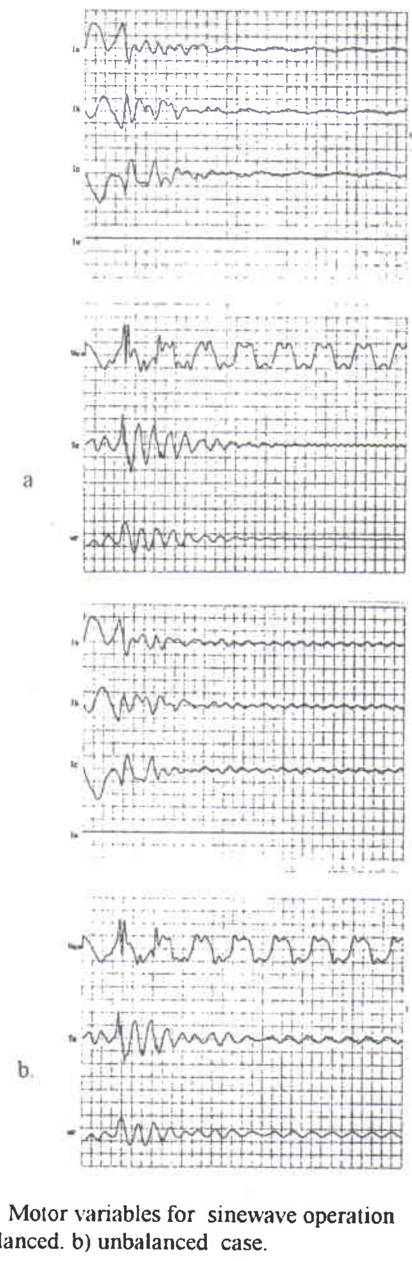

Fig.6. Motor variables for sinewave operation
a) Balanced. b) unbalanced case.

4- Solution of the Mathematical Model for Voltage Fed BDCM Operation

Following the general solution algorithm given above. another important operating mode of the machine as the voltage fed brushless d.c. motor can be analyzed. In this mode of operation. the machine is fed form voltage source. whose average d.c. voltage may be controlled properly. The switching circuit. whose switching sequence is kept synchronized to the position of the motor shaft is connected between motor and voltage source. Principal circuit of system is shown in Fig. 7. At the instants other than commutation takes place. two switches conduct. Assuming that at time zero. T1 and T4 is in conduction (see Table 1.). phase to neutral voltages of phase windings are determined as

$$V_{an}=(V_p-E_c')/2$$
$$V_{bn}=(-V_p-E_c')/2$$
$$V_{cn}=E_c' \qquad (6)$$

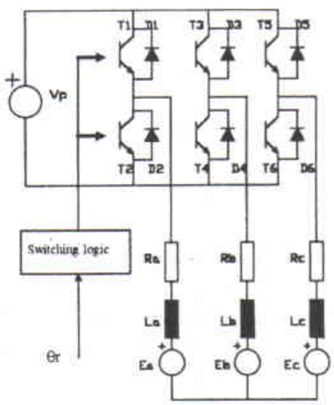

Fig.7. Schematics of BDCM system

Table 1. Switching sequence of inverter

rotor angle (deg)	0	60	120	180	240	300	360
conducting switches	T1 T4	T1 T6	T3 T6	T3 T2	T5 T2	T5 T4	T1 T4

where E_c' is the voltage induced across the phase "c" winding terminals due to its own speed voltage E_c and the mutual induction of phase a and b. Analytically. this voltage is given as.

$$E_c' = L_{ca}\frac{dI_a}{dt} + L_{cb}\frac{dI_b}{dt} + \omega_r\frac{\partial L_{cb}}{\partial \theta_r}I_b + \omega_r\frac{\partial L_{ca}}{\partial \theta_r}I_a + E_c \qquad (7)$$

For the simulation to be more realistic, this voltage should be strictly applied during the period of phase c is disconnected. Otherwise. a current would tend to flow. which is not the case in practical system. This situation is special to the salient pole-like PM machines. After rotation of 60 electrical degrees. phase b is disconnected and phase c is connected. Since D3 conducts until the current of phase b falls to zero. three switches are in conduction. for this period the terminal voltages are determined as,

$$V_{an}=(1/3)V_p$$
$$V_{bn}=(1/3)V_p$$
$$V_{cn}=-(2/3)V_p \qquad (8)$$

Since operation continues in similar manner. proper phase to neutral voltages are applied sequentaly.

Results of the simulation of a sample system is given in Fig. 8. where free running with constant voltage and load torque operation is analyzed. Apart from the starting transient. running is rather smooth and small oscillations of electromagnetic torque exist due to reluctance torque and harmonic terms of speed voltages. Note that, neutral and disconnected phase's current are well kept zero.

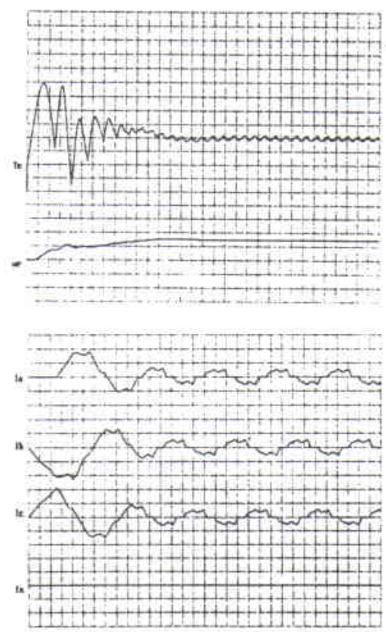

Fig.8. Results for the operation as BDCM

5- Solution of the Mathematical Model for Current Regulated (CR) Sinewave Operation

In this mode, the control method known as vector control is applied. Rotor frame current references (field and torque components, I_d* and I_q*) are converted into the three phase current references by rotor angle sine and cosine components obtained via a resolver. (I_a*,I_b*,I_c*) and a high frequency switching inverter is used for the actual stator currents to track this reference values. For normal operation, below the base speed, PM excitation is sufficient and therefore reference value for I_d is kept as zero. For the sake of simplicity, the hysteresis current control scheme is chosen to simulate. The main switching circuit is shown in Fig.9, where the current regulators simply act as a up - down connector or polarity selector, operating independently in the three phase. If the freewheeling periods are omitted, it can be said that, three switches in random sequence are in conducting state. According the connection composed by the switches, phase to neutral voltages of phase windings can be obtained similar to the operation analyzed above.

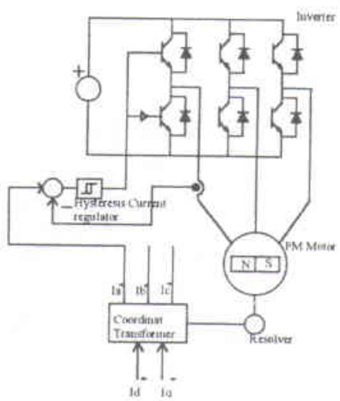

Fig 9. Schematics of the vector control system

The results of simulation of such operation is shown in Fig. 10. where no load running of machine is considered with I_d*=0, I_q*=10 A. Since considered machine is nonsinusoidal, sinewave currents don't produce a level torque. Here again, the harmonic components of speed voltages and inductance's results in torque harmonics. Fading in torque and currents are because of the saturation of current regulators as the speed voltage increases with speed. As predicted, the frequency of the three phase currents is locked with speed. Zero current in the neutral is slightly destroyed because of the high switching nature of the circuit.

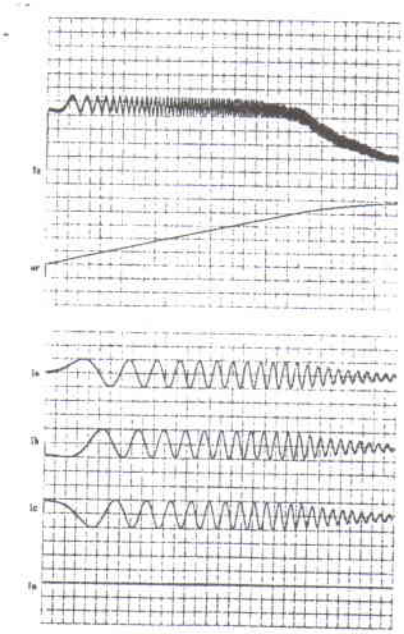

Fig.10. Results for operation as CRBDCM

6- Results and Discussion

A method of modeling is introduced which can be use almost any type of PM synchronous motor. Depending on the shape of the rotor and winding arrangement, flux linkages may contain space harmonics including the third, fifth etc. Similarly inductance matrix terms may contain significant harmonics which are negligible in classical a.a. machines. Solution of this model is analyzed on three different practical operation modes. It is shown that unbalanced nature of the motor can be compensated by some correcting terms. An universal inverse model which would be a basis for smooth torque in all of the modes of the operation is under resarch.

References

/1/ Duru T., Magnetic Analysis and Modeling of PMSM, (in Turkish),PhD Dissertation, Un. Kocaeli Inst. of Science, 1997.

/2/ Krause P. C.,Analysis of Electrical Machinery, Mc Graw Hill Book Comp.NY, 1986.